

Important concept is the risk premium consumers are willing to pay to avoid an uncertain income  $\tilde{y}$  or payment.

$$\tilde{y} = \bar{y} + \varepsilon, \text{ with } E(\varepsilon) = 0, E(\varepsilon^2) = \sigma^2$$

$U(y)$  is monotonically increasing and concave,

$U'(y) > 0$  (marginal utility of income is positive),

$U''(y) < 0$ , risk averse  $\Leftrightarrow$  diminishing marginal utility of income

Important question: what is the value of the risk premium  $k$  such that

$$U(\bar{y} - k) \equiv E\{U(\tilde{y})\}$$

$\bar{y} - k = y^{ce}$  is also called the certainty equivalent income.

Now  $k$  will be relatively small compared to  $\bar{y}$ , whereas  $\varepsilon$  will have a wider variation. Hence use a first order approximation of  $U()$  for the left hand side, and a second order Taylor series approximation on the right hand side. Here we replace the identity symbol ( $\equiv$ ) with the approximation symbol ( $\cong$ )

$$U(\bar{y}) + k U'(\bar{y}) \cong E\{U(\bar{y}) + \varepsilon U'(\bar{y}) + \frac{\varepsilon^2}{2} U''(\bar{y})\}$$

Moving the expectation operator inside the brackets and using  $E(\varepsilon) = 0, E(\varepsilon^2) = \sigma^2$

$$U(\bar{y}) + k U'(\bar{y}) \cong U(\bar{y}) + 0 + \frac{\sigma^2}{2} U''(\bar{y})$$

Solving for  $k$

$$k \cong -\frac{\sigma^2 U''(\bar{y})}{2U'(\bar{y})} \equiv \frac{\sigma^2}{2} R$$

Where  $R = -U''/U'$  is a measure of absolute risk aversion (ARA), originally defined by Pratt (1964).

It is readily verified that the exponential utility function (made famous by Tobin for stock return analysis) has the property of constant absolute risk aversion (CARA).  $U(y) = 1 - \exp(-\alpha y)$

Also useful is the concept of relative risk aversion (RRA) =  $ky = yU''/U'$ . Some functions have the property of constant relative risk aversion (CRRA).