

# Returns from investment in education in a dynamic model of international trade

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## Abstract

Data from the Conference Board and World Bank show that public expenditures on education and mean labor productivity are highly correlated and that educational expenditures Granger-cause labor productivity. However, static heterogeneous-firm models of international trade rely on exogenously determined distributions of labor productivity. I propose a dynamic intertemporal heterogeneous-firm model of trade, which incorporates public investment in education as a means of increasing productivity. The model shows that public investment in education can generate economic growth and increase net tax revenues by capturing a larger share of foreign markets, and thus increase the export share of GDP. In absolute terms, the greatest returns on investment in education are for small countries that are slightly less competitive than their trading partners. In relative terms, however, the greatest returns are for countries that are significantly less competitive than their partners. These findings are consistent with the experiences of Taiwan and South Korea.

## 1 Introduction

International trade has grown in magnitude and importance over the past six decades. Since 1950, global exports have risen at a faster rate than global output in every year except for 2001 and 2009.<sup>1</sup> For countries such as South Korea and Taiwan, exports to foreign markets have been a critical component of their astounding economic growth. Exports comprised

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<sup>1</sup>World Trade Organization - International Trade Statistics.

[http://www.wto.org/english/res\\_e/statis\\_e/statis.e.htm](http://www.wto.org/english/res_e/statis_e/statis.e.htm) - accessed 12/23/14.

approximately 9% of Taiwanese GDP in 1953, but had grown to nearly 53% by 2000 Wu (2004). In South Korea, exports have risen from 3% of GDP in 1960 to almost 54% of GDP in 2013.<sup>2</sup> This remarkable growth and rise in South Korean and Taiwanese living standards has prompted emulation from other developing countries and raises the question, can this success be imitated or were South Korea and Taiwan the products of fortunate circumstances?

Current international trade models, based on heterogeneous firms, are static in nature and depend on exogenously determined probability distributions of firm-level labor productivity (Eaton and Kortum, 2002; Melitz, 2003). The implication of such models is that success or failure is driven by luck; from this point of view, South Korea was the fortuitous recipient of benefits such as American military protection and political links which opened the way for exports of goods and services. If this is the case, then developing countries may have little opportunity to follow South Korea's path.

Yet there have been arguments put forth that suggest South Korea arrived at its current position by way of long-term planning and conscious decision-making (Rodrik, 1994, 1995). The South Korean government subsidized business by means of low or negative interest rate loans and also made a concerted effort to promote education, science and technology. There was also a large scale rise of public investment in physical capital. These actions created the conditions by which industry could develop and grow into a titan of trade.

In this paper, I first present data that demonstrate a strong correlation between productivity and investment in education. The results of the analysis of Granger causality (Appendix A.3) provide strong evidence that public expenditures on education Granger-cause labor productivity per worker, especially for longer time lags. In fact, there is also strong evidence that causality is bi-directional and that labor productivity per worker Granger-causes public educational expenditures, especially for shorter time lags.

I then develop a framework that incorporates investment in education into the existing models of international trade. This allows for an active role for the government through taxation and long-term planning. This model shows how, given an initial starting position, a country can either develop itself into an exporting powerhouse or lose significant market share to its foreign competitors. By adding intertemporal dynamics to a probabilistic

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<sup>2</sup>World Bank - World Development Indicators. <http://data.worldbank.org/data-catalog/world-development-indicators> - accessed 12/23/2014.

heterogeneous-firm model, this framework captures elements of explanations for South Korea's growth: through the initial probability distribution of unit labor requirements, the model allows for unforeseen circumstances, and through the dynamic investment in education the model incorporates the agency of long-term planning.

Early models of trade that enjoyed substantial empirical success were the so-called "Gravity" models. In its simplest incarnation, the gravity equation states that bilateral trade flows should be increasing in the incomes of the trading partners and decreasing in the geographic distance between them. More complex versions have also accounted for border crossings as well as linguistic and cultural differences. Many authors highlight the ability of the gravity equation to predict bilateral trade flows (Bergstrand, 1985; Deardorff, 1998) while others attempt to reconcile this empirical success with the lack of theoretical foundations by examining the consistency of theoretical models with the gravity equation results (Feenstra et al., 2001; Hummels and Levinsohn, 1995). They show that gravity equation results are only compatible with models of trade which feature product differentiation - either through monopolistic competition or via "Armington" national differentiation. Armington differentiation proposes that goods are characterized by the place of production and that there is some elasticity of substitution between identical goods from different countries. This allows an additional dimension of product differentiation than that implied by monopolistic competition amongst firms.<sup>3</sup>

Melitz (2003) and Eaton and Kortum (2002) developed theoretical frameworks that attempt to explain trade patterns as resulting from firm-level heterogeneity with respect to productivity. Eaton & Kortum characterize each firm as having a random draw from a known distribution of labor productivity where higher values imply greater productivity; variation in the distributions across countries represents varying levels of technology. They also include geographic determinants which originated in the earlier gravity models. Alvarez and Lucas (2007) further develop Eaton and Kortum (2002) by quantitatively analyzing the general equilibrium in this model of trade. These frameworks help explain the empirical findings dating back to Hilgerdt (1945), and more recently Blomstrom et al. (1989) who found that much trade occurs between developed countries. Earlier models of trade did not predict this; Heckscher-Ohlin models focused on relative factor endowments and predicted trade between capital-abundant (developed) countries and labor-abundant (developing) countries. By focusing on firm-level pro-

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<sup>3</sup>For more detailed analysis see Armington (1969)

ductivity differences, Melitz and Eaton & Kortum were able to explain the patterns of trade between developed countries as well as the coexistence of exporting firms and firms that only served domestic markets within a particular industry. More productive firms would be able to expand into foreign markets while the least productive ones would fill particular niches domestically. Krugman (1979) attempts to answer the trade pattern puzzle by developing a model featuring imperfect competition and increasing returns to scale. According to this formulation, countries would specialize in particular industries even if there was no basis for comparative advantage. This would then lead to high volumes of trade between developed countries. However these Ricardian models of trade are static in nature and do not allow for intertemporal dynamics which may be of interest to policy makers. The present paper extends the framework of Eaton & Kortum in a way that allows labor productivity distributions to vary over time and provides a suitable mechanism in the form of investment in human capital.

Baxter and King (1993) present a theoretical framework for how public investment (public capital) might play an important role in a frictionless real business cycle (RBC) model by including public capital in a Cobb-Douglas production function. Under this framework, they find much larger fiscal multipliers than in a standard RBC framework. Aschauer (1989) finds empirical evidence that public investment affects productivity when looking at annual US data from 1949 to 1985. Aschauer found that non-military expenditures such as ‘highways, streets, water systems, and sewers’ are significant in determining growth in productivity. However, Perotti (2004) finds that public investment crowds out private investment and is not particularly effective at increasing the level of output. Because of this, Perotti states that ‘there is no evidence that government investment pays for itself’. Conversely, Demetriades and Mamuneas (2000) find that for the 12 OECD countries in their sample, government investment has low short-run rates of return but has significant long-run impacts on output supply and input demand. None of these studies has applied the theory of public investment to the heterogeneous-firm models of international trade. In the present paper, the fusion of these two frameworks yields interesting dynamics and relevant implications for policy makers.

The remainder of this paper is organized as follows: Section 2 presents relevant data that motivate and justify some mechanisms of the model; Section 3 presents the basic model in a closed economy; Section 4 extends the model to an open economy; Section 5 shows dynamic conditions that characterize the optimal solution path; Section 6 presents numerical results for the baseline model and alternate parameterizations; discussion and conclusions

are in section 7.

## 2 Data

Hitherto, heterogeneous productivity has been modeled as resulting from a random draw from a known distribution. We wish to see if any link exists between average labor productivity and observable variables in the data. Figure 1 shows a distinct relationship between average labor productivity per worker and public educational expenditures. In this paper we maintain the approach used in Eaton and Kortum (2002), but modify it to include a deterministic component such that the distributions themselves are functions of an endogenous state variable - education or human capital.

Higher levels of education are expected to confer skills and knowledge which would translate into increased productivity. Academia is founded upon this premise. We would like to incorporate an empirical relationship between investment in education and worker productivity, should one exist in the data. Labor productivity data were obtained from The Conference Board's Total Economy Database (TED).<sup>4</sup> Labor productivity is in this case measured as labor productivity per worker (*LP/Worker*) which is defined as annual gross domestic product divided by the number of workers (employed) in a given year. The decision to select labor productivity per worker over labor productivity per hour was based upon data availability. However, as shown in Appendix A.1, these two measures of productivity are highly correlated. The measure (*LP/Worker*) is an average and doesn't take into account heterogeneity of productivity amongst workers in a given country. A mean measure such as this is susceptible to extreme outlying industries such as finance. Countries whose economy relies primarily on extraction of a valuable natural resource, such as oil, will also be outliers with regard to our adopted measure of labor productivity. In our analysis, we have omitted the most extreme outliers of this variety as education and manufacturing play a very different role in outliers than in a typical economy.<sup>5</sup> We also obtained GDP per capita (*GDP/Cap*) data from TED. Both of these variables are measured in 2013 dollars and are adjusted for inflation as well as purchasing power parity (PPP). All of the data from The Conference Board are publicly available.

We also obtained data on public expenditures on education from the

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<sup>4</sup><https://www.conference-board.org/data/economydatabase/>. Accessed Oct. 2014

<sup>5</sup>These omissions are five petrostates - Kuwait, Iraq, Qatar, Saudi Arabia, and United Arab Emirates.

World Bank’s World Development Indicators (WDI), given as a percentage of GDP ( $EE/GDP$ ). However, we want to normalize this in per-capita terms so we construct the following measure:

$$EE/Cap = EE/GDP \times GDP/Cap$$

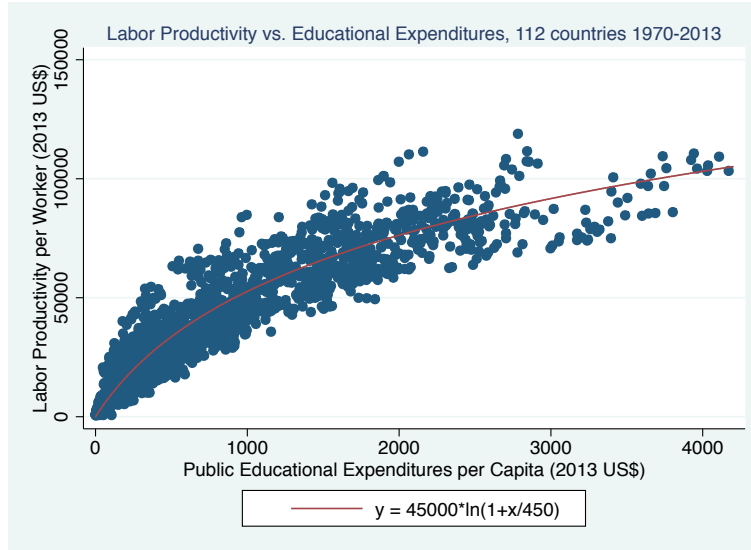


Figure 1: Labor productivity per worker vs. public educational expenditures per capita: 112 countries for the period 1970-2013,  $N = 2320$  observations. See Appendix A.2 for full list of countries and number of observations for each country.

Since we are interested whether there exists a relationship between  $LP/Worker$  and  $EE/Cap$ , we plot one against the other. Figure 1 shows a very close correlation between them. A reasonable linear fit can be imposed for low values of educational expenditures (such as might be the case for developing countries), but doesn’t properly reflect the tapering off that occurs at higher levels of expenditures. A linear fit would imply constant gains in productivity from ever-increasing levels of human capital, whereas Figure 1 shows evidence of diminishing returns. Economic theory suggests that diminishing returns may be appropriate; it takes more time, effort, and expenditures to achieve higher levels of education, which may offer progressively less benefit to a worker in a manufacturing sector. Initial increases in education (reading, writing, basic arithmetic) lead to significant gains in productivity

for workers who are now able to read manuals and work orders, and better understand the tools they are working with. While earning an advanced degree should confer an increase in productivity, there is little reason to suspect that the gains would be on par with the productivity boost from, say, a high school education.

There is a concern that the high correlation between labor productivity per worker and public educational expenditures does not imply causality between the two or that there might even be a causal relationship of the opposite direction from the one incorporated in this paper. One possible explanation is that increased labor productivity per worker generates higher output and more available resources for all public and private expenditures - including education. To test the framework developed in this paper, it is important to analyze Granger causality between these two variables – that is, do past values of one variable help predict the future values of the other? In fact, the results show significant bi-directional Granger causality, with the causality from *EE/Cap* to *LP/Worker* dominant for longer time lags and the causality from *LP/Worker* to *EE/Cap* dominant for short time lags. See Appendix A.3 for further details and analysis.

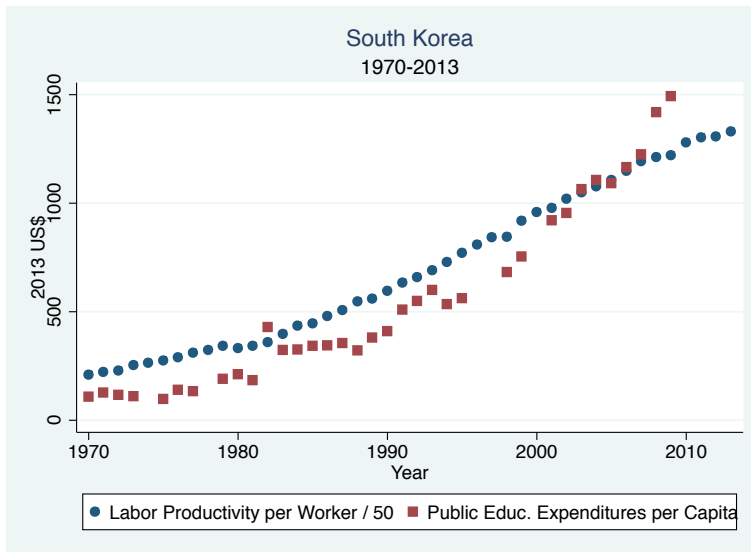


Figure 2: Historical Data for South Korea, 1970-2012

As the South Korean experience is a primary motivation for this paper, we turn our attention to South Korean data. Figure 2 shows historical

values of labor productivity per worker and per-capita public expenditures on education. The two variables of interest grow in tandem throughout the sample period. For countries (such as South Korea) not endowed with valuable deposits of natural resources, the primary way to achieve economic growth and to increase wealth is to raise productivity. Figure 3 shows the relationship between per-capita educational expenditures and per-worker labor productivity. A non-linear fit has been super-imposed to guide the eye; it does a remarkable job at fitting the data.

However, this fit does not do an equally good job for all countries. For developing countries with low productivity and low levels of educational expenditures, the data look noticeably more linear and likely reflects the fact that these countries have not yet reached the point of diminishing returns to education. This is especially true for low levels of educational expenditures. The vertical intercepts implied by the linear fit for data from Egypt and Sri Lanka suggest that productivity falls to almost zero as public educational spending becomes nil. The data for Tunisia and Portugal show a much higher intercept when public educational spending drops to zero - this may be due to significant private spending on education. This empirical evidence motivates the baseline parameterization discussed in Section 6. However, even when applying a linear fit to South Korean data, the vertical intercept is much higher than for Egypt and justifies the inclusion of two country specific parameters in the behavior of productivity and unit-labor requirements in the model.

As the Granger causality results show, labor productivity should not be presumed to depend explicitly on contemporaneous educational expenditures, but rather on past expenditures. The workers of today received their education and training in the past; today's spending is used to train the workforce of tomorrow. It is sensible to assume that labor productivity then depends on the sustained aggregate educational expenditures over some period of time. Indeed, current labor productivity per worker depends greatly on public educational expenditures over the past several years.

If we assume a particular country is close to its desired steady-state values, then it can be inferred that the per-capita stock of educational capital ( $E_{i,t}$  in the model) is equal to current per-capita educational expenditures ( $EE$ ) divided by the depreciation rate ( $\delta$ ). At the steady state, all current expenditures are used solely to offset depreciation rather than to accumulate



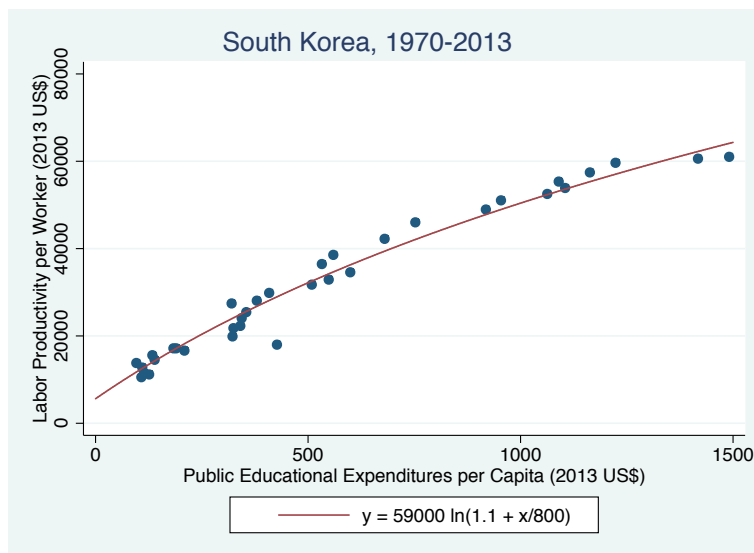


Figure 3: South Korean Data with Logarithmic Fit, 1970-2012

additional capital stock:

$$\begin{aligned}
 E_{i,t+1} &= E_{i,t} + EE_{i,t} - \delta E_{i,t} \\
 E_{i,t+1} &\approx E_{i,t} \\
 EE_{i,t} &\approx \delta E_{i,t}.
 \end{aligned}$$

Thus, it is reasonable to assume, on average, that  $E_{i,t} \approx (EE_{i,t})(1/\delta)$ . Therefore, educational expenditures can be considered a proxy for the level of the stock of educational capital. The treatment of education in the model is based on this assumption.

Although this paper focuses on the role of public expenditures on education and developing human capital, the framework described in the following sections is easily transferrable to describe public investment in physical capital such as roads, public utilities, telecommunications infrastructure and more. Data obtained on public gross fixed capital formation show a similar capacity to explain and predict variations in labor productivity per worker. This strengthens the model described in this paper as it can be tailored to address a variety of public investment options. See Appendix A.4 for data and further details.

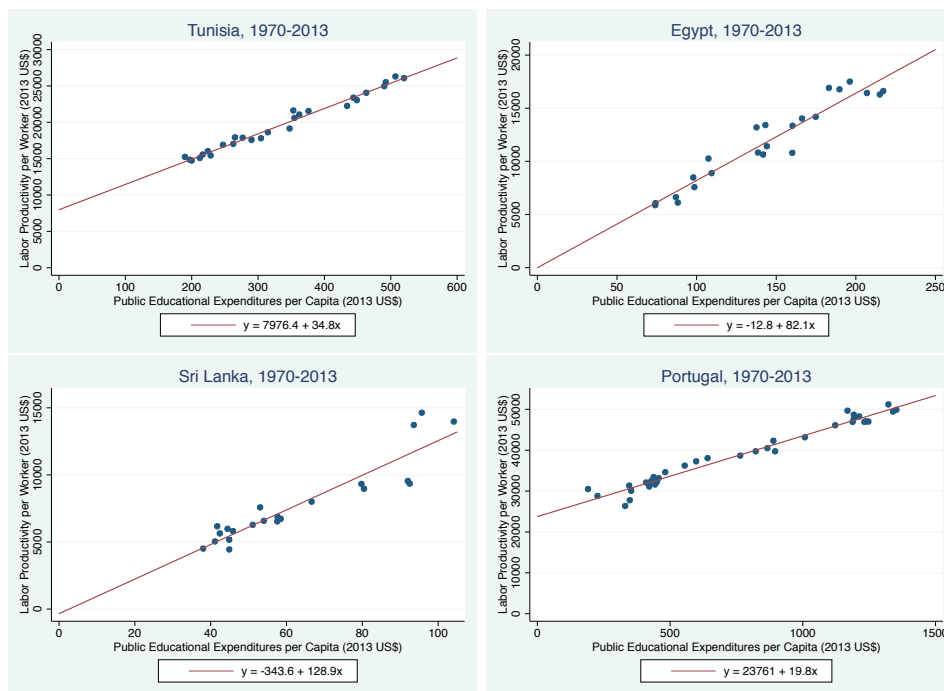


Figure 4: Linear relationship between LP/Worker and EE/Cap, 1970-2012

### 3 Model: Closed Economy

The basic setup is similar to the Alvarez and Lucas (2007) treatment of the Eaton and Kortum (2002) framework. We begin by describing a simplified closed-economy version of the model.

#### 3.1 Production

We consider a country with population  $N$  existing in autarky. There are three types of agents in the economy: firms, households, and a government. There are also two productive sectors - manufacturing and agriculture. Within the manufacturing sector there is a continuum of industries of mass 1, each of which produces a differentiated good. Within each industry, there is perfect competition amongst firms which all employ a common technology. Firms produce their good using only labor. Let  $x$  be the number of units of labor required to produce one unit of the differentiated good - thus labor productivity  $\propto 1/x$ . Industry-level productivity is heterogeneous with

$x$  randomly drawn from a probability distribution  $\phi(x)$ . For now, the only restrictions we impose on  $\phi(x)$  is that the support be limited to positive values only. The marginal cost of a firm in industry  $i$  is then:

$$MC_i = wx_i,$$

where  $w$  is the wage rate of labor and the price charged is:

$$p(x_i) = MC_i = wx_i. \quad (1)$$

Following Eaton & Kortum (2002), we assume that the agricultural sector produces a homogenous, costlessly-traded good which is sold at a fixed global price. We also assume that each producing country is small enough to act as a price taker and cannot directly influence the global price. There is a fixed productivity in this sector, that may vary across countries, which is unaffected by human capital. This fixed productivity in either country in turn determines the wage rate in both sectors of the economy since labor is perfectly mobile across sectors. The agricultural sector is able to absorb any excess labor which is unutilized by the manufacturing sector.

### 3.2 Consumers

Households derive utility from a composite consumption good which is defined according to the following constant elasticity of substitution (CES) aggregator function:

$$C = \left[ \int_0^1 q(u)^{\frac{\eta-1}{\eta}} du \right]^{\frac{\eta}{\eta-1}}. \quad (2)$$

where  $u$  is the index over the set of differentiated goods produced by industries in the manufacturing sector,  $q(u)$  is the quantity of good  $u$  consumed by the household, and  $\eta$  is the elasticity of substitution between differentiated goods. Households also supply labor inelastically without disutility. For now, we assume that demand is given exogenously. That is, households are assumed to demand some fixed amount of the composite consumption good and therefore their problem is reduced to a cost minimization problem. More formally, the household must choose quantities of each good,  $q(u)$ , in order to:

$$\begin{aligned} & \underset{q(u)}{\text{minimize}} && \int_0^1 p(u)q(u)du \\ & \text{subject to} && \left[ \int_0^1 q(u)^{\frac{\eta-1}{\eta}} du \right]^{\frac{\eta}{\eta-1}} \geq q. \end{aligned} \quad (3)$$

where  $q$  is the exogenously given demand;  $q \equiv N \times PCS$ , where  $N$  is the size of the country and  $PCS$  is per-capita spending. As each good enters the aggregate identically, the only defining characteristic of each good is the per-unit labor cost,  $x$ . We can therefore redefine the problem in terms of these productivity parameters. The problem of the household then becomes

$$\begin{aligned} & \underset{q(x)}{\text{minimize}} && \int_0^\infty p(x)q(x)\phi(x)dx \\ & \text{subject to} && \left[ \int_0^\infty q(x)^{\frac{\eta-1}{\eta}} \phi(x)dx \right]^{\frac{\eta}{\eta-1}} \geq q. \end{aligned} \quad (4)$$

where  $p(x)$  is the price of a good with productivity index  $x$ ,  $q(x)$  is the quantity demanded of that particular good, and  $\phi(x)$  is the probability density function (p.d.f) of the per-unit labor requirements. The first-order condition of this problem yields the following demand functions for a good with per-unit labor requirement  $x$ :

$$q(x) = p(x)^{-\eta} p^\eta q. \quad (5)$$

where  $p$  is the cost to obtain one unit of the final composite consumption good and is defined as:

$$p = \left[ \int_0^\infty p(x)^{1-\eta} \phi(x) dx \right]^{\frac{1}{1-\eta}}. \quad (6)$$

### 3.3 Government

The role of the government is simplified in the case of the closed economy, but we describe the problems and constraints the government faces for completeness. The government sets tax rates on production,  $T$ , in order to raise revenues. The tax applies to all production of goods by firms. This tax will be ultimately borne by consumers as firms earn zero profits due to the assumption of perfect competition within an industry. Thus, the effective price faced by households is:

$$p_{\text{eff}}(x) = wx(1 + T). \quad (7)$$

In the closed economy, total spending is:

$$\int_0^\infty p_{\text{eff}}(x)q(x)\phi(x)dx = p^\eta q [w(1 + T)]^{1-\eta} \int_0^\infty x^{1-\eta} \phi(x)dx. \quad (8)$$

of which a share  $\frac{T}{1+T}$  goes to the government and a share  $\frac{1}{1+T}$  goes to firms. The government must then decide what fraction of its revenues will be used to invest in the educational infrastructure or human capital. Investment in human capital will increase the long-term productivity of workers within the country. It will be assumed that the objective of the government is to maximize uncommitted revenue – that is, funds not earmarked for investment in education. Formally, the government’s problem may be written as:

$$\begin{aligned}
& \max_{T_t, I_t} \sum_{t=0}^{\infty} \beta^t [R_t - I_t] \\
\text{subject to } & I_t \leq R_t \\
& T_t \geq 0 \\
& E_t = (1 - \delta)E_{t-1} + I_t/N \\
& R_t = \frac{T_t}{1 + T_t} p^n q [w(1 + T_t)]^{1-\eta} \int_0^{\infty} x^{1-\eta} \phi_t(x) dx,
\end{aligned} \tag{9}$$

where  $I_t$  is the amount invested in human capital,  $\beta \in (0, 1)$  is the intertemporal discount factor,  $N$  is the size (population) of the country, the second constraint is the law of motion for the stock of human capital, and the third constraint is total government revenue. Here,  $E_t$  is the per-capita stock of educational capital within the country at time  $t$ . This can be interpreted as the current quantity and quality of public schools, libraries, telecommunications infrastructure, and universities. An increase in  $E$  would correspond to an increased effort to improve educational outcomes for students through appropriate channels. This could also be considered as any investment that improves the productivity of labor in the manufacturing sector. Similar to physical capital, this educational capital will depreciate over time as training becomes obsolete and as physical components, such as buildings, fall into a state of disrepair;  $\delta$  is the per-period depreciation rate in the stock of educational capital. We normalize the effect of investment on the accumulation of educational capital by the population of the country; an equally sized investment would be expected to yield much greater results in per-capita productivity in a smaller country than in a large country. We also denote the probability distribution,  $\phi_t(x)$ , as being dependent on time. This is because we will specify a relationship between the parameters of the distribution and  $E_t$  which is the mechanism through which educational expenditures and investment affect labor productivity for workers in the country. Thus, policymakers can, through tax and investment decisions, change productivity, and therefore will need to choose policy carefully in order to maximize the objective function stated in equation (9).

## 4 Model: Open Economy

We now consider the case where there are two countries, Home and Foreign, which we will denote  $H$  and  $F$ , respectively. The size and per-capita spending of each country are  $N_H, PCS_H$  and  $N_F, PCS_F$ . Each country has similar economic agents as in the closed economy. Specifically, in the manufacturing sector there are perfectly competitive producers of each differentiated good in each country. We assume that productivity draws for a particular good occur independently but not necessarily identically. That is, the distributions from which productivity parameters are drawn could differ across countries. The relevant space by which we identify goods is then  $x = (x_H, x_F)$  which is a vector of productivity draws for any given good. Due to the assumption of independence, the joint distribution of  $x$  is:

$$\phi_t(x) = \phi_{t,H}(x_H)\phi_{t,F}(x_F). \quad (10)$$

When there are two countries, consumers now have a choice of where to purchase a particular good. They can purchase it domestically or from a foreign supplier. As foreign and domestic producers of a particular good are offering an identical product, consumers will buy from whomever has the lowest price. Thus, for good  $x$ , the price faced by consumers in country  $H$  is:

$$p_H(x) = p_H(x_H, x_F) = \min \{w_H x_H(1 + T_H), w_F x_F(1 + T_F)\tau\}, \quad (11)$$

where  $\tau > 1$  is the iceberg bilateral trade cost associated with purchasing goods from abroad. Consumers will have the same demand functions for each particular good as in the closed economy but will have two options of where to buy each good. Let  $B_H^H$  be the set of goods that consumers in country  $H$  decide to buy from producers in country  $H$ , and let  $B_H^F$  be the set of goods that consumers in country  $H$  decide to buy from producers in country  $F$ . Formally, these sets are defined as follows:

$$B_H^H = \{(x_H, x_F) \in \mathfrak{R}_+^2 : w_H x_H(1 + T_H) \leq w_F x_F(1 + T_F)\tau\} \quad (12)$$

$$B_H^F = \{(x_H, x_F) \in \mathfrak{R}_+^2 : w_F x_F(1 + T_F)\tau \leq w_H x_H(1 + T_H)\}. \quad (13)$$

Then, the demand functions for households in country  $H$  are:

$$q_H^H(x) = q_H^H(x_H, x_F) = q_H^H(x_H) = p_H(x_H)^{-\eta} p_H^\eta q_H \quad \forall (x_H, x_F) \in B_H^H \quad (14)$$

$$q_H^F(x) = q_H^H(x_H, x_F) = q_H^H(x_F) = p_H(x_F)^{-\eta} p_H^\eta q_H \quad \forall (x_H, x_F) \in B_H^F, \quad (15)$$

where  $p_H$  is the price index in country  $H$  and is defined in the following manner:

$$p_H = \left[ p_H^{H1-\eta} + p_H^{F1-\eta} \right]^{\frac{1}{1-\eta}} \quad (16)$$

$$p_H^H = \left[ \int_{B_H^H} p_H(x_H)^{1-\eta} \phi(x) dx \right]^{\frac{1}{1-\eta}} \quad (17)$$

$$p_H^F = \left[ \int_{B_H^F} p_H(x_F)^{1-\eta} \phi(x) dx \right]^{\frac{1}{1-\eta}}. \quad (18)$$

In the open economy, the government's problem becomes more interesting. There is now incentive to carefully select investment in education and tax rates. Choosing sub-optimal tax rates could lead to loss of market share if domestic firms are undercut by foreign firms. As labor productivity is dependent on the stock of educational capital, investing too little in education could result in future losses if the other country invests heavily in education and is thereby able to increase the competitiveness of its manufacturing sector. If this occurs, the domestic government could lose a substantial portion of their tax base resulting in lower tax revenues.

In each period, the government in country  $i \in (H, F)$  must solve the following problem:

$$\begin{aligned} & \max_{T_{i,t}, I_{i,t}} \sum_{t=0}^{\infty} \beta^t (R_{i,t} - I_{i,t}) \\ \text{subject to} \quad & E_{i,t} = (1 - \delta)E_{i,t-1} + I_{i,t-1}/N_i \\ & I_{i,t} \leq R_{i,t} \\ & T_{i,t} \geq 0 \end{aligned} \quad (19)$$

where  $\beta \in (0, 1)$  is the intertemporal discount factor,  $\delta$  is the depreciation rate of the stock of human capital, and where total revenues are

$$\begin{aligned} R_{i,t} = \frac{T_{i,t}}{1 + T_{i,t}} & \left[ p_i^\eta q_i [w_i(1 + T_i)]^{1-\eta} \int_{B_i^i} x_i^{1-\eta} \phi_t(x) dx \right. \\ & \left. + p_j^\eta q_j [w_i(1 + T_i)\tau]^{1-\eta} \int_{B_j^i} x_i^{1-\eta} \phi_t(x) dx \right] \end{aligned} \quad (20)$$

and  $i, j \in (H, F), i \neq j$ . We assume that preferences are the same in each country and specifically that the elasticity of substitution between goods is identical for consumers in both the Home and Foreign country. This assumption is made for simplicity but will not dramatically affect the results of the model and therefore could be relaxed in future work. The first term in eq. (20) shows the tax revenue derived from sales to domestic consumers while the second term captures the tax income resulting from exports to consumers in the other country. This expression could be extended in a similar fashion to include additional countries in the model. Note that the educational stock,  $E_{H,t}$ , may increase (or decrease) between periods. The investment decisions that are made in the current period will determine the educational stock for the following period. The stock of educational capital is important because it will directly influence the country's distribution of firm productivity. Increasing the stock of educational capital will serve to raise the average productivity of all firms within the country, therefore lowering average unit-labor costs of manufactured goods. The details of this dependence, as well as the nature of the distribution of firm productivity, will be detailed in the following section.

In this paper we consider the case of a Home country such that its investment and taxation decisions do not lead to changes in the behavior of the Foreign country. This could be due to the small size of the country whereby we interpret the Home country as a small open economy, or it could be due to political constraints in the Foreign country whereby they are unable to adapt and respond to changes in policy in the Home country. The case of two adaptive and competing countries, each making optimal taxation and investment decisions, is considered in a subsequent paper (Draine 2015, in preparation).

## 5 Solution of the Model

In order to solve the model we must make some assumptions about the nature of the distribution of productivity. Following the existing literature (Garetto, 2013), we model per-unit labor requirements as following a Weibull distribution with shape parameter  $\kappa$  and scale parameter  $\lambda$  which is proportional to the average unit-labor requirement. We further assume that  $\kappa$  is the same across both countries, whereas  $\lambda_{i,t}$  is determined by the stock of educational capital in country  $i$  at time  $t$ . The Weibull distribution has the



following properties:

$$\begin{aligned}
\phi_{i,t}(x_i|\kappa, \lambda_{i,t}) &= \frac{\kappa}{\lambda_{i,t}} \left( \frac{x_i}{\lambda_{i,t}} \right)^{\kappa-1} e^{-(x_i/\lambda_{i,t})^\kappa} \text{ for } x_i \geq 0 \\
\Phi_{i,t}(x_i|\kappa, \lambda_i) &= 1 - e^{-(x_i/\lambda_{i,t})^\kappa} \text{ for } x_i \geq 0 \\
E[x_{i,t}] &= \lambda_{i,t} \Gamma(1 + 1/\kappa),
\end{aligned} \tag{21}$$

where  $\Phi_{i,t}$  is the cumulative density function and  $E[\cdot]$  is the expectation operator. Based on the data discussed in Sec. 2, we postulate the following law of motion for the average unit-labor costs.

$$\lambda_{i,t} = \frac{\Lambda_i}{\ln \left( c_i + \frac{E_{i,t}}{E_i^A} \right)}, \tag{22}$$

where  $c_i \geq 1$  is a country-specific constant. Thus,  $c_i$  establishes an upper-bound (or lack thereof) of average unit-labor requirements in the manufacturing sector in country  $i$  by governing the behavior of  $\lambda_{i,t}$  as  $E_{i,t} \rightarrow 0$ . As  $c_i$  increases, it effectively reduces the productivity loss resulting from low levels of educational capital. As  $c_i$  approaches one, the unit-labor requirements will approach infinity when there is no educational capital available. For the countries considered in this sample,  $c_i \in [1, 3]$ . We can therefore interpret  $c_i$  as being an endowment of natural ability of labor, or privately-funded educational capital. Here,  $\lambda_{i,t}$  is average unit-labor requirement or, alternatively,  $1/\lambda_{i,t}$  can be thought of as average labor productivity for workers in country  $i$  in period  $t$ . In this way, we can relate it to the observed data detailed in Section 2. Further,  $E_i^A$  can be interpreted as a scaling factor with regards to the level of educational capital. Higher values of  $E_i^A$  imply that higher levels of educational capital are required to achieve some given increase in productivity, even for countries with similar values of  $c_i$ . This could perhaps reflect cultural attitudes or bias regarding education. In a country or culture where education is highly valued, students may be able to achieve beneficial outcomes even with little material or educational capital. Conversely, in a country where education is viewed with disdain, even a large capital stock will provide relatively little benefit to the labor force. We will discuss the relationship between the distribution of labor costs and  $\lambda$  in the following section. The specification for the evolution of productivity, (22), is chosen in such a way as to be consistent with the empirical data and to provide both diminishing returns and a lower bound on productivity (upper bound on unit-labor requirements) proportional to the initial stochastic productivity draws (unit labor requirements). The parameter  $E_i^A$  acts as a measure of

educational inertia; high values of  $E_i^A$  make achieving higher levels of productivity require even greater stocks of educational capital and, therefore, more investment. Eq. (22) for the unit-labor cost includes a multiplicative parameter  $\Lambda_i$ .

Given the probability distributions of firm productivity and the dependence of  $\lambda$  on  $E_t$ , the variables of interest turn out to be functions of the ratio of average costs in each country. We thus define a new variable as this ratio:

$$z_{H,t} \equiv \frac{w_H \lambda_{H,t} (1 + T_{H,t})}{w_F \lambda_F (1 + T_F) \tau} = \frac{Z_H (1 + T_{H,t})}{\ln(c_H + \frac{E_{H,t}}{E_H^A})}, \quad (23)$$

where

$$Z_H \equiv \frac{w_H \Lambda_H}{w_F \lambda_F (1 + T_F) \tau}.$$

Here,  $z_{H,t}$  is the ratio of the average prices for consumers in country  $H$  for goods produced domestically relative to goods produced by the Foreign country. We further assume a static policy of educational investment on the part of the foreign country which implies a constant  $\lambda_F$ . By utilizing  $z_{H,t}$ , we can neatly express the optimality conditions for policy makers in the Home country.<sup>6</sup> As the optimality conditions and revenue depend on the ratio of average costs,  $z_{i,t}$ , we find that wages ( $w_i$ ) and the mean unit-labor requirement ( $\lambda_{i,t}$ ) become intertwined as their product appears prominently in the numerator of the ratio of interest. There are two first-order conditions for the two choice variables,  $T_{H,t}$  and  $E_{H,t}$ . The optimality condition with respect to production tax rates is:

$$f_H(z_{H,t}) + T_{H,t} z_{H,t} \frac{df_H(z_{H,t})}{dz_{H,t}} = 0, \quad (24)$$

where we define  $f_H(z_{H,t})$  as

$$f_H(z) = \left[ \frac{z}{(1 + z^\kappa)^{1+1/\kappa}} + \frac{(q_F \tau / q_H) z}{(1 + \tau^{2\kappa} z^\kappa)^{1+1/\kappa}} \right],$$

and the optimality condition with respect to investment and the educational capital stock is

$$\frac{T_{H,t}}{(1 + T_{H,t})^2} z_{H,t}^2 \frac{df_H(z_{H,t})}{dz_{H,t}} e^{-z_{H0}(1+T_{H,t})/z_{H,t}} = -\frac{E_H^A z_{H0} N_H}{R_{H0}} \left( \frac{1}{\beta} - 1 + \delta \right), \quad (25)$$

---

<sup>6</sup>See Appendix B for mathematical details of the derivation

where

$$R_{H0} \equiv q_H w_F (1 + T_F) \lambda_F \tau \left( \Gamma \left( \frac{1 + \kappa - \eta}{\kappa} \right) \right)^{1/(1-\eta)}. \quad (26)$$

These two equations specify the optimal solution path of  $T_{H,t}$  and  $E_{H,t}$  conditional on initial conditions, the model parameters, and the policies of the other country.

We propose a baseline parameterization of the model in which the two countries are initially symmetric in terms of their characteristics (size, initial distribution of unit-labor requirements) and we assume a simple tax and investment policy on the part of the foreign country. Specifically, we assume that the foreign country sets a fixed tax rate and only invests enough to maintain a constant stock of educational capital. It does not actively invest in a manner so as to alter its worker productivity over time. Our baseline parameterization is in Table 1.

Table 1: Baseline Parameterization

Parameter	Value	
$\kappa$	3	Shape parameter of productivity distribution
$\tau$	1.2	Bilateral iceberg trade cost
$N_H$	1	Size of Home country
$N_F$	1	Size of Foreign country
$PCS_H$	10	Per-capita spending of Home country
$PCS_F$	10	Per-capita spending of Foreign country
$w_H$	1	Wage in Home country
$w_f$	1	Wage in Foreign country
$\Lambda_H$	1	Multiplicative parameter for unit-labor requirements in Home country
$\lambda_F$	1	Average unit-labor requirements in Foreign country
$E_H^A$	1	Scale/Slope parameter of educational capital accumulation
$E_{H,1}$	1	Initial stock of educational capital in Home country
$T_F$	0.3	Production tax rate in Foreign country
$\beta$	0.96	Intertemporal discount factor
$\delta$	0.1	Depreciation rate of educational capital
$\eta$	2.5	Elasticity of substitution between differentiated goods
$c_H$	1	Intercept parameter

The calibrations of  $E_H^A$  and  $E_{H,1}$  have been chosen in a way such that the initial gains in Home productivity from increases in the educational stock are fairly linear. While not crucial to the results of the model, this calibration seems like a reasonable fit for developing countries. While additional investment in education will yield future payoffs, these payoffs will be diminishing in magnitude.

In the first period of the model, the initial educational capital stock is given and the government must only select an optimal tax rate. This is done by numerically solving (24). With the optimal tax rate, we can determine the value of  $z_{H,1}$  and thereby find total government revenue in the first period. For all subsequent periods, we can use (24) to express  $T_{H,t}$  in terms of  $z_{H,t}$ , and then substitute this into (25) and solve numerically for the optimal  $z_{H,t}$ . From this, we can then back out  $T_{H,t}$  and  $E_{H,t}$ . If  $E_{H,t}$  is feasible according to the resource constraint (investment does not exceed revenue in the previous period), then we can calculate total revenue and move to the next period. In the case where the desired educational capital stock isn't feasible, we set the investment rate to 100%, determine the new value of  $z_{H,t}$  conditional on the new  $E_{H,t}$  and numerically solve (24) to find the optimal tax rate. Alternatively, a situation could arise where the country wishes to decrease its educational capital stock; under such circumstances, we would set investment to 0%, determine the new value of  $Z_{H,t}$  conditional on the new  $E_{H,t}$  which survived depreciation and numerically solve (24) to find the optimal tax rate. We place such limitations because we do not allow for borrowing or active disinvestment, although these assumptions could be relaxed if desired.

Due to the nature and behavior of the optimality conditions, optimal solutions may not exist for some parameterizations.

## 6 Results

We first present the model dynamics under the baseline parameterization. The Foreign country is assumed to follow a policy of maintaining  $E_{F,t} = E_F$ , either because the Home country is too small to affect the decision-making process of the Foreign country or because the Foreign government faces some political or fiscal constraints. In the baseline case shown in Figure 5, we see that the Home country invests all earned revenue in the first period and almost all earned revenue in the second period in order to rapidly arrive at the desired steady state. This behavior is due to the linear nature of the government's objective function; realistically, there would likely be sociopolitical incentives to remain below an 100% investment rate. The rapid accumulation of educational capital also yields a significant increase in base revenue and in net income (defined as revenue less investment). The increased cost of depreciation due to more capital is outweighed by the large gains in revenue. As shown in Figure 5, convergence to steady state and steady state values depend critically on the wage rate. When wages are

higher, the rate of convergence is significantly delayed as initial revenue is substantially lower. Due to lower competitiveness, when wages are higher the home country prefers to set the tax rate lower in order to maintain market share. The optimal investment strategy also leads to an increase in exports as a whole and as a share of manufacturing GDP in the Home country.

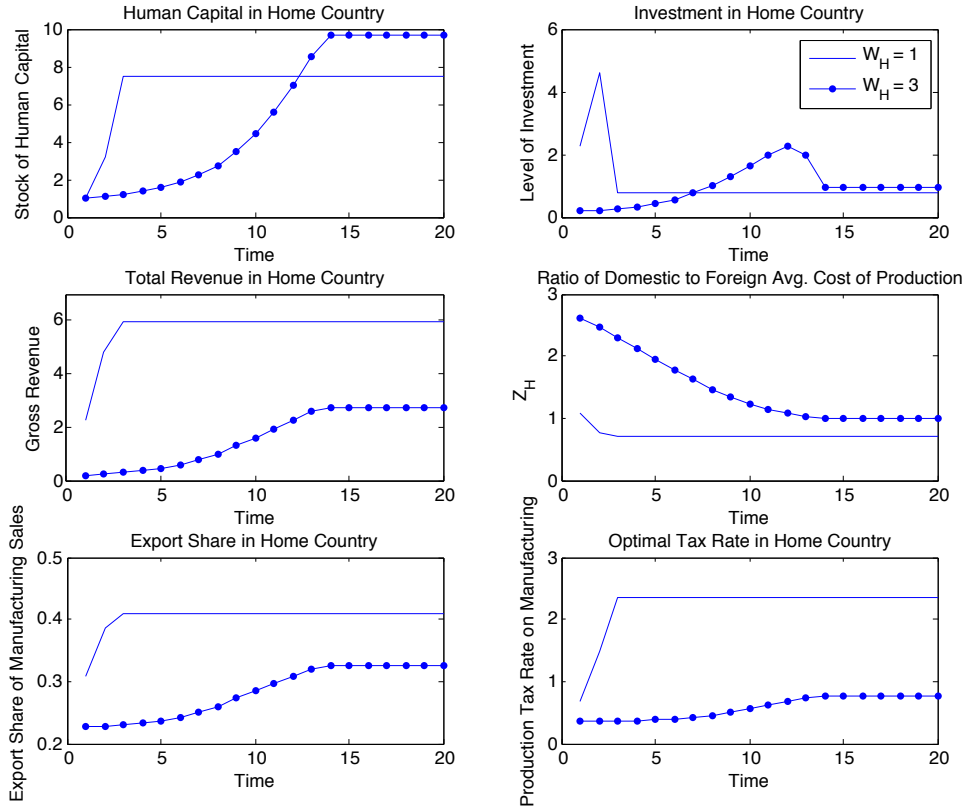


Figure 5: Model dynamics for the baseline parameterization.

The main metric by which we evaluate the optimal tax and investment policies described in the preceding section is on how they perform relative to what we will call a policy of 'passive investment'. By passive investment we mean choosing optimal tax rates in each period but only investing enough to offset depreciation; there will be no changes to productivity or the stock of educational capital from one-period to the next under passive investment.

We assume a policy of passive investment on the part of the foreign country under all circumstances as we are interpreting the Home country as a Small Open Economy which is unable to affect the decision-making process of the Foreign country. We will show the relative gains from optimal investment and how they are affected by the parameterization and starting conditions and describe any patterns which may be evident.

This metric is appropriate as total social welfare can be broken down into private and public welfare. Given the exogenous nature of household demand (fixed demand for the final composite consumption good) in the model, private welfare is held constant. If we assume that public welfare is a monotonic function of public consumption, then maximizing available government revenue is equivalent to maximizing public welfare and therefore total social welfare.

We measure the relative gain in present-value discounted net income from optimal investment defined in the following way:

$$G \equiv \frac{\sum_{t=0}^{\infty} \beta^t (R_{i,t}^* - I_{i,t}^*)}{\sum_{t=0}^{\infty} \beta^t (R_{i,t}^p - I_{i,t}^p)}, \quad (27)$$

where  $*$  indicates optimal policy and  $p$  indicates the values that would result from the “passive investment” trajectory (i.e., simply maintaining the initial stock of educational capital). In this case,  $I_{i,t}^p = I_{i,1}^p = \delta E_{i,1} N_i$ . We consider the above ratio in order to normalize any dependence of optimal returns on the size and wealth of the country; under certain parameterizations, the absolute revenues might be extremely large if the country is large and/or wealthy. Note that  $G$  should at least equal unity in the case where optimal policy is to maintain the existing educational stock.

Figure 6 shows  $G$  as a function of the wage in the Home country for three different specifications of  $\lambda_{i,t}$  and  $c_H$ . For  $c_H = 1$ , relative gains are increasing with wages. This is because the denominator of the ratio defined in (27) decreases more rapidly with increasing  $w_H$  than does the numerator. In essence, this is because the Home country can opt to invest to raise the per-worker productivity in order to maintain competitiveness when  $w_H$  is high. This can be seen in Figure 7 which plots both terms separately as a function of the domestic wage rate. In absolute terms, the increase in per-period revenue from pursuing a policy of optimal investment reaches a maximum when  $w_H = 1.3$ . This is found by calculating the difference in per-period revenue under optimal and passive investment policies. When wages are extremely low or high in the Home country, there is little benefit from

investment in educational capital. In the case where wages are low, there is no need to increase competitiveness because domestic firms dominate the foreign market already. When wages are extremely high, it would require astronomical levels of investment to improve productivity to the point where domestic firms can effectively capture segments of the market. The cost would vastly outweigh the benefit.

Figure 6 also shows how different parameterizations of  $c_H$  affect the ratio of gains from investment. The parameter  $c_H$  generates an upper-bound on unit-labor requirements and  $\lambda_H$ . When  $c_H = 1$  as in the baseline parameterization, then  $\lambda_H$  approaches infinity as  $E_H$  approaches zero. For values of  $c_H > 1$ , there is a well-defined upper-bound on  $\lambda$  that decreases as  $c_H$  increases. We find that the ratio of returns, which was strictly increasing under the baseline parameterization ( $c_H = 1$ ), now has a maximum for larger values of  $c_H$ .

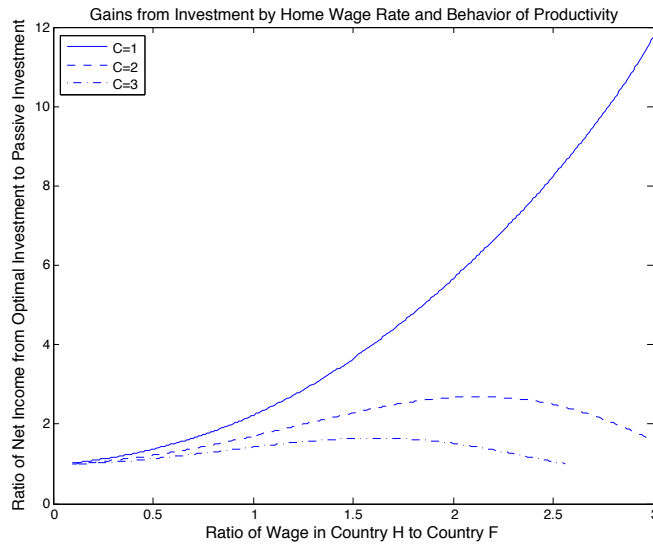


Figure 6: Gains from Investment Under Alternative Specifications of Productivity

When the firms in each country are similar (in terms of wages and average labor productivity), there are large potential gains from increasing productivity and capturing more of the market. We also find that  $G$  is decreasing in the size of the country. While a larger country means that there are more consumers and a larger market to conquer, it also proportionally increases

the cost of increasing the stock of educational capital. Small countries are able to cheaply develop the relevant infrastructure and boost productivity which allows them to poach foreign customers from foreign firms.

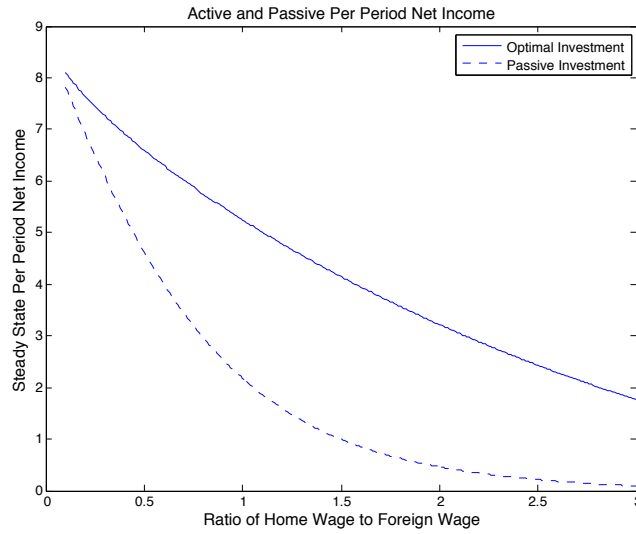


Figure 7: Per-Period Net Income under Passive and Active Investment Policies

We next turn our attention to how the gains from investment are affected by the Foreign production tax rate and the bilateral iceberg trade cost. These two variables behave in a similar fashion and it is therefore appropriate to describe them together. Figure 8 shows the gain as a function of iceberg trade cost,  $\tau$ , and Figure 9 shows how the gain depends on the foreign country production tax rate,  $T_F$ . We find that the gains are decreasing in each of these parameters for a fairly intuitive reason. The gains from investment are achieved through decreasing the value of  $z_{H,t}$ , which is the ratio of average costs in the home country of goods produced domestically and by a foreign country. In the baseline scenario, this is done by decreasing  $\lambda_{H,t}$  which reduces the numerator of  $z_{H,t}$ . However, when the foreign production tax rate rises or the bilateral iceberg trade cost increases, this serves an equivalent purpose from the perspective of the Home government. An increase in either of these terms causes the average cost of foreign goods to domestic consumers to rise and generates a fall in  $z_{H,t}$  (through an increase in the denominator). When this occurs, there is little to no need for the Home government to invest because they are already seeing the benefits at



no cost. The greatest gains are realized when the foreign tax rates are low and trade is costless or very cheap.

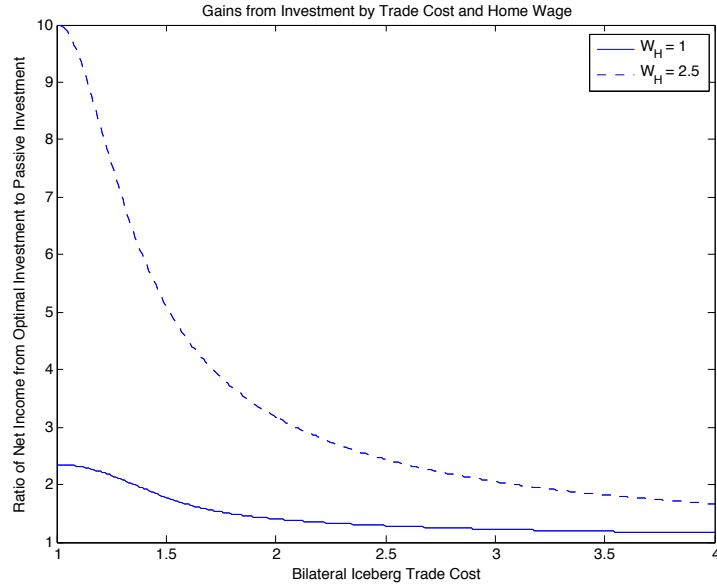


Figure 8: Gain from investment vs. iceberg trade cost.

We now consider how varying the exogenously given demand ( $q_H$ ) affects these results. From before, the exogenously given demand is the product of Home country size,  $N_H$ , and per-capita spending,  $PCS_H$ . Despite both of these variables jointly determining the overall level of demand, they each exhibit different effects on the gains from investment. Figure 10 shows that spending has an interesting effect on the ratio of gains from investment. As the citizens of the Home country become wealthier, there is a larger market for firms to sell to. Capturing a fixed percentage of the market by increasing productivity translates into even larger revenue gains as each customer is willing and able to purchase a larger quantity of goods. Although the absolute gains from active investment are larger as spending increases, the initial revenue from passive investment is also increasing at a similar rate. Thus, in percentage terms, there may not be much change in the gains as a function of wealth. Conversely, while having a larger population also increases the size of the market and the absolute level of gains in revenue, it also incurs a penalty as the government must invest more to achieve a fixed increase in the per-capita stock of educational capital, as shown in Figure 11.

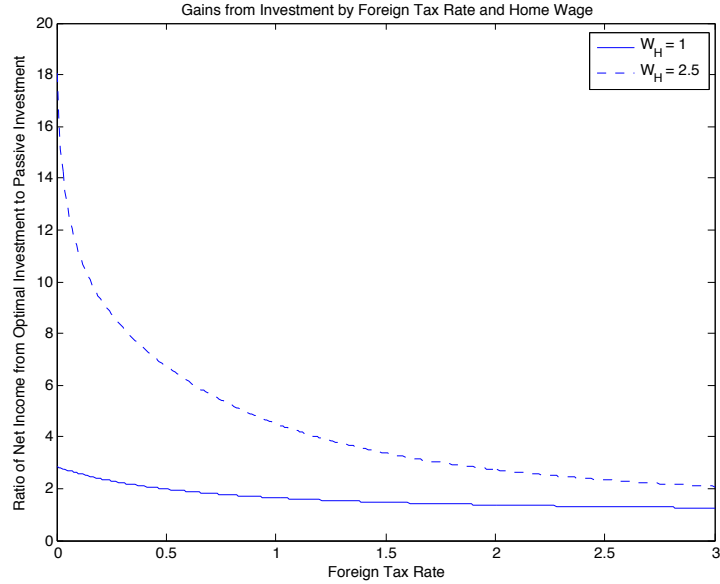


Figure 9: Gain from investments vs. foreign country production tax rate.

The increase in investment is proportional to the size of the country. A small country trading with a large country can achieve large gains at little expense while the large country must spend a lot to achieve considerably less. The cost of requiring additional investment to increase the stock of human capital outweighs the benefit of having a larger domestic market for the government to potentially tax. For lower wages, the penalty of having a larger population is mitigated by the lower level of investment needed to maximize revenue. For high wages, this effect is noticeably pronounced as the Home country needs to invest in education in order to gain competitiveness.

## 7 Discussion and Conclusions

This paper presents a new intertemporal dynamic model for competition between trading partners which includes the role of educational spending to improve productivity and hence competitiveness in one of the countries. The case of both countries investing in educational capital will be considered in a later paper. The model can be analyzed so that policymakers can determine the path of tax rates and educational investment rates to maximize total

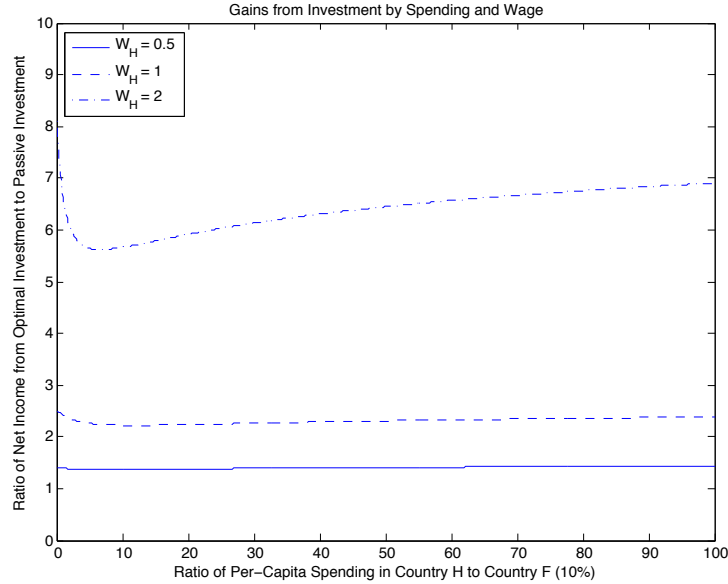


Figure 10: Gain from investment vs. per capita spending in country A.

government revenue available for non-education spending. If the government instead seeks to maximize some other quantity, the model can easily be modified to incorporate a different objective.

This model is motivated by empirical data that show a high correlation between labor productivity and public educational expenditures. The data show significant bi-directional Granger causality between these two variables. This is also seen in the model, in which increased spending on education increases future labor productivity and increased labor productivity leads to higher government revenue and to higher public investment in education.

The model shows that the greatest gains (in absolute terms) are achieved by small countries which are slightly less competitive than their trading partners. However, in relative terms, the greatest gains are achieved by countries which are significantly less competitive than their trading partners as even small gains constitute a large fraction of their original revenue.

Under the symmetric baseline parameterization, the Home country is able to achieve a roughly 120% increase in revenue available for spending by following the optimal tax and investment policies as shown in Figure 6. These policies consist of investing heavily in education and raising tax rates

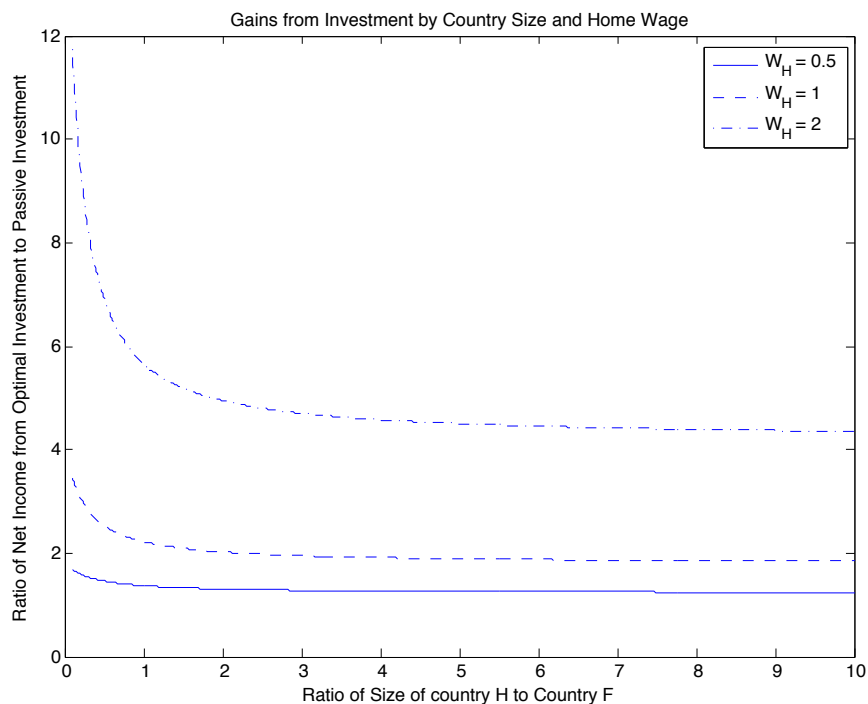


Figure 11: Gain from investment vs. ratio of country sizes.

as productivity rises. When the Home country is small and initially slightly less competitive compared to its trading partner, the government is able to achieve the greatest absolute gains in available revenue (see Figure 7 and Figure 11). When trade is free and costless (as a matter of geography and policy), the gain in available revenue increases to roughly 130% when following the optimal investment path. When trade becomes almost prohibitively costly (considered here to be when only 25% of shipped goods arrive at their destination), then the gain from optimal investment falls to approximately 15% when  $w_H = 1$  (see Fig 8). The proportional gain in available revenue is relatively constant when the wealth and spending habits of the domestic country vary. In absolute terms, the returns are increasing as a function of per-capita spending. Wages and productivity can be considered together as, within this model, they jointly determine the average costs of production in a particular country. The greatest absolute gain in available revenue is achieved when the home country is approximately 30% less competitive than

their trade rival (this gain amounts to a roughly 220% increase in available revenue as shown in Fig 7). All of the gains in available revenue are magnified when the Home country is small relative to its trading partner and this effect is further amplified when the domestic wage is high relative to the foreign wage rate. When the Home country is one hundredth the size of its trading partner and the two are symmetric in all other aspects, the government is able to achieve a staggering 250% increase in available revenue. This is primarily due to the fact that the Home country can capture foreign market share at a significantly lower cost.

So far, we have pinned down wages by means of a costlessly-traded and identical agricultural good and the assumption that the agricultural sector is large enough to readily absorb any excess labor not employed by firms in the manufacturing sector. However, both theory and observations indicate that wages should rise along with productivity. If the wage rate rose proportionally with the marginal product of labor, then any gains from increased productivity would be completely offset by an equivalent rise in wages. Under such a specification, investment in education would offer no benefit from the government's perspective. If wages were to rise at a rate less than the corresponding increase in labor productivity, then domestic firms would be able to capture market share both at home and abroad. However, these gains would be less than predicted in the present model, where wages are assumed to remain constant. Further, much of the gains found in this paper result from the fact that, conditional on a fixed wage rate, the government can raise taxes as labor productivity rises and still increase competitiveness. Thus the government is able to take a larger share of a larger pie, so to speak. If wages were to rise, then the government could maintain the same level of competitiveness as in this paper if taxes were to fall by an amount such that the product  $w(1+T)$  were to remain constant; this would result in lower tax revenues. A comprehensive treatment of wages should also include a mechanism by which increased wages and earnings would give rise to increased consumption spending. This could be achieved by means of a simple consumption rule in which some constant proportion of income is spent on the final composite consumption good. The standard mechanism for wage determination in trade models has been to use balanced trade conditions to pin down wages in the case of three or more countries or a ratio of wages in the case of two countries. Since we only consider two countries in this paper and government revenues depend critically on wages and costs, this method is unsatisfactory as it would not precisely pin down wages in both countries. Further, although in the long run it may be necessary to balance the current account with other countries, in the short and medium run, some countries

have consistently run trade surpluses or deficits with these being offset primarily by sales or purchases of public and private bonds; see Appendix A.5 for some examples. Both of these factors make the determination of wages via balanced trade conditions not supported by empirical data.

In this paper, we have only considered an active role for the Home country; we have assumed that the Foreign country is static and follows a passive investment strategy. In a future paper (Draine 2015, in preparation) we will consider a similarly-active role for the Foreign country and determine if there exists a stable Nash equilibrium for the two countries playing a simultaneous repeated game.

This paper included a tax on domestic production only, which has the effect of reducing contemporaneous competitiveness in a trade-off for future competitiveness. It will also be of interest to extend the model to include variation of import tariffs and the opportunity for punitive measures or even a full-blown trade war. Other lines of potential inquiry could include explicitly including borrowing and access to international capital markets, including some wage dynamics, and possibly including a role for corruption and anti-corruption measures in describing how government investment determines the stock of educational capital.

This paper shows the mechanisms behind the investment-led export-driven growth described in Rodrik (1994, 1995). There are slight differences between the two approaches, although they describe a similar cause and effect. Rodrik described Taiwan and South Korea as having a large boom in capital investment to augment an educated and highly-trained workforce. In contrast, the present paper models the effects of investment in education to enhance the productivity of the workforce. For both cases, the investment boom generates economic growth which is fuelled by trade and exports. This understanding suggests that other countries may be able to emulate the remarkable growth story of Taiwan and South Korea.

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## Appendices

### A Data

#### A.1 Measures of Labor Productivity

Labor productivity can be measured in several ways; two widely considered metrics are labor productivity per worker ( $LP/Worker$ ) and labor productivity per hour worked ( $LP/Hour$ ). Conceptually, labor productivity per hour should be better suited for cross-country comparisons than labor productivity per worker. An hour worked is an hour across the globe, while workers in different countries or even different industries may work varying lengths of time throughout the year. Some countries provide more generous vacation benefits while other countries promote a culture of overtime and self-sacrifice on the part of the employee. Thus, a concern is that using labor productivity per worker for cross-country comparisons may be misleading as we may be comparing the value added by a laborer working 40 hours per week vs. someone working 60 hours per week. Even if the two workers added the same amount of value per hour worked, the labor productivity per worker for the second laborer would be 50% higher due to longer hours worked. This example demonstrates that  $LP/Hour$  would be the best measure of labor productivity.



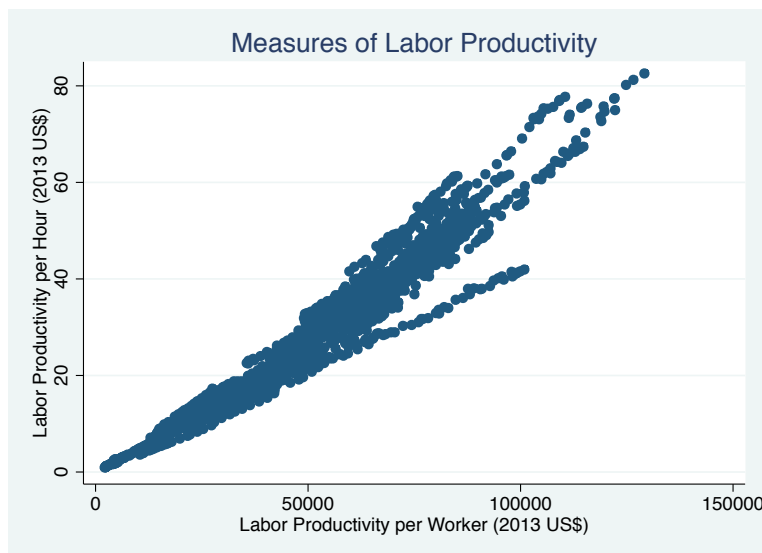


Figure 12:  $N = 2196$  and linear correlation coefficient  $\rho = 0.9739$ .

As seen in Figure 12,  $LP/Hour$  and  $LP/Worker$  are highly correlated, which suggests that the differences in hours worked across countries is less significant than might have been expected. In the data set from the Conference Board of 116 countries over the time period 1970-2013, every observation with recorded values for labor productivity per hour also had a recorded value for labor productivity per worker. For these 2196 observations, the correlation between  $LP/Hour$  and  $LP/Worker$  is 0.974 indicating that these two variables move in tandem. However, observations with recorded values for both  $LP/Hour$  and per-capita public educational expenditures ( $EE/Cap$ ) number far fewer than those with values for  $LP/Worker$  and  $EE/Cap$ . Therefore, due to the paucity of data on  $LP/Hour$ , we opt to use  $LP/Worker$  as the productivity variable in this paper.

Although we develop a framework to explain variations in labor productivity over time, it is useful to see how effective these two measures of labor productivity are at explaining variations in a common measure of output and prosperity, GDP per capita or  $GDP/cap$ . Table 2 presents the OLS regression results when considering  $LP/Hour$  and  $LP/Worker$  as regressors. Both variables perform equivalently when used as regressors for  $GDP/cap$ , but  $LP/Worker$  allows for the use of more than twice as many observations than  $LP/Hour$ .

Table 2: OLS Regression Results

	GDP/Cap (1)	GDP/Cap (2)	GDP/Cap (3)
LP/Worker	0.472*** (266.39)		0.393*** (29.71)
LP/Hour		800.39*** (143.34)	199.67*** (9.62)
Constant	-1581.4*** (-22.29)	149.1 (0.85)	-2846.4*** (-15.9)
N. Obs	4864	2196	2196
Adj. $R^2$	0.9359	0.9035	0.9312

Test statistics in parentheses. \* denotes significance at the 10% level, \*\* at the 5% level, and \*\*\* at the 1% level.

While theoretically  $LP/Hour$  would be the ideal measure of labor productivity, our analysis here shows that in practice both measures contain the same explanatory power and information. The majority of the observations for  $LP/Hour$  are from developed countries; adopting the use of  $LP/Hour$  would shift the distribution of remaining observations significantly into the developed world and we would lose important sources of data from developing countries. Thus, although theory suggests using  $LP/Hour$ , the current availability of data does not support this choice.

## A.2 Countries included in Sample

## A.3 Correlation and Causality

The data show a clear relationship between public expenditures on education and mean labor productivity (see Figures 1, 3, 4). The model of trade used in this paper implicitly assumes a directional causality in this relationship - namely that spending or investment in education will cause changes in labor productivity. However, there is a potential theoretical reason which might cause the relationship to run in the opposite direction. Suppose there was an exogenous increase in labor productivity. This increase in productivity would expand the production possibilities for the country and thereby generate a wealth effect as there are now more resources available for both consumption and investment. Assuming no changes in relative prices of consumption or investment goods (which is reasonable if the increase in labor productivity is neutral with respect to any particular good), then agents

in the country will spend more on all consumption and investment goods – including education. Thus, it could be argued that there is a potential endogeneity issue with these variables and an argument must be made for a particular causal relationship.

A common method of determining whether a causal relationship exists in the data is to investigate Granger causality between educational expenditures and labor productivity. However, certain issues complicate establishing Granger causality in the collected data. First, we are not examining two long-running time series but rather panel data. The second is that standard tests for Granger causality look for a linear relationship but the empirical relationship in the data is non-linear. This relationship will skew any analytical results regarding Granger causality.

The scatter plots (Figures 1, 3 and 4) of labor productivity per worker against per-capita educational expenditures show relatively little spread against a fitted concave function. This suggests that the relationship between labor productivity and educational spending is likely to be similar across countries in terms of both direction of causality and estimated coefficients. We can then test a homogenous non-causality hypothesis on the full sample which includes 112 countries over the period 1970-2013 for a total of 44 potential observations for each country cross-section. We consider a number of different lag lengths although the Akaike Information Criterion (AIC) suggests that the optimal lag length is four.

Table 5 shows that there is strong evidence of bi-directional causality between these two variables. At a 5% confidence level, we can rule out Granger non-causality in both directions for all lags up to 8: from *EE/Cap* to *LP/Worker* and from *LP/Worker* to *EE/Cap*. In the short-run (notably when one or two lags are used), there is stronger evidence for the claim that *LP/Worker* causes *EE/Cap*. For these lag lengths, we cannot reject the null that *EE/Cap* does not cause *LP/Worker* at the 1% significance level. This makes sense because the labor force is comprised of individuals spanning a wide range of ages and only a small fraction of them were the direct recipients of publicly-funded education in the past year or two; the only workers in the labor force whose productivity should depend on recent educational expenditures are those who have matriculated in the past year or two. However, when we increase the number of lags considered we are able to firmly reject the null that *EE/Cap* does not cause *LP/Worker*. When considering educational expenditures over the past 8 years, we are now measuring the impact on a very significant portion of the labor force – anyone who attended school within the past 8 years. This pattern of Granger causality from *EE/Cap* to *LP/Worker* suggests an effect on labor

productivity due to the accumulation of educational capital. One can reject Granger non-causality from *LP/Worker* to *EE/Cap* at a 1% confidence level for all lags up to 7 but not for 8 lags. Thus, in the short term, if *LP/Worker* increases, there are more resources to invest in education.

When we consider the optimal lag length as suggested by the AIC (4 lags), we find strong evidence of bi-directional causality with very low p-values. In fact, the model developed in this paper contains the bi-directional causality found in the data. The initial impetus in the model is increased spending on education which thereby increases future labor productivity. Higher labor productivity then allows firms to serve a larger market thereby expanding the tax base and overall tax revenues earned by the government. Increased tax revenues will lead to higher spending on education whether to accumulate additional educational capital so as to reach the optimal steady state or to offset depreciation of a higher capital stock. A more educated workforce is more productive; a more productive workforce produces more resources to be used for all purposes including education.

These results suggest an endogenous relationship between educational expenditures and labor productivity. If our goal was to estimate the exact relationship between these variables from the data, then it would be proper to run a Hausman test to formally test for endogeneity and then employ an estimation method using instrumental variables to correct for any bias in the estimator. However, for the purpose of this paper it is sufficient that the analysis of Granger causality shows that the causal relationship built into the model (whereby spending and investment in education determine the distribution of labor productivity in a particular country) has an empirical foundation.

#### **A.4 Effects of Public Educational Expenditures and Public Fixed Capital Formation on Labor Productivity**

The model outlined in this paper considers only the role of public expenditures on education and the accumulation of public 'educational capital'. However, the framework considered can also be applied to any form of public capital including, but not limited to, physical capital available for public use such as roads, infrastructure, and utilities. In this appendix we shall present data which support the application of the model to physical capital.

From the World Bank we obtained data on both total Gross Fixed Capital Formation (GFCF) and private Gross Fixed Capital Formation as a percentage of GDP. Subtracting private investment from total investment yields the public investment in fixed capital as a percentage of GDP. We then nor-

malize the data into per-capita terms to make cross-country comparisons meaningful.



Figure 13: Relationship between labor productivity per worker and fixed capital formation, both public and private. 112 countries, 1970-2013.  $N = 1872$  and  $\rho = 0.7923$  for public data;  $N = 1875$  and  $\rho = 0.8894$  for private data.

Figure 13 shows the relationship between fixed capital formation per capita, both public and private, with labor productivity per worker. We find that the slope is higher for public investment; private investment will benefit only workers employed by a particular firm or industry while public investment (roads, bridges, etc) benefits all workers in the country.

Figure 14 plots public GFCF per-capita against public educational expenditures per-capita. Despite significant clustering around the origin for countries with low values of public spending on either area, there is a positive relationship between the two variables. Countries which spend more on one public good tend to spend more on the other as well.

We wish to see how effective public GFCF is at predicting and explaining both labor productivity per worker and GDP per capita relative to educational expenditures. It is also of interest to determine whether or not these two variables are capturing the same effects or whether they contain distinct information.

Tables 6 and 7 show the regression output when labor productivity per worker and GDP per capita are the dependent variables, respectively. The results indicate that educational expenditures have better performance at explaining the variation in labor productivity and GDP per capita than public investment in fixed capital. However, utilizing both variables as regressors improves the fit for both dependent variables suggesting that there is

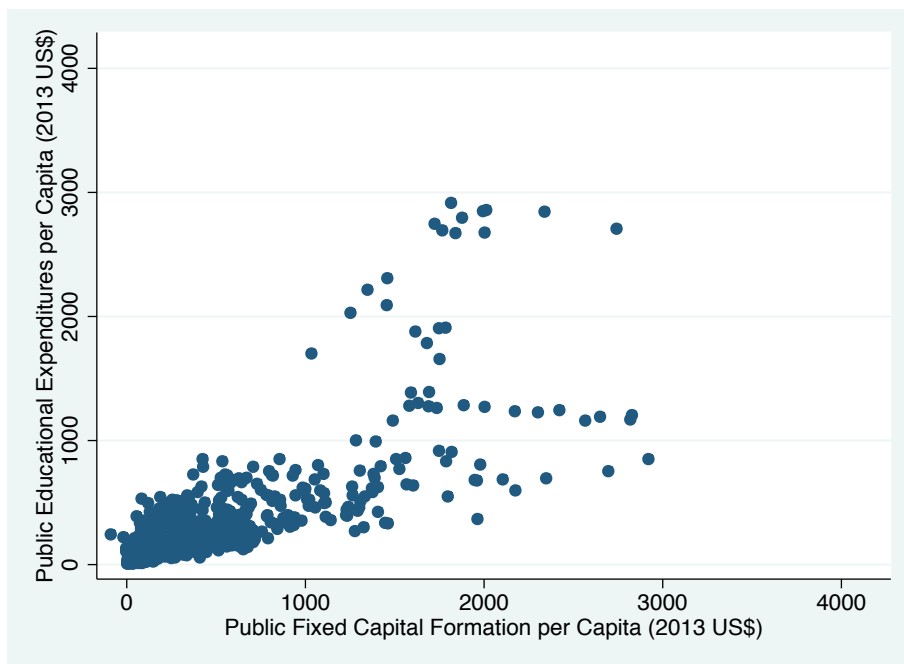


Figure 14: Relationship between educational expenditures and public capital investment for 112 countries, 1970-2013.  $N = 868$  and linear correlation coefficient  $\rho = 0.7792$ .

some unique information contained in each independent variable. However, there is certainly some overlap between educational expenditures and public investment in fixed capital. Although spending on textbooks and teacher salaries falls solely under educational expenditures, spending on construction and maintenance of school buildings and facilities would be counted under both measures of public spending. Further, much of the physical fixed capital formation which would affect labor productivity is done at the private level by firms and individuals. This is in contrast to educational spending which is done overwhelmingly by the public sector in most countries and which would affect nearly all workers in a country's labor force.

## A.5 Trade Balance

Models of international trade including heterogeneous firm-level labor productivity such as Eaton and Kortum (2002) and Garetto (2013) rely on

the use of balanced trade conditions to pin down equilibrium wages across countries.

The concern is that balanced trade conditions impose on the model a rigidity which is often lacking in data. The data on total exports and total imports of goods and services as a percentage of GDP are from the World Bank. The trade balance per capita is found by subtracting imports from exports and normalized to a per-capita basis. While no country can perpetually live beyond their means, many countries have consistently run large trade deficits or surpluses over at least a 20 year interval.

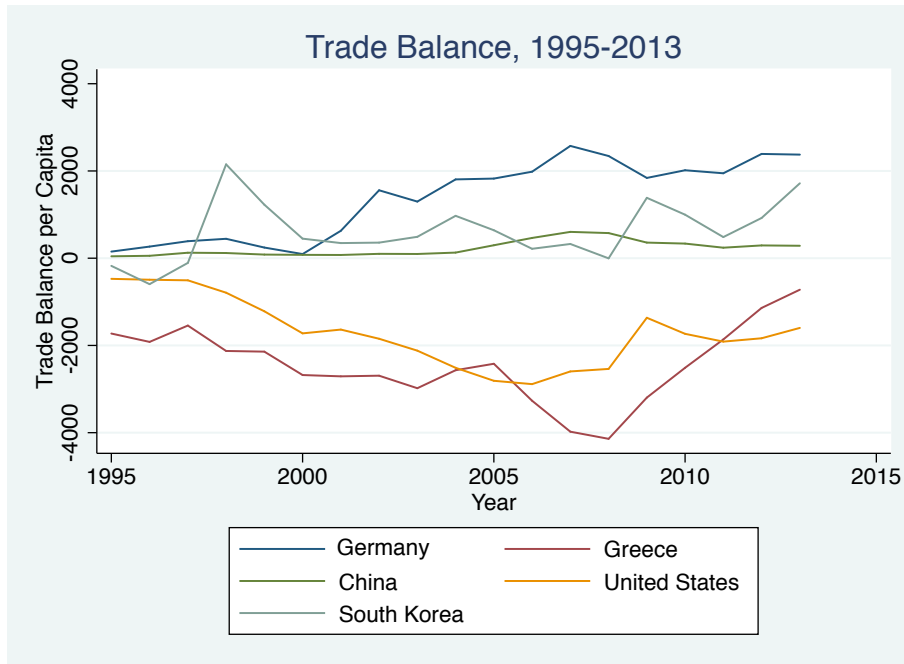


Figure 15: Trade balance per capita for selected countries, 1995-2013.

Figure 15 shows the historical data for five selected countries. Some countries tend to either consistently run a trade surplus or consistently run a trade deficit over this time frame. Earlier data on imports and exports were unavailable. While there is a significant shift occurring at 2008 and coinciding with the beginnings of the Great Recession, this does not seem to be a reversal of existing trends. Rather, it is simply the case of lackluster demand on the part of trading partners and the result of temporarily frozen credit lines as in the case of Greece.

The role of credit flows and financial assets should not be understated when discussing trade balances; countries which consistently run trade deficits, such as the United States and Greece, have been able to do so via loans and sales of assets and equity. The dramatic fall in Greek trade deficits beginning in 2008 is tied to the unavailability of Foreign credit during this time. Without loans from German and French banks, consumers in Greece were unable to import foreign goods and services. The model considered in this paper could be extended to explicitly include a role for bonds, both private and public, or some other interest-bearing asset. Given initial endowments of financial assets, it can be inferred that sales of such assets could finance trade deficits as they occur in the context of this model.

In the absence of financial markets in this model, the use of a balanced trade condition to determine equilibrium wages is not supported by data.

## B Mathematical Details

### B.1 Closed Form Solution

Here we outline the steps and assumption that are used to derive an analytical solution to the model. We make only one assumption for computational ease. Specifically, we assume that the shape parameter of the Weibull distribution, henceforth  $\kappa$ , is the same for both countries. We allow the scale parameter,  $\lambda$  to vary across countries and it is specifically through this parameter that one country can distinguish itself from the other in terms of productivity and labor costs.

Let  $\phi(x; \kappa, \lambda_H)$  be the density function of per-unit labor requirements in the home country and let  $\psi(y; \kappa, \lambda_F)$  be the corresponding density in the foreign country. Total revenues in period  $t$  for the home government are:

$$R_{H,t} = \frac{T_{H,t}}{1 + T_{H,t}} \left[ p_{H,t}^\eta q_H [w_{H,t}(1 + T_{H,t})]^{1-\eta} \int_{B_H^H} x^{1-\eta} \phi(x) dx \right. \\ \left. + p_{F,t}^\eta q_F [w_{F,t}(1 + T_{H,t})\tau]^{1-\eta} \int_{B_F^H} x^{1-\eta} \phi(x) dx \right], \quad (28)$$

where the first term represents sales to domestic consumers and the second term represents exports to foreign consumers. We can rewrite the integrals in this equation in the following form:



$$\begin{aligned}
C_{HH} &\equiv \int_{B_H^H} x^{1-\eta} \phi(x) dx = \int_0^\infty \psi(y) dy \int_0^{\alpha_t y} x^{1-\eta} \phi(x) dx \\
C_{FH} &\equiv \int_{B_F^H} x^{1-\eta} \phi(x) dx = \int_0^\infty x^{1-\eta} \phi(x) dx \int_{\theta_t x}^\infty \psi(y) dy,
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
\alpha_t &= \frac{w_{F,t}(1 + T_{F,t})\tau}{w_{H,t}(1 + T_{H,t})} \\
\theta_t &= \frac{w_{H,t}(1 + T_{H,t})\tau}{w_{F,t}(1 + T_{F,t})}.
\end{aligned} \tag{30}$$

We can define analogous integrals for the foreign country:

$$\begin{aligned}
C_{FF} &\equiv \int_{B_F^F} y^{1-\eta} \psi(y) dy = \int_0^\infty \phi(x) dx \int_0^{\theta_t x} y^{1-\eta} \psi(y) dy \\
C_{HF} &\equiv \int_{B_H^F} y^{1-\eta} \psi(y) dy = \int_0^\infty y^{1-\eta} \psi(y) dy \int_{\alpha_t y}^\infty \phi(x) dx.
\end{aligned} \tag{31}$$

Due to our assumption of Weibull-distributed labor requirements, these integrals have a closed-form solution. If we assume  $\kappa$  is constant across both countries, then these integrals may be simplified as follows:

$$\begin{aligned}
C_{HH,t} &= \lambda_{H,t}^{-\kappa} \gamma_{1,t}^{1+\kappa-\eta} G(\kappa, \eta) \\
C_{HF,t} &= \lambda_{F,t}^{-\kappa} \gamma_{2,t}^{1+\kappa-\eta} G(\kappa, \eta) \\
C_{FF,t} &= \lambda_{F,t}^{-\kappa} \gamma_{3,t}^{1+\kappa-\eta} G(\kappa, \eta) \\
C_{FH,t} &= \lambda_{H,t}^{-\kappa} \gamma_{4,t}^{1+\kappa-\eta} G(\kappa, \eta),
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
\frac{1}{\gamma_{1,t}^\kappa} &= \frac{1}{\lambda_{H,t}^\kappa} + \frac{1}{(\alpha_t \lambda_{F,t})^\kappa} \\
\frac{1}{\gamma_{2,t}^\kappa} &= \frac{1}{\lambda_{F,t}^\kappa} + \frac{\alpha_t^\kappa}{\lambda_{H,t}^\kappa} \\
\frac{1}{\gamma_{3,t}^\kappa} &= \frac{1}{\lambda_{F,t}^\kappa} + \frac{1}{(\theta_t \lambda_{H,t})^\kappa} \\
\frac{1}{\gamma_{4,t}^\kappa} &= \frac{1}{\lambda_{H,t}^\kappa} + \frac{\theta_t^\kappa}{\lambda_{F,t}^\kappa},
\end{aligned} \tag{33}$$

and

$$G(\kappa, \eta) = \kappa \int_0^\infty z^{\kappa-\eta} e^{-z^\kappa} dz = \Gamma\left(\frac{1+\kappa-\eta}{\kappa}\right). \quad (34)$$

In order for this term to be well-defined, we must impose the following constraint on the parameters of the model:

$$1 + \kappa - \eta > 0.$$

If this condition is satisfied, then we are able to find closed-form solutions for the integrals of interest conditional on choice variables  $(T_{i,t}, \alpha_{i,t})$ , the state variable  $(E_{i,t})$ , and initial conditions  $(\lambda_{i,0}, E_i^A)$ .

## B.2 Equilibrium and First Order Conditions

In the infinite horizon, the government's problem can be stated as maximizing the following sum:

$$\sum_{t=0}^{\infty} \beta^t [R_H(T_H(t), E_H(t)) - I_H(t)] \quad (35)$$

by choosing variables  $T_H(t)$  and  $I_H(t)$  subject to the following constraints:

$$\begin{aligned} E_H(t+1) &= (1-\delta)E_H(t) + I_H(t)/N_H \\ I_H(t) &\leq R_H(T_H(t), E_H(t)), \end{aligned} \quad (36)$$

where  $R_H(t)$  is revenue earned in period  $t$  and  $I_H(t)$  is investment in human capital in period  $t$ . We can therefore construct the following Lagrangian for this problem:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t [R_H(T_H(t), E_H(t)) - I_H(t)] \\ &+ \sum_{t=0}^{\infty} \pi(t) [(1-\delta)E_H(t) + I_H(t)/N_H - E_H(t+1)], \end{aligned} \quad (37)$$

where  $\pi(t)$  is the Lagrange multiplier corresponding to the constraint on  $E_H(t+1)$ . The first order conditions with respect to control variables are:

$$\frac{\partial L}{\partial T_H(t)} = \beta^{t-1} \frac{\partial R_H(T_H(t), E_H(t))}{\partial T_H(t)} = 0 \quad (38)$$

$$\frac{\partial L}{\partial I_H(t)} = -\beta^t + \pi(t)/N_H = 0, \quad (39)$$

or

$$\pi(t) = \beta^t N_H$$

and for the state variable

$$\frac{\partial L}{\partial E_H(t)} = \beta^t \frac{\partial R_H(T_H(t), E_H(t))}{\partial E_H(t)} + (1 - \delta)\pi(t) - \pi(t - 1) = 0. \quad (40)$$

Combining the previous two equations we get

$$\frac{\partial R_H(T_H(t), E_H(t))}{\partial E_H(t)} = \left[ \frac{1}{\beta} - (1 - \delta) \right] N_H. \quad (41)$$

We can further simplify this analysis by introducing a new variable:

$$z_H(t) \equiv \frac{w_H(1 + T_H(t))\lambda_H(t)}{w_F(1 + T_F)\lambda_F\tau}. \quad (42)$$

This allows us to express revenue in the following way:

$$R_H(t) = R_{H0} \frac{T_H(t)}{1 + T_H(t)} f_H(z_H(t)), \quad (43)$$

where

$$R_{H0} = q_H w_F (1 + T_F) \lambda_F \tau G^{1/(1-\eta)}, \quad (44)$$

and

$$f_H(z) = \left[ \frac{z}{(1 + z^\kappa)^{1+1/\kappa}} + \frac{(q_F \tau / q_H) z}{(1 + \tau^{2\kappa} z^\kappa)^{1+1/\kappa}} \right]. \quad (45)$$

Since

$$\lambda_H(E_H(t)) = \frac{\Lambda_H}{\ln(c_H + E_H(t)/E_H^A)}, \quad (46)$$

we can write

$$z_H(t) = Z_H \frac{1 + T_H(t)}{\ln(c_H + E_H(t)/E_H^A)}, \quad (47)$$

where

$$Z_H = \frac{w_H \Lambda_H}{w_F (1 + T_F) \lambda_F \tau}. \quad (48)$$

Therefore,

$$\frac{\partial z_H(t)}{\partial T_H(t)} = \frac{z_H(t)}{1 + T_H(t)}, \quad (49)$$

and

$$\frac{\partial z_H(t)}{\partial E_H(t)} = -\frac{z_H^2(t)}{E_H^A Z_H(1+T_H(t))} e^{-Z_H(1+T_H(t))/z_H(t)}. \quad (50)$$

It should be noted that  $R_{H0}$  and  $Z_H$ , while independent of control and state variables for the Home country, could be functions of time if the Foreign country decides to change its control and state variables.

From equations (38) and (43), we get

$$f_H(z_H(t)) + T_H(t)z_H(t) \frac{df_H(z_H(t))}{dz_H(t)} = 0. \quad (51)$$

From equations (41) and (50), we get

$$\frac{T_H(t)}{(1+T_H(t))^2} z_H^2(t) \frac{df_H(z_H(t))}{dz_H(t)} e^{-Z_H(1+T_H(t))/z_H(t)} = -\frac{E_H^A Z_H N_H}{R_{H0}} \left( \frac{1}{\beta} - 1 + \delta \right). \quad (52)$$

Together with the following constraints:

$$E_H(t+1) = (1-\delta)E_H(t) + I_H(t)/N_H \quad (53)$$

$$I_H(t) \leq R_H(t), \quad (54)$$

and the initial condition,  $E_H(1)$ , Eqs. (51) and (52) specify the optimal path  $(T_H^*(t), E_H^*(t))$  for the Home country, conditional on the policies chosen by the Foreign country.

We first solve Eq. (51) for  $T_H^*(1)$ , since the initial value of educational capital is given. From  $T_H^*(1)$  we can compute  $R_H^*(1)$ . We then simultaneously solve Eqs. (51) and (52) to determine the optimal values of  $T_H^*(2)$  and  $E_H^*(2)$ . If the stock of human capital is feasible for country H, that is

$$E_H^*(2) \leq (1-\delta)E_H(1) + R_H^*(1) \quad (55)$$

then we can find  $I_H^*(1)$  as

$$I_H^*(1) = N_H (E_H^*(2) - (1-\delta)E_H(1)) \quad (56)$$

If  $E_H^*(2)$  is not feasible, then we set  $I_H^*(1) = R_H^*(1)$  and therefore

$$E_H(2) = (1-\delta)E_H(1) + R_H^*(1)/N_H \quad (57)$$

and we must solve for  $T_H^*(2)$  by equation (51) conditional on  $E_H(2)$ . Due to the linear nature of the objective function and lack of Inada-type conditions,

we find that the optimal behavior of the government is to invest as much as possible until reaching the desired steady state. Once they reach the steady state, they will invest only as much as is necessary to offset depreciation. We have not included provisions for borrowing; if borrowing were possible, then we would need to include borrowing, outstanding debt, and relevant interest rates in the Lagrangian for the government's problem. The optimal solution would depend on the quantity borrowed, the increase in revenue from being at the desired steady state, and the cost of borrowing.

Table 3: List of Countries in Sample and No. Observations

Country	N. Obs	Country	N. Obs	Country	N. Obs
Albania	14	Germany	10	Pakistan	37
Algeria	3	Ghana	23	Peru	15
Angola	9	Greece	26	Philippines	18
Argentina	31	Guatemala	13	Poland	18
Armenia	14	Hong Kong	29	Portugal	35
Australia	24	Hungary	40	Romania	9
Austria	39	Iceland	18	Russia	8
Azerbaijan	16	India	11	Senegal	12
Bahrain	3	Indonesia	16	Serbia-Montenegro	5
Bangladesh	24	Iran	35	Singapore	25
Barbados	18	Ireland	38	Slovak Republic	18
Belarus	12	Israel	35	Slovenia	15
Belgium	25	Italy	29	South Africa	20
Bolivia	14	Jamaica	27	South Korea	35
Brazil	13	Japan	32	Spain	31
Bulgaria	19	Jordan	21	Sri Lanka	22
Burkina Faso	12	Kazakhstan	12	St. Lucia	15
Cambodia	8	Kenya	29	Sweden	22
Cameroon	24	Kyrgyz Republic	17	Switzerland	29
Canada	35	Latvia	21	Syria	25
Chile	26	Lithuania	16	Tajikistan	17
China	25	Luxembourg	20	Tanzania	9
Colombia	20	Macedonia	6	Thailand	37
Costa Rica	18	Madagascar	29	Trinidad-Tobago	17
Cote d'Ivoire	26	Malawi	19	Tunisia	27
Croatia	7	Malaysia	26	Turkey	18
Cyprus	32	Mali	17	Uganda	16
Czech Republic	18	Malta	28	Ukraine	14
Denmark	31	Mexico	19	United Kingdom	36
Dominican Republic	14	Moldova	15	United States	19
Dem. Rep. Congo	10	Morocco	31	Uruguay	19
Ecuador	14	Mozambique	5	Venezuela	20
Egypt	23	Netherlands	38	Vietnam	2
Estonia	15	New Zealand	32	Yemen	3
Ethiopia	16	Niger	19	Zambia	30
Finland	37	Nigeria	2	Zimbabwe	13
France	40	Norway	38		
Georgia	15	Oman	22		

Table 4: Testing for Granger Non-Causality

No. Lags	N. Obs	Null Hypothesis	P-value
1	1867	LP/Worker does not cause EE/Cap	2E-07
		EE/Cap does not cause LP/Worker	0.0344
2	1526	LP/Worker does not cause EE/Cap	1E-05
		EE/Cap does not cause LP/Worker	0.0197
3	1252	LP/Worker does not cause EE/Cap	4E-06
		EE/Cap does not cause LP/Worker	4E-05
4	1020	LP/Worker does not cause EE/Cap	1E-05
		EE/Cap does not cause LP/Worker	3E-06
5	849	LP/Worker does not cause EE/Cap	1E-05
		EE/Cap does not cause LP/Worker	2E-06
6	707	LP/Worker does not cause EE/Cap	7E-04
		EE/Cap does not cause LP/Worker	3E-05
7	591	LP/Worker does not cause EE/Cap	0.0034
		EE/Cap does not cause LP/Worker	4E-04
8	493	LP/Worker does not cause EE/Cap	0.014
		EE/Cap does not cause LP/Worker	5E-04

Table 5: Testing for Granger Non-Causality

Lags	N. Obs	Direction (Null of No Causality)	F-Stat	P-Val	Direction (Null of No Causality)
1	1867	$EE/Cap \rightarrow LP/Worker$	4.4797	0.0344	$LP/Worker \rightarrow EE/Cap$
2	1526	$EE/Cap \rightarrow LP/Worker$	3.9390	0.0197	$LP/Worker \rightarrow EE/Cap$
3	1252	$EE/Cap \rightarrow LP/Worker$	7.6901	4E-05	$LP/Worker \rightarrow EE/Cap$
4	1020	$EE/Cap \rightarrow LP/Worker$	7.9671	3E-06	$LP/Worker \rightarrow EE/Cap$
5	849	$EE/Cap \rightarrow LP/Worker$	6.9143	2E-06	$LP/Worker \rightarrow EE/Cap$
6	707	$EE/Cap \rightarrow LP/Worker$	5.2255	3E-05	$LP/Worker \rightarrow EE/Cap$
7	591	$EE/Cap \rightarrow LP/Worker$	3.8949	4E-04	$LP/Worker \rightarrow EE/Cap$
8	493	$EE/Cap \rightarrow LP/Worker$	3.5876	5E-04	$LP/Worker \rightarrow EE/Cap$

Table 6: OLS Regression Results

	LP/Worker (1)	LP/Worker (2)	LP/Worker (3)
EE/Cap	31.389*** (105.54)		27.563*** (33.80)
Pub. GFCF/Cap		30.066*** (56.16)	14.919*** (22.41)
Constant	11678*** (35.83)	5634*** (17.54)	3883*** (14.50)
N. Obs	2320	1872	868
Adj. $R^2$	0.8277	0.6276	0.8924

Test statistics in parentheses. \* denotes significance at the 10% level, \*\* at the 5% level, and \*\*\* at the 1% level.

Table 7: OLS Regression Results

	GDP/Cap (1)	GDP/Cap (2)	GDP/Cap (3)
EE/Cap	15.848*** (136.84)		16.219*** (58.48)
Pub. GFCF/Cap		12.460*** (52.09)	3.481*** (15.37)
Constant	3089*** (24.35)	1389*** (9.68)	581*** (6.38)
N. Obs	2320	1872	868
Adj. $R^2$	0.8898	0.5918	0.9369

Test statistics in parentheses. \* denotes significance at the 10% level, \*\* at the 5% level, and \*\*\* at the 1% level.