Social Network Of Indirect Favor Exchange

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Abstract

I develop a game theoretical model of indirect favor exchange networks where one can request favors through a chain of contacts based on Jackson et al (2012)'s model of direct favor exchange where one can only request favors from his direct contacts. I study the cooperative behavior fostered by potential sanction in a network. First, I provide a full characterization of "renegotiation-proof" networks and then propose a robustness condition. Under this specific robustness refinement, when the maximum length of contact chains is larger than 3, only star-shaped networks achieve highest robustness. I explore data from Indian rural villages to show that networks of social links are less assortative than the relative networks, and also have lower support levels.

1 Introduction

Favor exchange is a form of cooperation that we observe at every level of our life, from small to large favors. Although many favors such as borrowing and returning a large amount of money and giving priority seats to those in need are sometimes regulated by social, religious, legal and political norms, most of the favors remain informal. Another interesting thing to notice is that in many occasions favor exchanges are not bilateral behaviors. For example, if one person refuses to lend a small loan, friends of the borrower may refuse to perform favors for this person. In this case, punishment for not performing a favor is carried out by other members in the network. Lastly, it is common for a person to seek a favor through a chain of contacts, instead of from his direct friends. For example, one may need to go through several contacts to set up a job interview opportunity.

Although there is a large literature on social capitals, there is relatively few literature that studies the game theoretical foundation of such behaviors. Such literature focuses on the empirical behaviors instead of the relation between social structures and individual motivations. These literature rooted from the sociological literature including Simmel (1950), Coleman (1988) and Krackhardt (1996).

The characteristics mentioned above, punishment through other members and chain of contacts, are very relevant to the study of networks. Understanding structures of social interaction networks can help understand and facilitate diffusion of information and resources. For example, promotion and management of microfinance can rely on the network structure. For highly clustered structures, promotion should avoid wasting resources on targeting too many agents in the same cluster. For a highly centralized network, promotion should target agents with high centrality.

I develop a game theoretical model of *indirect* favor exchange networks where one can request favors through a chain of contacts based on Jackson et al (2012)'s model of *direct* favor exchange where one can only request favors from his direct contacts. I study how their methods may extend to this new scenario and how equilibrium refinements may require modification.

In this paper I provide a game theoretical foundation of informal favor exchange in social networks. In particular, I study situations where simple bilateral enforcement is not sufficient to sustain cooperation and it is the pressure from the network that fosters cooperation. Bilateral interactions may be infrequent enough that they cannot self-enforce cooperation or favor exchange.

I examine the setting where opportunities for one agent to perform a favor for another agent arrive randomly over discrete periods. An agent can go through a chain of linked agents of an exogenously set length to seek a favor. Performing a favor is costly and is not instantly repaid. But the benefit for the recipient exceeds the cost occurred to who performs the favor, so it is efficient for social welfare if all favors are performed when needed. However, since providing a favor is not instantly compensated, it could be possible that it is not in an agent's selfish interests to provide a favor even if that means he will never receive favor from the recipient again. A social network makes it possible for an agent to lose multiple relationships over declining one favor and thus can provide enough incentive for an agent to keep performing favors. One thing to notice is that a wide range of equilibrium can be supported by a "grim-trigger" strategy, where once anyone fails to provide a favor, everyone stops providing favors forever. This sort of equilibria are not realistic or desirable, and they can be ruled out by the definition of renegotiation-proof equilibrium borrowed from Jackson et al (2012). This definition requires that any continuation equilibrium after a failure to provide a favor must not be Pareto dominated by another feasible equilibrium continuation. This captures the credibility of threats to punish noncooperative behaviors.

In section 4 I characterize the set of all renegotiation-proof equilibria as a set of renegotiation-proof networks. I find that while the characterization is similar in methodology to Jackson et al (2012), their result, social quilts, are not renegotiation-proof in my setting. Then in section 5 I extend the robustness concept in their research to indirect favors to refine the set of renegotiation-proof equilibria. The level of robustness in Jackson et al (2012) is impossible to reach in my setting. In particular, under this specific robustness refinement, when the length of contact chains are larger than 3, only star-shaped networks achieve highest robustness. In section 6, I compare networks constructed from social links to the naturally induced kinship networks and show that they are less assortative and also have lower support levels. This supports the conclusion in section 5.

2 Related Literature

The current literature on networks can be roughly divided into two types: exogenous networks and endogenous networks.

The former type focus on networks that are exogenously determined, for example by geological locations. In this stream, applications of network studies are made on social learning (Bala and Goyal, 1998), crime and social interaction (Glaeser, Sacerdote and Scheinkman, 1996) and social coordination (Morris, 2000).

This paper is located in the second stream. This part of literature mainly focuses on which networks provide efficiency and stability when agents have the capability to alter the network structure (form or sever links). Some examples are Dutta and Mutuswami (1997), Bala and Goyal (2000) and Jackson and Watts (2002).

Coleman (1988) discusses three forms of social capital: Obligations, Expectations, and Trustworthiness of Structures, Information Channel and Norms and Effective Sanction. However, no game theoretical structure or systematic study of incentives and enforcements are pursued.

Some closely related previous literature in terms of the theoretical analysis of a repeated game on a network includes a series of papers that study prisoners' dilemmas in network settings, including Raub and Weesie (1990); Ali and Miller (2009); Lippert and Spagnolo (2011); and Mihm, Toth, Lang (2009). In particular, Raub and Weesie (1990) and Ali and Miller (2009) show how completely connected networks shorten the travel time of contagion of bad behavior, which can quicken punishment for deviations. There are several differences between their study and mine. In those settings, individuals do not have information about others' behavior except through what they observe in terms of their own interactions. Thus, punishments travel through the network only through contagious behavior and the main hurdle to enforcing individual cooperation is how long it takes for someone's bad behavior to come to reach their neighbors through chains of contagion. This emphasizes on the information diffusing property of networks. My setting is different, where individuals have complete information and the network provides a mechanism of sanction. My model focuses on how the network affect individual incentives.

3 Model

3.1 Network, Favors and Payoffs

Consider the setup of Jackson et al (2012). A finite set $N = \{1, 2, ..., n\}$ of agents are linked in a social network, described by a undirected graph. The set of agents, N, is fixed throughout my analysis and a network is represented by the set of links, g. For simplicity, let ij be the link $\{i, j\}$ and $ij \in g$ means that agent i and j are directly linked in the network g. Let $g - ij$ denote a network obtained from g by deleting the link ij. I define distance of any i and j as the minimum number of links on a path connecting i and j and denote the distance as $d(i, j)$. The set of neighbors of an agent i within distance l is denoted by $N_i^l(g) = \{j | d(i, j) \leq l\}$.

Time proceeds in discrete periods indexed by $t \in \{0, 1, ...\}$ (infinitely repeated).

At the beginning of each period, a random draw takes place, where there is a chance of $n(n-1)p < 1$ that exactly one favor is needed, otherwise no favor is needed. If a favor is needed, then each ordered pair of agents (first being provider and the latter being recipient) have equal chance to be assigned this favor. In this setting, the chance that an agent i needs a favor from an agent j is p. In this setting, at most one favor may be needed in each period. Note that p does not depend on the structure of the network. I characterize this way since the favors I study here are delivered indirectly through the network and the need for favors doesn't depend on whether two agents are indirectly connected. For example, when a job-seeker needs a favor to set up an interview to a specific position, this need for favor exists regardless whether this job-seeker is connected to the hiring team.

Here is where my setting and Jackson et al (2012) differ. Say that agent i and j are *l-connected* (indirectly) if there exists a path of links with length smaller than or equal to l from i to j. When a favor is delivered through such path, the agent doing the favor pays an amount c of effort, the agent receiving the favor gains v and everyone else incurs no cost nor benefit. I assume that $v > c$ here, which makes favor exchanging ex post Pareto efficient. I examine favor exchanges that cannot be self-enforcing. In such cases, contracts may be too costly to write up or too difficult to be complete. For example, it can be extremely difficult or costly to write up a contract regulating what kinds of favors and when one should return after one receives a favor, especially when such return of favor involves asking for favors from other friends or the need for favor occurs randomly. Also, some behaviors cannot be written into and regulated by contracts. For example, according to Bian (1997), in last century's China, jobs were assigned by government officials. An important type of favor then was assigning a good job based on personal relationships and connections, which was illegal and thus cannot be contracted.

Similar to Jackson et al (2012), agents discount payoffs over time by $0 < \delta < 1$. For any agent i and j, in each period, there is a chance of p that i needs a favor from j and a chance of p that j needs a favor from i. If i and j are l-connected and on the equilibrium path all favors are performed when needed and no link gets deleted, the payoff of a bilateral relationship starting from next period is $\sum_{t=1}^{\infty} \delta^t p(v-c) = \frac{\delta p(v-c)}{1-\delta}$. If $\frac{\delta p(v-c)}{1-\delta} < c$, then an agent have incentive to decline to perform a favor when called upon as the benefits from the future doesn't overcome the cost incurred in the current period. In such case, any link is not sustainable if the two agents are in isolation. As shown in the payoff function, low marginal benefit from each favor and low frequency of interaction leads to low discounted payoff. I will only study such cases here since bilaterally enforced cooperation under this setting would be a trivial and uninteresting case.

A society is described by (N, p, v, c, δ), which will be fixed throughout my analysis.

3.2 The Game

The favor exchange game I model is described as follows:

- The game begins with an initial network, g_0.
- Period t begins with network g_{t-1}.
- Agents simultaneously announce the links they are willing to keep, $L_i \subset N_i^1(g_{t-1})$, leading to $g'_t = \{ij | i \in L_j, j \in L_i\}$. Note that agents cannot form links but only can choose to break existing links.
- There is a chance of $n(n-1)p$ that exactly one favor is needed and otherwise no favor is needed. If a favor is needed, the favor is randomly assigned to any ordered pair of agents with equal probability. If agent i needs a favor from agent j, and there is no path connecting i and j, then this opportunity is lost. Note that I assume $n(n-1)p \leq 1$.
- If i and j are l-connected, agent i chooses a path with length smaller than or equal to l to reach out to j and j chooses whether or not to perform the favor. If a favor is performed, then j incurs cost c and i gains benefit v. If j declines to perform the favor, j loses its link to the last member of the path, i.e. the agent that's directly linked to j in the path connecting i to j. The resulting network is g_t.

There are several things to note about this model.

- I do not consider the formation of the network but only consider what networks are sustainable. Agents get to react to other agents' behavior by dropping links. Since links that are lost cannot be rebuilt again, this constitutes the sanction against declining to perform a favor. Essentially, the agents' decisions are which links to maintain. This simplifies the structure of the model and avoids complicated strategies such as punishing for a finite number of periods and then rebuilding links.

- Agents do not pay for each favor in monetary terms. An agent cannot just pay the cost c to the favor performing agent. This is consistent with observations in reality. Many favors cannot be repaid in money. Using the same example as above, assigning a good job cost the official political capital. Receiving money would be risky and other political favors were a more realistic way to repay the favor.

- This model is a complete information game. This may not seem desirable when the size of the network is large. However, I keep it simple to derive more explicit results. However, in robust equilibria, information does not need to flow through more than l links to enforce the equilibria.

3.3 Equilibrium

Denote $A_i^l(g) = \{j | j\, is\, l - connected(indirectly)\, with\, i\}$. Any network with $c < |A_i^l(g)| \frac{\delta p(v-c)}{1-\delta}$ for all i can be sustained by a so-called "grim-trigger" strategy, where all links are kept and all favors are performed on the equilibrium path, and if any agent declines to perform a favor, all agents drop all links. However many of these equilibria supported by the "grim-trigger" strategy are not so desirable, as in reality it is extremely impossible for the whole society or even just a group of people to dissolve if one person declines to do one favor and deletes one link. I borrow from Jackson, Rodriguez-Barraquer and Tan (2012) the definition of renegotiation-proof equilibrium and renegotiation-proof networks. The logic behind this equilibrium concept is that when people can communicate, it is more likely that they would rather reach a Pareto efficient continuation than letting the entire society collapse. I only consider pure strategies here. Also, note that if deleting a link does not impact an agent's utility, the agent will delete it. This follows from the logic that if a link ij can be deleted by i without affecting i's utility, then when j requests a favor from i directly, i will decline it and lose the link ij.

Definition. *Renegotiation-proof equilibria are defined by induction. Denote RPN_k to be the set of renegotiation-proof networks (those networks that are sustained in renegotiation-proof equilibria) with k links. Let G_k denote the set of networks with k links.*

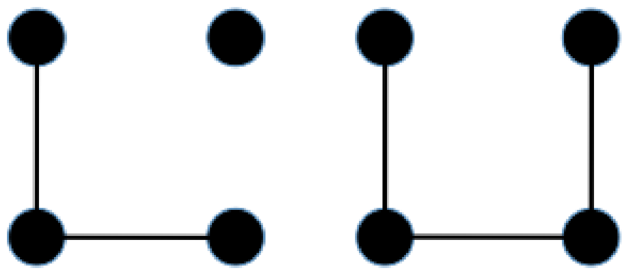
- *Let $RPN_0 = \{\varnothing\}$.*

- *Let RPN_k denote the subset of G_k such that $g \in RPN_k$ iff beginning with $g_0 = g$ gives a pure strategy subgame perfect equilibrium such that 1) on the equilibrium path g is always sustained (all favors performed if possible and all links kept), and 2) in any subgame starting with some network $g' \in G_{k'}$ with $k' < k$, if g'' is reached with a positive probability (depends on the order of favors needed) and played in perpetuity in the continuation, then $g'' \in RPN_{k''}$ for some k'' and there doesn't exist $g''' \subset g'$ such that $g''' \in RPN_{k'''}$ and have utilities $u_i(g''') \geq u_i(g'')$ for all i with strict inequality for some i.*

4 Characterization of Renegotiation-proof Networks

The analysis of the network structure essentially depends on how many l-connected neighbours are needed to incentivize an agent to maintain a link. Let m be an integer such that $(m-1)\frac{\delta p(v-c)}{1-\delta} < c < m\frac{\delta p(v-c)}{1-\delta}$. Without loss of generality, I use strict inequality on both sides. As shown in the following section, all the parameters δ, p, v, c can be summarized by m and the set of equilibria is determined by n and m only.

Here is an example to illustrate renegotiation-proofness of the equilibria.



As shown above, suppose $n = 3$, $m = 2$. We further suppose $l = 3$. The network on the left is a renegotiation-proof equilibrium since if any agent declines a favor and deletes a link, the rest of the three agents form a two-agent network. Because $m = 2$, bilateral relations cannot be sustained and then the remaining agents will delete the link left. In this way, any agent who deletes a link will lose 2 l-neighbours. Since $m = 2$, this will be enough to keep them from deviating. Thus it is a renegotiation-proof equilibrium.

Now look at the network on the right. The bottom-right agent can delete the link to the top-right agent and induces a network identical to the one on the left. Since the left network is a renegotiation-proof equilibrium, agents have to follow a continuation that is not Pareto dominated by it, by the definition of renegotiation-proof equilibria. In this case, it has to be network on the left itself. Then the bottom-right agent who deletes a link will lose access to only 1 agent and thus will delete the link. Therefore the network on the right is not a renegotiation-proof equilibrium. But note if all agents chooses the "grim trigger" strategy, the network on the right is indeed a Nash equilibrium.

In this section, I characterize the set of renegotiation-proof equilibria.

First, I propose a definition of "m,l-critical networks". Let $G(m,l) = \{g | \forall i, |N_i^l(g)| \geq m, or\, |N_i^l(g)| = 0\}$.

Definition. *A network g is called a m,l-critical network if (i) $g \in G(m,l)$; (ii) For any i and $ij \in g$, there doesn't exist $g' \subset g - ij$ such that $|N_i^l(g')| > |N_i^l(g)| - m$ and $g' \in G(m,l)$.*

Proposition 1. *Any nonempty network $g \in G(m,l)$ contains a nonempty m-l-critical network, and m-l-critical networks are renegotiation-proof.*

Definition. *Transitively m-l-critical networks are defined as follows. Let G_k denote the set of networks with k links. Given an integer m satisfying $(m-1)\frac{\delta p(v-c)}{1-\delta} < c < m\frac{\delta p(v-c)}{1-\delta}$, let $TC_k(m,l) \subset G_k(m,l)$ denote the set of m-l-transitively critical networks with k links.*

- *$TC_0(m,l) = \{\emptyset\}$.*
- *Inductively on k, $TC_k(m,l) \subset G_k$ is such that $g \in TC_k(m,l)$ if and only if for any i and $ij \in g$,*

there exists $g' \subset g - ij$ such that $g' \in TC_{k'}(m,l)$ for some $k' < k$, $|N_i^l(g')| \leq |N_i^l(g)| - m$, and there is no $g'' \in TC_{k''}(m,l)$ such that $g'' \subset g - ij$ and $|N^l(g'')| > |N^l(g')|$.

A network g is transitively m-l-critical if when some link ij of i's is deleted, then the next largest transitively m,l-critical network that is a subset of $g - ij$ involves i losing access to at least $m - 1$ more agents. Note that this definition doesn't involve any incentive, but only the structure of the networks themselves. This can be seen as an algorithm to find all the renegotiation-proof equilibria, as I will show next in Theorem 1.

Theorem 1. *A network is renegotiation-proof if and only if it is transitively m,l-critical.*

This theorem transforms the problem of incentives to a problem of the network structure itself and essentially can be used as an algorithm to find all renegotiation-proof networks.

5 Robustness

A desirable property of a network is that when an agent deletes a link, the network doesn't totally collapse to an empty network, unless it is small enough. Here I propose a robustness concept, which is an extension to the robustness concept in Jackson et al (2012).

Definition. *A network g is robust to degree r, iff it is renegotiation-proof and sustained as part of a pure strategy subgame perfect equilibrium with $g_0 = g$ such that in any subgame continuation from any renegotiation-proof $g' \subset g$, and for any i and $ij \in g'$, if i declines a favor when called upon and then ij is deleted and $g'' \subset g' - ij$ is reached with positive probability and then played in perpetuity in the continuation, then if $hl \in g'$ but $hl \notin g''$ then $h,l \in N_i^r(g') \cup \{i\}$.*

In words, in a network that is robust to degree r or any of its renegotiation-proof subnetworks, if a link ij is deleted, further deletion must be contained among i and i's r-neighbours.

One thing to note here is that, the robust equilibrium in Jackson et al (2012), the social quilts, are no long renegotiation-proof networks here, though under certain conditions, similar structures (decentralized union of centralized small groups) may be found in this setting too.

Definition. *A network g is called a star network if there exists an node i such that i is linked with all other agents and no other links exist.*

Denote a star network of size s with $SN(s)$. It is easy to see that a star network of size $mt + 1$ for some integer t is robust to degree 2 for any $l \geq 2$ (by simple induction on t).

Proposition 2. *Any network cannot be robust to degree 1 if $l > 1$.*

Hence, for $l > 1$, robust to degree 2 is the highest robustness possible. Actually, robust to degree 2 can be achieved for any l.

Theorem 2. *For $l \geq 4$, a network g is renegotiation-proof and robust to degree 2 if and only if g is $SN(mt+1)$ for some integer t.*

The star networks, when $l \geq 4$, are the only networks that are robust to degree 2. Also note that in a star network, all favors, between whichever two agents, are performed and thus the society achieves maximum social welfare. While the whole society being organized into a star network may seem unrealistic, note that this result is derived given the assumption of complete information. It is plausible to assume that for groups that are "small" enough for in-time and efficient information sharing, a star network is an efficient and robust structure. In Bala and Goyal (2000), they find that in the two-way case (undirected graph), limiting networks converge to either empty or center-sponsored star network. This result is consistent with my finding here.

6 Empirical Evidence

In this section, I explore the data used by Jackson et al (2012). The data is collected from 75 Indian rural villages through questionaires in a microfinance program. Here I focus on the assortativity coefficients (henceforth denoted AC) and support levels of the networks.

Assortativity measure the preference of nodes in a network to be linked with other nodes of similar degrees. The assortativity coefficient is the Pearson correlation coefficient of degree between pairs of linked nodes and it is given by $r = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma_q^2}$, where q_j is the distribution of the remaining degree (degree minus one) and e_{jk} is the joint probability distribution of the remaining degrees of the two vertices. The assortativity coefficient ranges from -1 (disassortative, i.e. high degree nodes tend to link with low degree nodes) to 1 (assortative, i.e. high degree nodes tend to link with high degree nodes). A social quilt based on Jackson et al (2012) has a positive assortativity coefficient and a star network has assortativity coefficient equal to -1.

A link ij in a network is called supported if there exist another node k which is a neighbour of both i and j. The support level of a network is the ratio of the number of supported links to the number of all links. This measure is introduced in Jackson et al (2012). A social quilt based on Jackson et al (2012) has a support level equal to 1 and a star network has a support level equal to 0.

Note that the dataset only contains information of approximately half of the households in each village. This may bias both measures and the direction of bias is unknown.

Now, I define the links in the data. There are 11 different relationships surveyed in the data and I define five types of links based on those. The 11 relations are as follows: borrowing money (henceforth denoted M1), lending money (M2), borrowing rice (F1), lending rice (F2), giving advice (A1), receiving advice (A2), visiting (V1), getting visits (V2), getting medical help (ME), going to temple together (T) and being related (R). I define five types of links as shown in table (1) below.

Link Type	Denotation	Description
Relative	R	R
Social	S	$M1 \cup M2 \cup F1 \cup F2 \cup A1 \cup A2 \cup V1 \cup V2 \cup ME \cup T$
Strict Social	SS	$(M1 \cap M2) \cup (F1 \cap F2) \cup (A1 \cap A2) \cup (V1 \cap V2) \cup ME \cup T$
Union	U	$S \cup R$
Strict Union	SU	$SS \cup R$

(1)

The links studied in my model fits the definition of non-strict links since agents don't necessarily reciprocate favors bilaterally. But the strict links, which are similar to the link definition used in Jackson et al (2012), have the advantage of reducing measurement error, since some villager may be mistaken about the relationships he/she has with another villager.

While I find that all the village has positive assortativity and high support level with all these types of links, I argue that the reason is that these networks are largely dominated by the relative relation. The following table shows the probability of each type of links exists for a related pair of agent compared to a unrelated pair (note for U and SU, the first row is 100% since relative relation is included in their definition). Related agents are far more likely to have a link of S and SS than unrelated agents. Related agents have more frequent interactions with each other and likely engage in risk sharing and even property sharing and etc beyond favor exchange. The relative networks demonstrate highly positive assortativity coefficients (average of 0.688 and median of 0.667) and high support levels (average of 0.9605 and median of 0.9611). It is impossible to retrieve a network that is purely built for social favor exchange and all the networks' assortativity coefficient and support levels are greatly skewed towards 1.

Link Type	S	SS	U	SU
Rel%	92.3%	90.4%	100%	100%
NonRel%	0.25%	0.16%	0.25%	0.16%

(2)

For groups of larger scale, limitation of information sharing may be a reason preventing such structure from emerging. For example, if two parts of a networks have no or little information flow, then they actually can be considered separate and structures like this can emerge from my setting: g_1 and g_2 share the node i, which is the center node of g_1 and a non-center node in g_2. Then the essential takeaways of a star network are that low-degree nodes are connected through high degree nodes (low assortativity) and few small loops exist (low support level). These two features determines that a star network efficiently diffuses information with minimum number of links. From here I predict that social links will skew networks' assortativity coefficient towards -1 and support levels towards 0.

Prediction I. All the networks constructed by social links have a lower assortativity than the network induced by the relative links. In other words, favor-related links skew the network's assortativity measure towards -1.

The following table shows the average and median of the assortativity ratio of each of the four types of networks to the relative link network.

Link Type	R	S	SS	U	SU
Average #Links	2620.4	3587	3095.2	3784.9	3347.5
Mean	0.6880	0.3732	0.4846	0.3560	0.4302
Median	0.6666	0.3556	0.4628	0.3425	0.3966
Standard Deviation	0.1250	0.0884	0.1292	0.0782	0.1155
Average Ratio to R	-	0.5456	0.7019	0.5220	0.6225
Median Ratio to R	-	0.5417	0.7051	0.5185	0.6218
SD of Ratio to R	-	0.0914	0.1056	0.0866	0.0871

(3)

Table (3) shows that networks of links S, SS, U and SU demonstrate a lower assortativity coefficient than R. In particular, this is true for each of the 75 villages for S, SS, U and SU.

Notice that the R networks have less links than the others. Amongst all the R networks, the correlation coefficient of assortativity coefficients and link numbers is 0.5923. Therefore the difference of assortativity is not simply due to the difference in number of links.

The following table shows regressions of assortativity coefficients on number of villagers, number of links and subsets of dummy variables for S, SS, U and SU , with fixed effect for each village.

	#Villagers	#Links	S	SS	U	SU
I	0.0637^{**}	-0.0524^{**}	-0.2641^{**}	-0.1785^{**}	-0.2710^{**}	-0.2197^{**}
	(0.0047)	(0.0134)	(0.0150)	(0.0100)	(0.0174)	(0.0124)
II	0.0683^{**}	0.0656^{**}	-0.2513^{**}	-0.1722^{**}	-	-
	(0.0075)	(0.0202)	(0.0215)	(0.0131)	-	-
III	0.0623^{**}	-0.0485^{**}	-	-	-0.2754^{**}	-0.2225^{**}
	(0.0063)	(0.0177)	-	-	(0.0223)	(0.0155)

(4)

(**: significant at 99% confidence. #Villagers are of units of 100. #Links is of units of 1000.)

As shown in Table (4) above, each of the dummy variables has a negative regression coefficient with 99% significance.

Prediction II. All the networks constructed by the social links have a lower support level than the network induced by the relative links. In other words, favor-related links skew the network's support level towards 0.

The following table shows the average and median of the support level ratio of each of the four types of networks to the relative link network.

Link Type	R	S	SS	U	SU
Average #Links	2620.4	3587	3095.2	3784.9	3347.5
Mean	0.9605	0.9069	0.8923	0.9170	0.9080
Median	0.9611	0.9074	0.8939	0.9169	0.9071
Standard Deviation	0.0140	0.0182	0.0195	0.0172	0.0170
Average Ratio to R	-	0.9443	0.9291	0.9548	0.9454
Median Ratio to R	-	0.9440	0.9311	0.9552	0.9462
SD of Ratio to R	-	0.0166	0.0207	0.0147	0.0156

(5)

Table (5) above shows that networks of links S, SS, U and SU demonstrate a lower support level than R. In particular, this is true for each of the 75 villages for S and U and 74 out of 75 for SS and SU.

Again the R networks have less links than the others. Amongst all the R networks, the correlation coefficient of support levels and link numbers is 0.4131. Therefore the difference of support level is not simply due to the difference in number of links.

The following table shows regressions of support levels on number of villagers, number of links and subsets of dummy variables for S, SS, U and SU, with fixed effect for each village.

	#Villagers	#Links	S	SS	U	SU
I	0.0567^{**}	-0.0061^{**}	-0.0476^{**}	-0.0653^{**}	-0.0363^{**}	-0.0480^{**}
	(0.0009)	(0.0026)	(0.0030)	(0.0020)	(0.0034)	(0.0024)
II	0.0584^{**}	-0.0113^{**}	-0.0426^{**}	-0.0628^{**}	-	-
	(0.0015)	(0.0043)	(0.0046)	(0.0028)	-	-
III	0.0584^{**}	-0.0110^{**}	-	-	-0.0307^{**}	-0.0445^{**}
	(0.0011)	(0.0030)	-	-	(0.0038)	(0.0026)

(6)

(**: significant at 99% confidence. #Villagers are of units of 100. #Links is of units of 1000.)

As shown in Table (6) above, each of the dummy variables has a negative regression coefficient with 99% significance.

Therefore social networks, both with and without relative links, demonstrate a lower assortativity coefficient and support level and both predictions are not rejected.

7 Conclusion

In this paper I extend Jackson et al's model by allowing agents to seek favor indirectly through a chain of contacts. While the algorithm for finding renegotiation-proof equilibria remains analogous, the resulting renegotiation-proof equilibria class becomes vastly different. Social quilts are no longer renegotiation-proof networks and their robustness criteria (robust to degree 1) is not feasible for any $l > 1$.

In this setting, however, decentralized unions of smaller units still constitute an important class of renegotiation-proof networks. Among them, one particular class, star networks demonstrate the lowest degree of robustness (most robust) and full efficiency. This is consistent with results from Bala and Goyal (2000) and also matches observation of groups of relatively small sizes. As for groups of larger scale, limitation of information sharing may be a reason preventing such structure from emerging. For example, if two parts of a networks have no information flow, then they actually can be considered separate and structures like this can emerge from my setting: g_1 and g_2 share the node i, which is the center node of g_1 and a non-center node in g_2.

My empirical analysis shows that networks of social links are less assortative than the relative networks, and also have lower support levels.

Meanwhile, other refinement criteria may lead to other interesting and tractable structure. Equilibria with information restriction may look drastically different. The implication of relaxing these assumptions and refinements calls for future investigations.

8 Appendix

Proof of Proposition 1.

Let g' be a smallest nonempty network (in the sense of set inclusion) that is a subset of g and lies in $G(m,l)$. Such a network exists (possibly g itself) since $g \in G(m,l)$ and is thus m,l-critical. The second statement follows Theorem 1, which is proven below. **QED.**

Proof of Theorem 1.

The proof is similar to the proof of Theorem 1 in Jackson et al (2012).

Fix m and l. Then in the following proof I will simplify notations involving m and l.

I first show that if $g \in RPN_k$ then $g \in TC_k$. I prove by induction in k. The result is obvious when $k = 0$. So, suppose that it holds through $k-1$ and consider some k.

Given a network $g \in RPN_k$, by the definition it follows that g is sustainable on the equilibrium path. So for any i and $ij \in g$, if i is called upon to do a favor for j and declines, then at least one possible continuation must lead to a network $g' \subset g - ij$ such that $g' \in RPN_{k'} = TC_{k'}$, $|N_i^l(g')| \leq |N_i^l(g)| - m$, and there is no $g'' \subset g - ij$ such that $g'' \in RPN_{k''}$ and $|N^l(g'')| > |N^l(g')|$. Otherwise, then if i did not perform the favor, he or she would save the cost c and lose access to at most $m-1$ agents in any continuation. Thus, i would benefit from deviating and not performing the favor since $(m-1)\frac{\delta p(v-c)}{1-\delta} < c < m\frac{\delta p(v-c)}{1-\delta}$, which contradicts the fact that g is sustained as a renegotiation-proof equilibrium. Thus, for every i and ij, there exists $g' \subset g - ij$ such that $g' \in TC_{k'}$ for some k', $|N_i^l(g')| \leq |N_i^l(g)| - m$, and there is no $g'' \subset g - ij$ such that $g'' \in TC_{k''}$ and $|N^l(g'')| > |N^l(g')|$. Therefore, $g \in TC_k$.

Next, I show that if $g \in TC_k$ then $g \in RPN_k$. Again, I prove this by induction on the number of links in a network. Note in this part, strategies need to be considered. As such, I work with a stronger induction hypothesis, with the induction indexed by k, number of links in the network. The induction hypothesis is that starting from any node and any $g_0 \in G_k$, there exists a pure strategy subgame perfect equilibrium continuation such that (i) there is a unique network $g_1 \in RPN_{k_1}$ for some $k_1 \leq k$ that is reached and sustained in perpetuity in the continuation, with $g_1 = g_0$ if $g_0 \in TC_k$, such that (ii) on the equilibrium continuation path a favor is performed if and only if it corresponds to a link in g_1; and (iii) in any subgame starting with some network $g' \in G_{k'}$ with $k' \leq k$ if g'' is played in perpetuity with some probability in the continuation then $g'' \in RPN_{k''}$ for some k'' and there does not exist any $g''' \subset g'$ such that $g''' \in RPN_{k'''}$ and $u_i(g''') \geq u_i(g'')$ for all i with strict inequality for some i.

First, note that $RPN_0 = \{\emptyset\} = TC_0$. Also note that starting from $g_0 = \emptyset$ there is a unique subgame perfect equilibrium continuation (no favors can be supplied and no links can be sustained) and so it follows directly that conditions (i)–(iii) are satisfied.

Now assume that the induction hypothesis holds for all $k' < k$. I show that the same is true for k. Begin with the case such that $g_0 = g \in TC_k$. On the equilibrium path, have all agents maintain all links (so $L_i(g_t) = N_i^1(g_t)$ whenever $g_t = g_0 = g$) and perform all favors. The off-the-equilibrium-path strategies are described as follows. If an agent i is called upon to provide a favor for an agent j such that $d(i,j) \leq l$ and does not do the favor, then the continuation is as follows. Given that $g \in TC_k$, by the definition of transitive m,l-criticality, there exists $g' \subset g - ij$ such that $g' \in TC_{k'} = RPN_{k'}$, $|N_i^l(g')| \leq |N_i^l(g)| - m$ and there is no $g'' \subset g - ij$ such that $g'' \in TC_{k''}$ for any k'' and $|N^l(g'')| > |N^l(g')|$. Denote this network by $g(i,j) = g'$. Following i's failure to perform a favor to j, let the continuation be such that $L_h(g - ij) = N_h^1(g(i,j))$ for all h. This results in the network$g(i,j) \in RPN_{k'}$ following the link announcement stage, and so from then on there is a pure strategy subgame perfect equilibrium sustaining $g(i,j)$ and satisfying (i)–(iii) by the induction step, and so let agents play the strategies corresponding to such an equilibrium in that continuation. At all other nodes off the equilibrium path for which strategies are not already specified the nodes are necessarily at a network with fewer links, and so pick a pure strategy equilibrium continuation that satisfies (i)–(iii), which is possible by the induction hypothesis. This satisfies (i)–(iii) by construction.

To check that this is a subgame perfect equilibrium, by the specification of the strategies above, it suffices to only check that no agent wants to deviate from the equilibrium path, and also that following some i's failure to provide a favor to j, no agent h wants to deviate from $L_h(g - ij) = N_h^1(g(i,j))$. By construction, an agent i who is called upon to do a favor for an agent j and deviates will end up losing access to at least m agents, and this cannot be an improving deviation. Next, consider, some agent h's incentive to deviate from $L_h = N_h^1(g_0)$ if g_0 is still in play, or else from $L_h(g - ij) = N_h^1(g(i,j))$ following some i's failure to provide a favor to j. By not deviating, the agent gets the payoff from g_0 or $g(i,j)$ in perpetuity. By deviating, the agent h will end up with a continuation starting from a network $g'' \subset g_0$ or $g'' \subset g(i,j)$, respectively, where the agent has not gained any access and may have lost some access. Since each link has a positive future expected value, this cannot be an improving deviation.

Next, I show that from any node in the subgame from some initial $g_0 \notin TC_k$ there exists a pure strategy subgame perfect equilibrium continuation satisfying (i)–(iii). There are two types of nodes to consider. One is a node at which some agent i is called upon to provide a favor for an agent j such that $d(i,j) \leq l$, and another is a node where agents announce the links they wish to sustain.

First, consider starting at g_0 and a node where agents announce the links that they wish to sustain. Find some g' that has the maximal $k' < k$ of links such that $g' \in RPN_{k'}$ and $g' \subset g_0$. For each h set $L'_h = N_h^1(g')$ and then from g' play a continuation satisfying (i)–(iii) (by the induction step). If any agent deviates, to L such that $L'_h \subset L$, then play the same continuation, as this will not affect the network formed. Otherwise, the continuation will lead to some g'' with strictly fewer links for l and since $g' \in RPN$, the continuation will necessarily result in a lower expected continuation payoff. This establishes the claim for this type of nodes.

Next, consider a node at which some agent i is called upon to provide a favor for an agent j such that $d(i,j) \leq l$. There are two cases that can follow: one where i performs the favor and so the resulting network is then g_0. In that case, I have just shown that there is a pure strategy subgame perfect equilibrium continuation satisfying (i) to (iii). Let g´ be the network sustained on the equilibrium path in one of these that has the largest $|N_i^l(g')|$ for i. If i does not perform the favor, then $g-ij \in G_{k-1}$ is reached. By the induction hypothesis again there is a pure strategy subgame perfect equilibrium continuations satisfying (i)–(iii), and let g'' be a network sustained by one of these that has the largest $|N_i^l(g')|$ for i. Now, based on those two continuations, let i choose a pure strategy best response. The claim follows. **QED**.

Proof of Proposition 2.

Assume that a network g is robust to degree 1 with $l > 1$. By Proposition 1, there exist a smallest m,l-critical network $g' \subseteq g$ and there doesn't exist any m,l-critical network g^* such that $g^* \subset g'$. Since g is robust to degree 1, so is g'. Now if any agent i in g' deletes a link, all links are lost. Since g' is robust to degree 1, only links that can be deleted are links among i's 1-neighbors. Therefore all agents in g' must be linked with each other (since g's is minimally critical and doesn't contain isolated nodes). Now delete any link ij and the remaining network g" is still in $G(m,l)$ and therefore must contain a nonempty m,l-critical network that is a strict subset of g'. This contradicts with the fact that g' is a smallest m,l-critical network. **QED.**

Proof of Theorem 2.

First, we show that if g is a star network of size $mt+1$ for some integer t then g is renegotiation-proof and robust to degree 2.

We prove this by induction that $SN(mt+1)$ are renegotiation-proof and star network of all other sizes are not. For $t=1$, the assumption clearly holds. Assume for all $t < t'$, the assumption holds.

Now consider t'. For any star network of size s where $m(t'-1)+1 < s < mt'+1$, the center agent can delete any link and then reach $SN[m(t'-1)+1]$. He will lose access to at most m-1 agents and therefore such star network is not renegotiation-proof. For star network of size $mt'+1$, if the center agent deletes any link, then the subsequent continuation will be $SN[m(t'-1)+1]$ and i will lose access to m agents. Then other agents will not delete any link either. Thus $SN(mt'+1)$ is renegotiation-proof and the induction assumption holds for t'.

Second, we show that if a network g is renegotiation-proof and robust to degree 2 then g must be $SN(mt+1)$ for some integer t.

First, we consider any minimally critical network g_c that is robust to degree 2. By robustness to degree 2, there can't exist any $i \neq j$ such that $distance(i,j) > 2$. Also obviously $g_c \in G(m,l)$.

Fix linked agent i,j .

If $N_i^1(g_c) \cap N_j^1(g_c) \neq \emptyset$, then suppose $k \neq i,j$ such that $k \in N_i^1(g_c) \cap N_j^1(g_c)$. Then consider $g_c - ij$. The distance between any two nodes will increase by at most 1. Since $l \geq 4 > 2+1$, $g_c - ij \in G(m,l)$ and by **Proposition 1,** $g_c - ij$ contains a m,l-critical network, which contradicts with the fact that g_c is a minimally critical network.

Now suppose $N_i^1(g_c) \cap N_j^1(g_c) = \emptyset$. If that there exists $k,r \neq i,j$ such that $k \in N_i^l(g_c)$ and $r \in N_j^l(g_c)$, then again consider $g_c - ij$. The distance between any two nodes will increase by at most 2. Since $l \geq 4 = 2+2$, $g_c - ij \in G(m,l)$ and by **Proposition 1,** $g_c - ij$ contains a m,l-critical network, which contradicts with the fact that g_c is a minimally critical network.

Therefore only one of $N_i^l(g_c)$ and $N_j^l(g_c)$ can be nonempty. Suppose $N_i^l(g_c) \neq \emptyset$. Then $\forall k$ such that $k \in g_c$ and $k \neq i$, $N_k^l(g_c)$ must be empty. Then obviously g_c is $SN(m+1)$ since the only critical star-networks are $SN(m+1)$.

Therefore if a network g is renegotiation-proof and robust to degree 2 then g must only contain minimally critical networks of $SN(m+1)$.

Suppose g contains a $SN(m+1)$ subnetwork g_c. If g_c has links to outside is on its non-center node, then suppose one of those links is jk with $j \in g_c$ and $i \in g_c$ is another non-center node. Then $(g_c - i) \cup jk \in G(m,l)$ and it must contain a m,l-critical network, which cannot be $SN(m+1)$. This contradicts with the conclusion above.

Now suppose g contains a $SN(m+1)$ subnetwork and none of the $SN(m+1)$ subnetwork(s) has links to outside on its non-center node. Then g must be a star network. By the proof above, a star network can be renegotiation-proof iff it is $SN(mt+1)$ for some integer t. Therefore, g must be $SN(mt+1)$ for some integer t. **QED.**

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