The Origins of Aggregate Fluctuations in a Credit Network Economy*

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October 16, 2016

Abstract

I show that inter-firm lending plays an important role in business cycle fluctuations. I first build a network model of the economy in which trade in intermediate goods is financed by supplier credit. In the model, a financial shock to one firm affects its ability to make payments to its suppliers. The credit linkages between firms then transmit financial shocks across the economy, amplifying their effects on aggregate output. To calibrate the model, I construct a proxy of inter-industry credit flows from firm- and industry-level data. Counterfactual exercises suggest these credit network effects can be a powerful amplification mechanism. I estimate aggregate and idiosyncratic shocks to industries in the US and find that financial shocks are a prominent driver of observed cyclical fluctuations: more than two-thirds of the drop in industrial production during the Great Recession is accounted for by financial shocks. Furthermore, idiosyncratic financial shocks to a few key industries can explain a considerable portion of these effects. In contrast, while productivity shocks played a meaningful role before 2007, they had a decidedly negligible impact during the Great Recession.

*I am very grateful to my advisors Stefania Garetto, Simon Gilchrist, and Adam Guren for their guidance. I also thank Giacomo Candian, Maryam Farboodi, Mirko Fillbrunn, Illenin Kondo, Fabio Schiantarelli, and participants of the BU macro workshop, Federal Reserve Board seminars, and BU-BC Green Line Macro Meeting, INET seminar at Columbia University, Econometric Society NASM, and EEA-ESEM for comments which substantially improved this paper. All errors are my own.

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Introduction

The recent financial crisis and ensuing recession have underscored the importance of external finance for the real economy. Generally, firms obtain most of their short-term external financing from their suppliers, in the form of delayed payment terms for their purchases. In spite of its importance, the aggregate implications of these lending relationships remain poorly understood.

In this paper, I show that inter-firm lending plays an important role in business cycle fluctuations. To this end, I introduce supplier credit into a network model of the economy and show, analytically and quantitatively, that the credit network of an economy amplifies the effects of financial shocks. I then use my framework to empirically shed light on the origins of observed business cycle fluctuations in the US.

My approach involves three steps. First, I provide intuition with a stylized model in which trade in intermediate goods is financed by supplier credit. In this model, a shock to one firm's liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects. Second, I calibrate the model to assess the quantitative importance of these credit network effects. For this, I construct a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data. Finally, I estimate shocks to these industries using two empirical approaches: by estimating an identified VAR, and by performing a structural factor analysis of the model. Accounting for the propagation mechanisms of credit and input-output interlinkages reveals the central importance of financial shocks as a driver of US business cycle fluctuations.

The credit linkages that I model take the form of trade credit, or delayed payment terms, that suppliers of intermediate goods often extend to their customers. Trade credit is the single most important source of short-term external finance for firms, and facilitates most inter-firm trade. In the US, trade credit was three times as large as bank loans and fifteen times as large as commercial paper outstanding on the aggregate balance sheet of non-financial corporations in 2012.[1] In most OECD countries, trade credit accounts for more than half of firms' short-term liabilities and more than one-third of their total liabilities.[2] All of these facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the financial distress of their customers. Anecdotal evidence has long suggested that when firms play a dual

[1]During the Great Recession, the dry-up of trade credit was comparable to that of bank lending, with a peak-to-trough decline of about 25 percent. See the Federal Reserve Board's Flow of Funds.

[2]A large empirical literature documents the pervasiveness of trade credit. Generally, trade credit contracts are last for as short as 15 days to as long as several months. See Petersen and Rajan (1997) for more information.

role of supplier and creditor, delays in payment may transmit financial distress from firms to their suppliers.[3] There is growing evidence to suggest that this intuition is empirically relevant. A number of studies - including Jacobson and von Schedvin (2015), Boissay and Gropp (2012), Raddatz (2010), and Kalemli-Ozcan et al. (2014) - have found that firm- and industry-level trade credit linkages propagate financial shocks from firms to their suppliers. In spite of this evidence, the macroeconomic implications of trade credit have been largely overlooked in the literature. I therefore develop a framework for understanding how inter-firm trade and credit interact in response to changing credit conditions.

I consider an economy similar to that of Bigio and La'O (2016), in which firms are organized in a production network and trade intermediate goods with one another. Limited enforcement problems require firms to make cash-in-advance payments to their suppliers before production takes place. However, firms can delay part of these payments by borrowing from their suppliers. I assume that, to obtain this credit, a firm can credibly pledge some fraction of its future cash flow to repay its suppliers. Importantly, this implies that the cash-in-advance payments collected by each firm are endogenous to the model, and depend on the prices of its customers' goods. As it turns out, endogenous changes in firms' cash-in-advance constraints are crucial for how the economy behaves in response to shocks.

When one firm is hit with an adverse shock to its cash on hand, there are two channels by which other firms in the economy are affected. First is the standard input-output channel, which has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2016): the shocked firm cuts back on production, reducing the supply of its good to its customers. Second is a new credit linkage channel which tightens the financial constraints of upstream firms. That is, when the shocked firm cuts back on production, the price of its good rises. This increases the collateral value of its future cash flow, allowing the firm to reduce the cash-in-advance payments it makes to its suppliers. Being more cash-constrained, these suppliers may be forced to cut back on their own production, and reduce the CIA payments to their own suppliers (and so on and so forth). In this way, these credit network effects amplify the firm-level shock in a manner which depends on the underyling structure of the credit network.

Next, I evaluate the quantitative relevance of the mechanism. In order to calibrate the model, I first construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis. With this, I produce a map of the credit network of the US economy at the three-digit NAICS level of detail. I calibrate the model to match this proxy and the input-output matrices of the US. I also allow for substitutability between cash and bank credit, so

[3] For example, the government bailout of the US automotive industry in 2008 was precipitated by an acute shortage of liquidity, which came about largely due to extended delays in payment for goods already delivered.

that firms can partially offset a loss in customer payments with increased bank borrowing.

Counterfactual exercises reveal that the propagation mechanism is quantitatively significant - in response to an aggregate financial shock, the credit network of the US amplifies the fall in GDP by 28 percent. Furthermore, the aggregate impact of an idiosyncratic (industry-level) financial shock depends jointly on the underlying structures of the credit and input-output networks of the economy. Based on this analysis, certain industries emerge as systemically important to the US economy.

In the empirical part of the paper, I use this theoretical framework to investigate which shocks drive cyclical fluctuations once we account for the network effects created by credit interlinkages. My framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks. To address these issues, I use two methodological approaches.

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data. To do this, I first construct quarterly measures of bank lending based on data from Call Reports collected by the FFIEC. I then augment an identified VAR of macro and monetary variables with this measure of bank lending, and with the excess bond premium (EBP) of Gilchrist and Zakrajsek (2012), which reflects the risk-bearing capacity of the financial sector. I then construct financial shocks as changes in bank lending which arise from orthogonalized innovations to the EBP. [4] For productivity shocks, I use the quarterly, utilization-adjusted changes in TFP estimated by Fernald (2012).

Feeding these estimated shocks into the model, I find that, before 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production (IP). However, during the Great Recession, productivity shocks had virtually no adverse effects on IP - in fact, they actually *mitigated* the downturn. On the other hand, two-thirds of the peak-to-trough drop in aggregate IP during the recession can be accounted for by financial shocks, with the remainder unaccounted for by either shock. In addition, the credit network of these industries played a quantitatively important role in exacerbating the effects of financial shocks, accounting for nearly a fifth of the fall in IP.

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster, Sarte, and Watson (2011), using data on the output and employment growth of US IP industries. I

[4] I construct the measure of bank lending in such a way that changes in the demand for bank lending are largely netted out. Therefore, changes in my measure of bank lending mostly reflect supply-side changes.

first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use standard factor methods to decompose each of these shocks into an aggregate component and an idiosyncratic component.

Through variance decomposition I show that, while the idiosyncratic component of productivity shocks can explain a substantial fraction of aggregate volatility before 2007, it played virtually no role during the Great Recession. Rather, nearly three-quarters of the drop in IP during the recession can be accounted for by aggregate financial shocks. In addition, the remainder can be accounted for by idiosyncratic financial shocks to a few systemically important IP industries - namely the oil and coal, chemical, and auto manufacturing industries. Furthermore, the credit and input-output linkages between industries played a significant role in propagating these industry-level shocks across the economy.

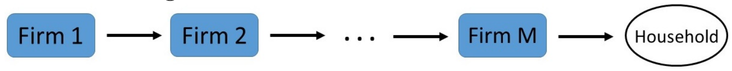
The broad picture which emerges from these two empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession. While shocks to aggregate TFP have long been relied upon as a principal source of cyclical fluctuations, the lack of direct evidence for such shocks has raised questions about their empirical viability. On the other hand, the credit and input-output interlinkages of firms can create a powerful mechanism by which a shock to one firm's financial constraint propagates across the economy. When we account for this amplification mechanism, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

Related Literature

This paper contributes to several strands of the literature. A growing literature examines the importance of network effects in macroeconomics, including Acemoglu et al. (2012), Shea (2002), Dupor (1999), Horvath (2000), Acemoglu et al. (2015), Baqaee (2016), and Carvalho and Gabaix (2013). These abstract away from financial frictions. The notable work of Bigio and La'O (2016), explore the interaction between financial frictions and the input-output structure of an economy. However, they do not explicitly model any credit relationships between firms. Luo (2016) embeds an input-output structure in the framework of Gertler and Karadi (2011), with a role for trade credit. However, trade credit linkages do not propagate shocks across the economy *per se*.[5] Kiyotaki and Moore (1997) study theoretically how a shock to a firm in a credit chain can cause a cascade of defaults in a partial equilibrium framework. Gabaix (2011), Foerster et al. (2011), and Stella (2014) evaluate the contribution of idiosyncratic shocks to aggregate fluctuations, the latter two using a structural

[5] Credit linkages only affect the interest rate that the bank charges firms. As such, all network effects are due to input-output linkages, as in Bigio and La'O (2016).

Figure 1: Vertical Production Chain



factor approach. Jermann and Quadrini (2012) evaluate the importance of financial shocks by explicitly modelling the tradeoff between debt and equity financing.

The rest of the paper is organized as follows. In section I, I introduce the stylized model and derive the analytical results. In sections II-IV, I generalize the production network structure, discuss the construction of my proxy for credit flows and calibration, and summarize the quantitative results. In section V, I perform the empirical analyses.

I. Stylized Model: Vertical Production Structure

In this section, I build intuition with a simple model. The stylized nature of the production structure of the economy permits closed-form expressions for equilibrium variables. I will later generalize both the production structure and preferences.

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are M goods, each produced by a continuum of competitive firms with constant returns-to-scale in production. We can therefore consider each good as being produced by a representative, price-taking firm. Each good can be consumed by the household or used in the production of other goods.

The representative household supplies labor competitively to firms and consumes a final consumption good. . It has preferences over consumption C and labor N given by $U(C, N)$, and a standard budget constraint, where w denotes the competitive wage earned from working, and π_i the profit earned by firm i.

$$U(C, N) = log\, C - N \qquad C = wN + \sum_{i=1}^{M} \pi_i \tag{1}$$

There are M price-taking firms who each produce a different good, for now arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with $i = M$ denoting the producer of the final good.

The production technology of firm i is Cobb-Douglas over labor and intermediate goods, where x_i denotes firm i's output, n_i its labor use, and x_{i-1} its use of good $i-1$, z_i denotes firm i's total factor productivity, η_i the share of labor in its production (and $\eta_1 = 1$), and $\omega_{i,i-1}$ the share of good $i-1$ in firm i's total intermdiate good use (equal to 1 for now). Let p_s denote the price of good s.

$$x_i = z_i n_i^{\eta_i} x_{i-1}^{\omega_{i,i-1}(1-\eta_i)} \tag{2}$$

Limited enforcement problems between firms create a need for *ex ante* liquidity to finance working capital. The household cannot force any debt repayment. Therefore, firm i must pay the full value of wage bill, wn_i, up front to the household before production takes place. In addition, each firm i must pay for its intermediate goods purchases, $p_{i-1}x_{i-1}$ up front to its supplier. Thus, firms are required to have some funds at the beginning of the period before any revenue is realized.

Firm i can delay payment to its supplier by borrowing some amount τ_{i-1} from its supplier, representing the trade credit loan given from $i-1$ to i. In addition, each firm can obtain a cash loan b_i from the bank. The net payment that firm $i-1$ receives from its customer at the beginning of the period is therefore $p_{i-1}x_{i-1} - \tau_{i-1}$. Firm i's cash-in-advance constraint takes the form

$$\underbrace{wn_i}_{wage\ bill} + \underbrace{p_{i-1}x_{i-1} - \tau_{i-1}}_{CIA\ payment\ to\ supplier} \leq \underbrace{b_i}_{bank\ loan} + \underbrace{p_i x_i - \tau_i}_{CIA\ from\ customer} \tag{3}$$

Thus, the cash that firm i is required to have in order to employ n_i units of labor and purchase x_{i-1} units of intermediate good $i-1$, is bounded by the amount of cash that firm i can collect at the beginning of the period. Note that trade credit appears on both sides of the constraint.

Firms face borrowing constraints on the size of loans they can obtain from their suppliers and the bank. Firm i can obtain the loan b_i from the bank at the beginning of the period by pledging a fraction B_i of its total end-of-the-period revenue $p_i x_i$, and a fraction α of its accounts receivable τ_{i+1}, where $\alpha \,\epsilon\, (0, 1]$.[6]

$$b_i \leq B_i p_i x_i + \alpha \tau_i \tag{4}$$

Firms are also constrained in their ability to obtain trade credit from their suppliers. In particular, firm i can credibly pledge a fraction θ_i of its end-of-the-period revenue to repay its supplier.

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{5}$$

[6] I will later show that α parameterizes the degree of substitutability between cash and bank credit.

How do firms choose how much to lend to their customers and borrow from their suppliers? Recall that representative firm i is actually comprised of a continuum of competitive firms with CRS production. Perfect competition amongst these suppliers forces them to offer their customers the maximum amount of trade credit permitted by the constraint. This result holds even when these suppliers are cash-constrained in equilibrium.[7] (I leave the proof of this to an online appendix.) While this pins down the supply of trade credit, I study firms' demand for trade credit below.

We can re-write firm i's cash-in-advance constraint as

$$wn_i + p_{i-1}x_{i-1} \leq \underbrace{\chi_i p_i x_i}_{liquid\ funds} \tag{6}$$

where

$$\chi_i \equiv \underbrace{\frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i}}_{debt/revenue\ ratio} + \underbrace{1 - \frac{\tau_i}{p_i x_i}}_{cash/revenue\ ratio} \tag{7}$$

Therefore, a firm's expenditure on inputs is bounded by the amount of funds it has at the beginning of the period. The variable χ_i describes the tightness of firm i's cash-in-advance constraint, and will play a key role in the mechanism of the model. The tightness of a firm's cash-in-advance constraint is comprised of the firm's debt-to-revenue ratio and its cash-to-revenue. These describe how much of the firm's revenue is financed by debt, and how much of its revenue is collected as a cash-in-advance payment, respectively. Notice that χ_i is decreasing in $\frac{\tau_i}{p_i x_i}$, the amount of i's output sold on credit: the more credit that i gives its customer, the less cash it collects at the beginning of the period.

Firm i chooses its input purchases n_i and x_{i-1}, and how much trade credit to borrow τ_{i-1}, to maximize its profits subject to its cash-in-advance constraint. (Recall that because of perfect competition, the firm takes its trade credit lending τ_i as given.)

$$\max_{n_i,\, x_{i-1},\, \tau_{i-1}} \quad p_i x_i - wn_i - p_{i-1}x_{i-1}$$

$$s.t. \;\; wn_i + p_i x_{i-1} \quad \leq \chi_i(\tau_{i-1}) p_i x_i \tag{8}$$

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{9}$$

Denote by τ^*_{i-1} firm i's choice of how much trade credit to borrow from its supplier. I show in online appendix O.A1 that if firm i's cash-in-advance constraint (8) is binding in equilibrium, then it borrows the maximum amount of trade credit offered by its supplier, pinning down $\tau^*_{i-1} = \theta_i p_i x_i$. For much of this paper, I consider this more interesting case in which firms are

[7]These results are supported by micro-level evidence on trade credit: competition amongst suppliers is is often sufficiently high that they are forced to offer their customers extended payment terms, even when they are cash-constrained. See, for instance, Barrot (2015).

constrained in equilibrium.[8]

If firms are constrained in equilibrium, we can re-write the tightness χ_i of a firm's constraint using firms' binding borrowing constraints to replace τ_i and b_i.

$$\chi_i = \underbrace{B_i + \theta_i}_{debt/revenue\ ratio} \quad \underbrace{+1 - (1-\alpha)\theta_{i+1}\frac{p_{i+1}x_{i+1}}{p_i x_i}}_{cash/revenue\ ratio} \tag{10}$$

Crucially, equation (4) shows that χ_i is an *equilibrium object* - it is an endogenous variable which depends on the firm's forward credit linkage θ_{i+1} and the revenue of its customer.[9] Hence, changes in the price of its customer's good affect the tightness of firm i's cash-in-advance constraint. [10] Here, the endogeneity of χ_i will be a critical determinant of how the economy responds to shocks.

Firm i's optimality conditions equate the ratio of expenditure on each type of input with the ratio of their share of production. I show in Appendix A3 that firm i's cash-in-advance constraint (3) binds in equilibrium if and only if $\chi_i < 1$. Combining the first order conditions with the cash-in-advance constraint yields the optimality conditions below.[11]

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}\ , \qquad p_{i-1} = \phi_i \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}} \tag{11}$$

Here, $\phi_i \equiv \min\ \{1,\ \chi_i\}$ describes firm i's shadow value of funds.[12] ϕ_i is strictly less than one if and only if firm i's cash-in-advance is binding in equilibrium. Equations (5) says that, if binding, the cash-in-advance constraint inserts a wedge $\phi_i < 1$ between the marginal cost and marginal benefit of each input, representing the distortion in the firm's input use created by the constraint. A tighter cash-in-advance (lower χ_i) corresponds to a greater distortion, and lower output. Through χ_i , ϕ_i *endogneously* depends on shadow value funds of downstream firms ϕ_{i+1}, reflecting that firms' constraints are interdependent due to trade credit.

Note that there are two types of interlinkages between firms: input-output linkages, represented by input shares $\omega_{i,i-1}$ in production; and credit linkages, represented by the borrowing limits θ_i between firms. Each of these interklinkages will play a different role in generating network effects from shocks.

[8]Nevertheless that (9) binds in equilibrium is not crucial for the qualitative results, and may in fact understate the quantitative results.

[9] Notice that the firm's debt-to-revenue ratio is fixed, because firms collateralize their end-of-period revenue for borrowing.

[10]This a key difference with Bigio and La'O (2016), in which the tightness of each firm's cash-in-advance is an exogenous parameter because there is no inter-firm lending.

[11]Since τ_{i-1} is important only insofar as it affects the tightnesses of firms' constraints, it shows up in firm i's first order conditions only through ϕ_i.

[12]More precisely, the shadow value of funds of firm i is given by $\frac{1}{\phi_i} - 1$.

A. Equilibrium

I close the model by imposing labor and goods market clearing conditions $N = \sum_{i=1}^{M} n_i$ and $C = Y \equiv x_M$.

Definition: An equilibrium is a set of prices $\{p_{i_{i\epsilon I}}, w\}$, *and quantities* x_i, n_i, $\tau_{i_{i\epsilon I}}$ *that (i) maximize the representative household's utility, subject to its budget constraint; (ii) maximize each firm's profits subject to its cash-in-advance, bank borrowing, and supplier borrowing constraints; and (iii) clear goods markets and the labor market.*

Equilibrium aggregate output in the economy is determined by each firm's production function and financial constraint. To see this, let $\bar{Y}$ denote the aggregate output that would prevail in a frictionless input-output economy (à la Acemoglu et al. (2012)), given by $\bar{Y} \equiv \prod_{i=1}^{M} \tilde{\eta}_i^{\tilde{\eta}_i} z_i^{\tilde{\omega}_i}$.[13] Define aggregate liquidity in the economy as $\bar{\Phi} \equiv \prod_{i=1}^{M} \phi_i^{\sum_{j=1}^{i} \tilde{\eta}_j}$, an aggregation of all firm's shadow value of funds. Then an analytical expression for equilibrium aggregate output, derived in Appendix A5, shows output to be log-linear in $\bar{Y}$ and the aggregate liquidity in the economy.

$$Y = \bar{Y}\bar{\Phi} \tag{12}$$

Intuitively, (12) says that equilibrium aggregate output is constrained by aggregate liquidity - the funds available to all firms to finance working capital at the beginning of the period. Note that if all firms are unconstrained, then $\bar{\Phi} = 1$ and $Y = \bar{Y}$. If one firm i is constrained, aggregate output depends on how its constraint affects the supply of intermediate good i for all downsream firms, given by $\sum_{j=1}^{i} \tilde{\eta}_j$.[14]

To summarize, firms' financial constraints distort production in a way which depends on the credit and input-output structures of the economy. The tightness of each firm's constraint in turn depends on trade credit, and is therefore determined by the underlying structure of the credit network of the economy. At this stage, it is worth discussing how this economy compares to that of Bigio and La'O (2016). A novelty of Bigio and La'O (2016) is to show how firm-level financial constraints affect aggregate output through the input-output structure of the economy. However, because all payments between firms are settled at the end of the period after production takes place, there is no role for trade credit in Bigio and La'O (2016), and so financial constraints ϕ_i are fixed exogenously. As I show in the next section, when

[13] Here, $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1}$ denotes firm i's share in total intermediate good use, and $\tilde{\eta}_i \equiv \eta_i \tilde{\omega}_i$ denotes firm i's share of labor in aggregate output.

[14] Note that the credit network of the economy - i.e. the set $\{\theta_i\}_{\forall i \epsilon I}$ - shows up implicitly in (12) through each ϕ_i.

these constraints are determined endogenously by trade credit relationships, the economy can behave qualitatively very differently in response to shocks.

B. Aggregate Impact of Firm-Level Shocks

I now examine how the economy responds to firm-level financial shocks and productivity shocks. I model a *financial shock* to firm i by a change in B_i, the fraction of firm i's revenue that the bank will accept as collateral for the bank loan. This is a reduced-form way to capture a reduction in the supply of bank credit to firm i, and represents an exogenous tightening in firm i's financial constraint.[15]

If firm i is unconstrained in equilibrium, a marginal financial shock $d\ B_i$ has no effect on its production - the firm has deep pockets and can absorb the shock. However, if the firm is constrained, then it is forced to reduce production as it can no longer finance as many inputs with up front payments. In addition to this direct effect, there are two types of network effects by which the shock affects other firms in the economy: input-output channel and the credit linkage channel.

Network Effects: Standard Input-Output Channel: Through the first channel, which I call the *standard input-output channel*, the shock propagates through input-output interlinkages, increasing firms' input costs. This is the standard channel analyzed in in the input-output literature, including Acemoglu et al. (2012) and Bigio and La'O (2016). The reduction in firm i's output increases the price p_i of good i. This acts as a supply shock to the customer downstream (firm $i+1$), who is now faced with a higher unit cost of its intermediate good. In response, firm $i+1$ cuts back on production, which causes the p_{i+1} to increase, etc. Thus, as a result of the shock to firm i, all firms downstream experience a supply shock to their intermediate goods, and cut back on production. This amplifies the shock because as firms reduce production, they cut back on employment which, in turn, reduces the wage and household consumption.[16] In addition, the shock travels upstream as suppliers adjust their output to respond to the fall in demand for their intermediate goods.

Network Effects: Credit Linkage Channel: There is also a is a new, additional channel of transmission - which I call the *credit linkage channel* - which describes how the financial

[15] In the general network model in the following section, each firm sells some portion of its output directly to the household. In this setting, one could alternatively interpret the fall in B_i as a failed payment by final consumer. In either case, these are idiosyncratic shocks to the firm's liquid funds such that $\frac{d\chi_i}{d B_i} > 0$, and are not well-represented by a change in its productivity or technology.

[16] This channel is ultimately driven by the input specificity in each firm's production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good i in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

constraints of upstream firms are tightened endogenously in response to the shock.

Recall that when firm i cuts back on production, the price p_i of its good rises. This increases the collateral value of its future cash flow, allowing it to delay payment for a larger fraction of its purchase from supplier $i-1$.[17] As a result, supplier $i-1$'s cash/revenue ratio falls, meaning the fraction of its revenue collected as up front payment falls. This tightens its cash-in-advance constraint - i.e. χ_{i-1} falls.[18][19]

$$\chi_{i-1} \downarrow \equiv \underbrace{B_{i-1} + \theta_{i-1}}_{debt/revenue\ ratio} + \underbrace{1 - \frac{\tau_{i-1}}{p_{i-1}x_{i-1}}}_{cash/revenue\ ratio\ \downarrow} \tag{13}$$

Thus, with less cash on-hand, the supplier $i-1$ is now faced with a tighter financial constraint itself. The supplier may therefore be forced to reduce production further, and thereby pass the shock to its own suppliers and customers. (This continues up the chain of firms). In this manner, the initial effect of the shock is amplified as upstream firms experience tighter financial conditions.

But why doesn't firm $i-1$ reduce the trade credit it extends to i in order to increase its cash holdings and relax its own financial constraint? Recall that representative firm $i-1$ actually consists of a continuum of perfectly competitive firms, and that competition amongst these firms forces them to offer their customers the maximum trade credit loan allowed by the borrowing constraint, even when they are themselves cash-constrained in equilibrium. [20] As a result, they are unable to reduce trade credit loans to increase their cash holdings. This mechanism is in line with strong empirical evidence that firms in financial distress reduce the up-front payments they make to their suppliers, thereby transmitting the financial distress to their suppliers.[21]

Note the role that α plays in mitigating the transmission of the shock via the credit linkage channel. The higher that α is, i.e. the more that firm $i-1$ can collateralize its trade credit τ_{i-1}, the less that χ_{i-1} falls in response to the shock to i. Although $i-1$ receives a smaller cash-in-advance payment from its customer, it can collateralize a higher fraction of its trade credit to obtain more credit from the bank. This reduces the loss in liquidity that it

[17]This is true even though the volume of trade credit τ_{i-1} may actually fall in response to the shock.

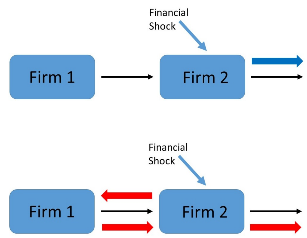
[18]Recall that firms' debt/revenue ratios are fixed in equilibrium.

[19] More precisely, there are three effects on χ_{i-1}, the tightness of $i-1$'s constraint, all of which imply that χ_{i-1} falls unambiguously in response to the shock $d\ B_i$. Recall from (10) that firm $i-1$'s cash/revenue ratio depends inversely on $\frac{p_i x_i}{p_{i-1}x_{i-1}}$. First, the shock increases p_i, as discussed above. Second, the fall in firm i's output increases the ratio $\frac{x_i}{x_{i-1}}$ due to the decreasing returns to x_{i-1} (since $(1-\eta_i) < 1$). And third, the fall in i's demand reduces the price p_{i-1} of good $i-1$. Each of these effects reduces χ_{i-1}.

[20]For instance, Barrot (2014) shows that competition amongst suppliers may force even cash-constrained firms to offer trade credit to their customers.

[21]See Jacobson and von Schedvin (2015), Raddatz (2010), and Boissay and Gropp (2012).

Figure 2: Feedback Effect



suffers due to the smaller cash payment. Therefore, α parameterizes the degree to which each firm can substitute lost cash-in-advance payments for a higher bank loan. I later explore the quantitative relvance of α.

Feedback Effect Created by Transmission Channels: Importantly, the two transmission channels produce a feedback effect which amplifies the shock, as illustrated in Figure 2. Suppose that firm 2 is hit with an adverse financial shock, causing its cash-in-advance constraint to become tighter, and forcing it to cut back on production. The standard input-output channel, represented by the blue arrow, transmits the shock downstream in the form of a higher intermediate good price.

In addition, the credit linkage channel tightens the constraints of upstream firms, as firm 2 reduces the cash-in-advance payments it makes to its supplier. With a tighter financial constraint the supplier is forced to reduce production, which feeds back to firm 2 again in the form of higher price for the intermediate good. Thus, firm 2 is hit not only with a tighter financial constraint, but also endogenously higher input costs, (which it passes on to its customer, and so on). In this manner, the two channels interact to create a feedback loop represented by the red arrows, which exacerbates the initial shock.[22]

C. Impact of Firm-Level Shock on Aggregate Output

In light of these mechanisms, I now derive analytical expressions for how a firm-level financial shock affects aggregate output, and show that the credit network effects amplify the shock in a manner which depends on the structure of the credit linkages.

From (12), I decompose the change in aggregate output due to a financial shock to firm i into components reflecting the standard input-output channel and the credit linkage channel.

$$\frac{d\,log\,Y}{d\,B_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\,log\,\phi_j}{d\,B_i} \tag{14}$$

[22]A firm-level financial shock to in my model therefore is isomorphic to an *aggregate* financial shock to all firms in a model with fixed constraints, e.g. Bigio and La'O (2016).

Here, the terms $\frac{d\,log\,\phi_j}{d\,B_i}$ capture the credit linkage channel, and reflect how the financial shock to firm i affects the shadow value of funds of every other firm j in the network. The terms $\bar{v}_j$ capture the standard input-output channel, and map these changes in each ϕ_j into aggregate output. ($\bar{v}_j \equiv \sum_{k=1}^{j} \tilde{\eta}_k$ depends on the share of labor in aggregate output of each firm.) This decompostion will allow me to quantify the aggregate effects of each channel later on.

In an economy without the credit linkage channel, such Bigio and La'O (2016), each ϕ_j is fixed so that $\frac{d\,log\,\phi_j}{d\,B_i} = 0$ for all $j \neq i$. In words, financial constraints would not respond endogenously to a shock. Therefore, (14) would reduce to $\frac{d\,log\,Y}{d\,B_i} = \bar{v}_i$.

However, credit network effects *amplify* the effects of the firm-level financial shock on aggregate output. This is because $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ and therefore $\frac{d\,log\,Y}{d\,B_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\,log\,\phi_j}{d\,B_i} > \bar{v}_i$ (proved in Appendix A2. In addition, the credit network effects $\frac{d\,log\,\phi_j}{d\,B_i}$ are weakly increasing in θ_{jk} for all firms i,j, and k. This implies that the stronger is the credit linkage between any two firms i and j - either directly or through other firms - the stronger is the transmission of the financial shock upstream. Thus, the aggregate impact of the financial shock depends on the location of firm i within the networks, and the strength of input-output and credit linkages between firms.

D. Impact of Firm-Level Productivity Shock on Aggregate Output

Now consider a productivity shock to firm i, represented by a fall in i's total factor productivity (TFP) z_i. It turns out that, due to Cobb-Douglas production, each firm's cash/revenue ratio, and therefore the tightness of their constraint ϕ_j, is independent of the productivity of firms z_i.[23] As a result, $\frac{d\,log\,\phi_j}{d\,B_i} = 0$ for all firms i,j. Thus, the credit linkage channel plays no role in propagating productivity shocks. However, the standard input-output channel amplifies the productivity shock just as in Acemoglu et al. (2012). Thus, the response of aggregate output is given by $\frac{d\,log\,Y}{d\,z_i} = \frac{\tilde{\omega}_i}{z_i}$, where $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1}$ represents firm i's share in the total intermediate good use of the economy.

Summary of Theoretical Results: To summarize, three main insights emerge from the model. First, when firms are suppliers of intermediate goods as well as the creditors who finance the transactions of these goods, firm-level shocks can endogenously generate large changes in the aggregate liquidity available for trade in intermediate goods. This creates a multiplier effect which amplifies the aggregate effects of firm-level shocks. Second, the aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

Is this mechanism likely to be quantitatively relevant? Until now, the structure of the

[23] Acemoglu, Akcigit, and Kerr (2015) argue that Cobb-Douglas is a good approximation for production at the industry level.

networks was assumed to be a straight line. So to answer this question and take the model to the data, I now generalize certain features of the model.

II. General Model

To capture more features of the economy, I now allow for an arbitrary network structure so that each firm may trade with and borrow from or lend to any other firm in the economy.

I assume that each of the M goods can be consumed by the representative household or used in the production of other goods. The household's total consumption C is Cobb-Douglas over the M goods, and it has GHH preferences.[24]

$$U(C,\, N) = \frac{1}{1-\gamma}\left(C - \frac{1}{1+\epsilon}N^{1+\epsilon}\right)^{1-\gamma} \quad , \qquad C \equiv \prod_{i=1}^{M} c_i^{\beta_i} \tag{15}$$

Here, ϵ and γ respectively denote the Frisch and income elasticity of labor supply. The household maximizes its utility subject to its budget constraint (1). This yields optimality conditions which equate the ratio of expenditure on each good with the ratio of their marginal utilities, and the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \ , \qquad N^{1+\epsilon} = C \tag{16}$$

Each firm can trade with all other firms. Firm i's production function is again Cobb-Douglas over labor and intermediate goods.

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}}\right)^{1-\eta_i} \tag{17}$$

Here, x_i denotes firm i's output and x_{ij} denotes firm i's use of good j. Since ω_{ij} denotes the share of j in i's total intermediate good use, I assume $\sum_{j=1}^{M}\omega_{ij} = 1$ so that each firm has constant returns to scale. The input-output structure of the economy can be summarized by the matrix Ω of intermediate good shares ω_{ij}.[25]

[24] Quantitatively similar results hold for preferences which are additively separable in aggregate consumption C and labor N.

[25] This is simply a generalization of the input-output structure in the stylized model. In that case, the Ω would be given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & & \\ \vdots & & \ddots & \\ \omega_{M1} & & & \omega_{MM} \end{bmatrix}$$

Note that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence, Ω describes how firms would trade with each other in the absence of frictions.

Each firm's cash-in-advance constraint takes the same form as in the stylized model, with the exception that each firm has M suppliers and M customers instead of just one of each. τ_{is} denotes the trade credit loan that firm i receives from each of its suppliers s.

$$wn_i + \underbrace{\sum_{s=1}^{M} (p_s x_{is} - \tau_{is})}_{net\ CIA\ payment\ to\ suppliers} \leq b_i + \underbrace{p_i x_i - \sum_{c=1}^{M} \tau_{ci}}_{net\ CIA\ received\ from\ customers} \tag{18}$$

Firm i faces borrowing constraints with each of its suppliers, to which it can pledge fractions θ_{is} of its future cash flow to repay the loans. Each firm can also borrow b_i from the bank by pledging B_i of its revenue and α of its accounts receivable $\sum_{c=1}^{M} \tau_{ci}$.

$$\tau_{is} \leq \theta_{is} p_i x_i \qquad b_i \leq B_i p_i x_i + \alpha \sum_{c=1}^{M} \tau_{ci} \tag{19}$$

As before, competition amongst suppliers in industry s forces them to offer the maximum trade credit permitted by the limted enforcement problem, so that the trade credit borrowing constraint always binds when industries are cash-constrained in equilibrium. The structure of the credit network between firms can be summarized by the matrix of θ_{ij}'s.

$$\Theta \equiv \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & & \\ \vdots & & \ddots & \\ \theta_{M1} & & & \theta_{MM} \end{bmatrix}$$

Plugging the binding borrowing constraints into (18) yields a constraint on i's total input purchases, where χ_i describes the tightness of i's cash-in-advance constraint.

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i \tag{20}$$

Just as in the stylized verision, χ_i is an an equilibrium object, where firm i's cash/revenue ratio depends on the prices p_c of its customer's goods and its forward credit linkages θ_{ci}.

$$\chi_i = \underbrace{B_i + \sum_{s=1}^{M} \theta_{is}}_{debt/revenue\ ratio} + \underbrace{1 - (1-\alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}}_{cash/revenue\ ratio} \tag{21}$$

Firms choose labor and intermediate goods to maximize profits subject to their cash-in-advance constraint. Again, firm i's contraint inserts a wedge ϕ_i between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad x_{ij} = \phi_i (1-\eta_i) \omega_{ij} \frac{p_i}{p_j} x_i \tag{22}$$

where the wedge $\phi_i = min\{1\,,\ \chi_i\}$ is determined by the firm's shadow value of funds. Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^{M} n_i \qquad x_i = c_i + \sum_{c=1}^{M} x_{ci} \tag{23}$$

The equilibrium conditions of this generalized model take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model.

For the remainder of the paper, I consider the case in which all industries are constrained in equilibrium.[26] This is to ensure that a marginal financial shock to each industry has a non-zero effect.[27]

Relationship Between Firm Influence and Size

A well-known critique of frictionless input-output models such as Acemoglu et al. (2012) is that the size of a firm, as measured by its share s_i of aggregate sales, is sufficient to determine the aggregate impact of a shock to sector i, and one does not need to know anything about the underlying input-output structure of the economy. All relevant information about the input-output structure is captured by s_i. As a result, an idiosyncratic shock to firm i is isomporphic to an aggregate TFP shock weighted by s_i. This makes it impossible to distinguish the role of idiosyncatic versus aggregate shocks in generating aggregate fluctuaions, as the two are observationally equivalent.

Bigio and La'O (2016), however, show that this isomorphism breaks down when the economy has financial frictions. To determine the aggregate impact of an idiosyncratic shock,

[26]The parameters B_i are set as free parameters to ensure that the calibration is consistent with this case of the equilibrium. This is discussed in the calibration section.

[27]If there are at least some firms in an industry who are constrained, then a credit supply shock will have real effects. Therefore, assuming an industry is unconstrained will bias the results.

one needs to know the full input-output structure structure of the economy, summarized by the matrix Ω, and the vector $\vec{\phi}$ indicating the degree to which each industry is financially constrained.

My model shows that when firms are linked by credit relationships, knowing Ω and $\vec{\phi}$ is no longer sufficient to measure the aggregate impact of a shock to a sector or firm i. Because these credit linkages propagate shocks across firms, the aggregate impact of an idiosyncratic shock also depends not only on Ω and $\vec{\phi}$, but also on the underlying structure of the credit network of the economy, summarized by the matrix Θ.

Solving the General Model

The equilibrium of the general model is the solution to system of $M^2 + 5M + 2$ nonlinear equations in the same number of unknowns, listed in Appendix A6. For any set of model parameters

$$\left\{\left\{z_i,\, B_i,\, \eta_i,\, \beta_i,\, \{\theta_{ij},\, \omega_{ij}\}_{j\epsilon I}\right\}_{i\epsilon I},\, \alpha,\, \epsilon,\, \gamma\right\}$$

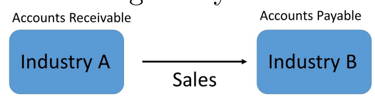
there is a unique solution to the system. Since the model is one period, the behavior of the system in response to shocks can be modeled by comparative statics. In particular, I am interested in the *change* in the economy that results form a perturbation of one or more of the model parameters $\{B_i,\, z_i\}_{i\epsilon I}$, representing financial and productivity shocks, respectively. I therefore log-linearize the system of nonlinear equations around a point $\{B_i^*,\, z_i^*\}$. In the quantitative analysis, I calibrate this point (and the remainder of the parameters), to match data for the US economy. I thus obtain a log-linear approximation for the response of the equilibrium variables to firm-level financial and productivity shocks.

It is worth clarifying one point about productivity shocks. It turns out from the Cobb-Douglas specification of firm production functions that the equilibrium is already log-linear in each z_i. Therefore, the log-linearized response of the equilibrium variables to a change in z_i is independent of the level of z_i. Therefore, I do not need to calibrate the parameters $\{z_i\}_{i\epsilon I}$ to approximate a response in the economy to a productivity shock. Indeed, when one log-linearizes the equilibrium system around $\{B_i^*,\, z_i^*\}$, z_i^* drops out of the log-linear equations.

III. Quantitative Analysis

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and

Figure 3: Constructing Proxy for Trade Credit Flows



examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

A. Mapping the US Credit Network

Calibration of the trade credit parameters θ_{ij} requires data on credit flows between industry pairs; but data on credit flows at any level of detail is scarce. To overcome this paucity of data, I construct a proxy for trade credit flows τ_{ij} between industry pairs using industry-level input-output data and firm-level balance sheet data. I use input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the period 1997-2013. The BEA publishes annual input-output data at the three-digit NAICS level, at there are 58 industries, exlcuding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to $p_j x_{ij}$ in my model for every industry pair $\{i, j\}$. Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm's total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample.

My strategy for constructing the proxy is illustrated in Figure 3. From the payables and receivables data, I observe how much, on average, firms in each industry have borrowed from all of their suppliers collectively, and lent to all of their customers collectively.[28] However, I do not observe how an industry's stock of trade credit and debt breaks down across each of its suppliers and customers. Therefore, I combine the input-output data with the payables and receivables data to approximate the fraction of sales from firms in industry j to firms in industry i made on credit, on average, yielding a proxy for trade credit flows τ_{ij} between each industry pair.

B. Calibration

[28]The vast majority of accounts receivables and payables of US corporations consists of trade credit.

With the proxy for trade credit flows at hand, I calibrate the general model to match US data. I calibrate technology parameters η_i and ω_{ij} to match the BEA input-output tables of the median year in my sample, 2005. From firm i's optimality conditions (10), we can write the firm's total expenditure on inputs as

$$\begin{aligned} wn_i + \sum_{j=1}^{M} p_j x_{ij} &= \left(\eta_i + [1 - \eta_i] \sum_{j=1}^{M} \omega_{ij} \right) \phi_i p_i x_i \\ &= \phi_i p_i x_i \end{aligned}$$

where the second equality holds due to the constant returns to scale of i's production technology. This implies that

$$\phi_i = \frac{wn_i + \sum_{j=1}^{M} p_j x_{ij}}{p_i x_i} \tag{24}$$

The right-hand side of (12) is directly observable from the BEA's Direct Requirements table.

Looking through the lense of the model, the observed input-output tables reflect both technology parameters and distortions created by the liquidity onstraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm i's optimality conditions for each input and my calibrated ϕ_i's

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \qquad \omega_{ij} = \frac{p_j x_{ij}}{(1 - \eta_i) \phi_i p_i x_i} \tag{25}$$

Again the ratios $\frac{wn_i}{p_i x_i}$ and $\frac{p_j x_{ij}}{p_i x_i}$ are directly observable from the Direct Requirements tables for every industry i and j.

I calibrate the parameters θ_{ij}, representing the credit linkages between industries j and i, to match my proxy of inter-industry trade credit flows $\hat{\tau}_{ij}$ using industry i's binding borrowing constraint.

$$\theta_{ij} = \frac{\hat{\tau}_{ij}}{p_i x_i} \tag{26}$$

Industry i's total revenue $p_i x_i$ is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005).

To calibrate B_i, the parameters reflecting the agency problem between an and the bank, recall the definition of ϕ_i given by (11), which depends on the technology parameters (calibrated as described above) and the tightness χ_i of each industry's cash-in-advance, where

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1 - \alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \tag{27}$$

The total revenue of each industry $p_i x_i$ is observable from the Uses by Commodity tables, and ϕ_i and θ_{is} for all s were calibrated as described above. I therefore use (13) and (11) to back out B_i for each industry. Thus, the calibration of B_i ensures that $\phi_i < 1$, so that all industries are constrained to some degree in equilibrium.[29]

I follow the standard literature and set $\epsilon = 1$ and $\gamma = 2$, which represent the Frisch and income elasticity, respectively. I therefore set $\alpha = 0.2$ in my baseline calibration, but check the sensitivity of the quantitative results to varying α.[30]

IV. A Quantitative Exploration of the Model

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and financial shocks. I first ask how much aggregate fluctuations does the credit network of the US economy generate?

It is instructive to first discuss how the transmission mechanism outlined in the stylized model maps into this more general setting. In addition to the feedback effects described in Section I, there are now additional spillover effects arising from the additional linkages between each industry. To illustrate, consider the petroleum and coal manufacturing industry and the utilities industry in the US. Each is linked by a common supplier, the oil and gas extraction industry, as illustrated in Figure 4. Suppose that firms in petroleum and coal manufacturing experience an exogenous tightening of their financial constraints, forcing some to reduce production. This raises the price of petroleum and coal products for the rest of the economy, corresponding to the standard input-output channel represented by the blue arrow.[31] But in the absence of the credit linkage channel of transmission, firms in the utilities industry will remain largely unaffected by the shock.

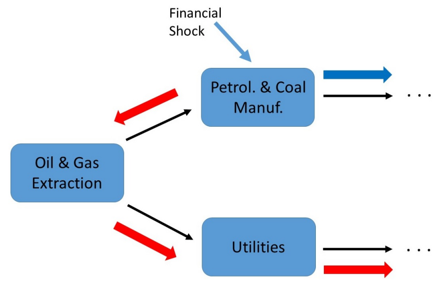
However, through the credit linkage channel, the shock causes petroleum and coal manufacturers to reduce the up front payments they make to their oil and gas suppliers. As a result, these oil and gas firms are also faced with tighter financial conditions themselves, and may be forced to further cut back on production. This reduction in the supply of oil and gas causes utilities firms to face higher oil and gas prices, who pass these effects downstream in

[29]Recall that for a credit supply shock to have any effect on industry i, a necessary condition is that the industry be constrained in equilibrium.

[30] Recall that α is the fraction of receivables that industries can collateralize to borrow from the bank. Omiccioli (2005) finds that the median Italian firm in a sample collateralizes 20 percent of its accounts receivable for bank borrowing.

[31]In addition, the suppliers in the oil and gas industry will face lower demand from their customers, and reduce production accordingly.

Figure 4: Transmission Mechanism in the General Model



the form of higher energy prices. These additional network effects further amplify the effects of the shock.

How large are these credit network effects likely to be? To answer this, I hit the US economy with an aggregate financial shock, and industry-level financial shocks.

A. Response to an Aggregate Financial Shock

Suppose that the economy is hit with a one percent aggregate financial shock: each industry i's cash-in-advance constraint is tightened by one percent. [32] Under my baseline calibration, I find that US GDP falls by 2.92 percent - a large drop. To assess how much of this fall in GDP is generated by the propagation of shocks via the credit network, I perform the same exercise, but shutting down the credit linkage channel. To do so, I impose that financial constraints do not respond endogenously to shocks - i.e. $\frac{d\, log\, \phi_j}{d\, B_i} = 0$ for all $j \neq i$.

With the credit linkage channel shut off, GDP falls by only 2.28 percent in response to the same aggregate shock - a difference of 0.64 percetage points. Put differently, the credit network effects amplify the fall in GDP by about 28 percent, suggesting that the credit network of the US can play an important role in generating aggregate fluctuations in GDP from financial shocks.

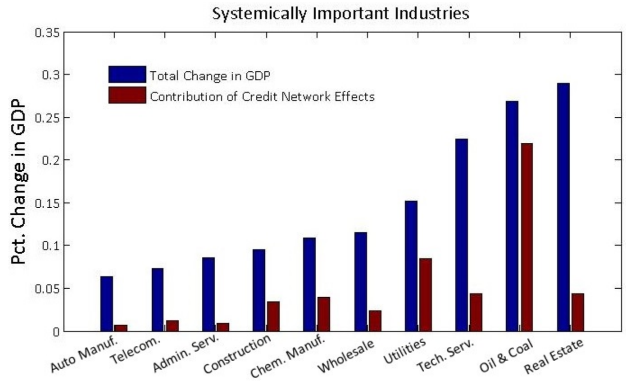
Table 3 in the appendix reports the sensitivity of these results to the specification of $\alpha = 0.2$, the parameter controlling the substitutability of cash and bank credit. Recall that a higher α mitigates the transmission of financial shocks, as firms are better able to offset a lower cash/revenue ratio with a higher loan from the bank.[33] In each case, the aggregate impact of the shock is quite large. While the multiplier effect of the credit network indeed falls as α approaches 1, credit network effects are quantitatively significant for reasonable values of α.[34]

[32] More specificially $d\, B_i = 0.01$ for all industries i. This can be interpreted as a one percent fall in the aggregate supply of credit.

[33] When $\alpha = 1$, the two are perfect substitutes and financial shocks have no credit network effects.

[34] Recall that according to Omiccioli (2005), the median Italian firm collateralizes about 20 percent of its

Figure 5:



Notes: This chart shows the ten most systemically important US industries based on the counterfactual drop in GDP in response to a 1 percent industry-level financial shock. The contribution of credit network effects is computed numerically. This exercise is done excluding financial and services indsutries.

B. Response to Industry-Level Financial Shocks

Next, I ask which industries are likely to be systemically important to the US economy, in light of these network effects. I measure the systemic importance of industry i by the how much GDP falls in response to a 1 percent financial shock to industry i. Figure 5 shows a bar graph of the ten most systemically important industries in the US, based on this exercise. For each industry i, the blue bars show the elasticity of GDP with respect to B_i, or the percentage change in GDP in response to a 1 percent financial shock to industry i.

The model implies that an industry-level financial shock can have a strong impact on US GDP. For example, although the technical services industry accounts for only 0.069 percent of US GDP, a one percent financial shock this industry causes a fall in GDP of 0.19 percent - a multiplier of 2.75. (Put differently, in the absence of input-output or credit linkages, GDP would fall by only 0.069 percent in response to the shock). Thus, the network effects generated by the interaction of input-output and credit interlinkages of industries can create a substantial amplification mechanism.

C. Summary of Quantitative Analysis

The quantitative analysis showed that i) the credit linkages between US industries play a quantitatively significant role in amplifying aggregate and industry-level financial shocks, even when allowing for substitutability between bank credit and cash payments; and ii) the

receivables, implying $\alpha = 0.2$. Nearly a third of these firms collateralize less than 10 percent of their receivables, and 55 percent less than half. Given the higher dependence of Italian firms on bank lending relative to US firms, these numbers could be substantially lower for the US.

systemic importance of an industry depends on the input-output and credit network effects of shocks. Therfore, an understanding of the role that credit linkages play in propagating idiosyncratic shocks introduces a new notion of the systemic importance of firms or industries based on their place in the credit network.

D. Mapping the Model to the Data

In order to map the model to the data, I extend the static model to be a repeated cross-section. Let X_t, N_t, B_t, and z_t denote the M-by-1 vectors of output growth, employment growth, financial shocks, and productivity for each industry respectively, in quarter t. The log-linearized model yields closed-form expressions for how the output and employment of each industry respond to financial and productivity shocks. (These are derived in the online Technical Appendix.)

$$X_t = G_X B_t + H_X z_t \qquad N_t = G_N B_t + H_N z_t \tag{28}$$

The M-by-M matrices G_X and H_X (G_N and H_N) map industry-level financial and productivity shocks, respectively, into output growth (employment growth), and capture the effects of input-output and credit interlinkages in propagating shocks across industries. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct the observed, quarterly cyclical fluctuations in the output $\hat{X}_t$ and employment $\hat{N}_t$ of US industrial production industries using data from the Federal Reserve Board's Industrial Production Indexes, which includes data on the output growth of these industries, and the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, from which I observe the number of workers employed by each of these indutries. At the three-digit NAICS level there are 23 such industries.[35] For each dataset, I take 1997 Q1 through 2013 Q4 as my sample period, and seasonally-adjust and de-trend each series. In the empirical analysis to follow, I use this data and the expressions (28) to decompose observed cyclical fluctuations into various components.

V. Empirical Analyses

[35]Hours worked is not directly available at this level of industry detail and this frequency.

In the empirical part of the paper, I use my theoretical framework to investigate which shocks drive observed cyclical fluctuations in the US, once we account for the network effects created by credit and input-output linkages between industries. The framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks.

The model allows one to disentagle the contribution of two supply-side shocks to observed fluctuations: financial and productivity shocks. Furthermore, the model allows for network effects to generate fluctuations in economic aggregates from idiosyncratic (industry-level) shocks. This permits a disctinction of the roles of idiosyncratic versus purely aggregate shocks in driving aggregate fluctuations. To decompose observed cyclical fluctuations into components along these two dimensions, I use two methodological approaches. In the first, I identify shocks without imposing the structure of my model on the data; in the second, I identify shocks using a structural estimation.

First Method: Estimating Shocks without the Model

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data - the identifying assumptions are completely independent of the model. An added advantage of this method is that it permits the estimation of a residual component of observed fluctuations - a component which is not explained by either shock. However, the shocks estimated using this method are assumed to be common to all industries.

A. Estimating financial shocks

To identify credit supply shocks to the US economy, I estimate an identified VAR using a similar approach as Gilchrist and Zakrajsek (2011). To do this requires first constructing a measure of bank-intermediated business lending.

I construct a measure of aggregate business lending by US financial intermediaries using quarterly Call Report data collected by the FFIEC. To capture lending to the business sector, I use commercial and industrial loans outstanding.[36] But as Gilchrist and Zakrajsek (2011) show, this type of on-balance sheet lending reacts to financial market disruptions only with a signficant lag. In contrast, a cyclically-sensitive component of bank lending is unused commitments, representing off-balance sheet lines of credit to businesses.[37] While changes in unused commitments after a financial shock mostly reflect a tightening in the supply of lines

[36]This is a conservative estimate of bank lending to US firms.

[37]The authors show that the contraction in unused loan commitments was concomitant with onset of the financial crisis in 2007, while business loans outstanding contracted only with a lag of about four quarters.

of credit[38], they could also reflect borrowers drawing down the unused portion of their lines of credit - a change in the demand for bank loans. To net out any demand-side changes in bank lending, I construct a quarterly measure of aggregate bank lending, called the *business lending capacity* of the financial sector, as the sum of unused commitments and commercial and industrial loans outstanding in each quarter.[39]

To empirically identify credit supply shocks, I augment a standard VAR of macroeconomic and financial variables with the measure of business lending capacity, and the excess bond premium of Gilchrist and Zakrajsek (2012) - a component of corporate credit spreads designed to capture changes in the risk-bearing capacity of financial intermediaries.[40] The endogenous variables included in the VAR, ordered recursively, are: (i) the log-difference of real business fixed investment; (ii) the log-difference of real GDP; (iii) inflation as measured by the log-difference of the GDP price deflator; (iv) the quarterly average of the excess bond premium; (v) the log difference business lending capacity (vi) the quarterly (value-weighted) excess stock market return from CRSP; (vii) the ten-year (nominal) Treasury yield; and (viii) the effective (nominal) federal funds rate. The identifying assumption implied by this ordering is that stock prices, the risk-free rate, and bank lending can react contemporaneously to shocks to the excess bond premium, while real economic activity and inflation respond with a lag. I estimate the VAR using two lags of each endogenous variable.

To map the orthogonalized innovations in the excess bond premium into the financial shocks $\tilde{B}_t$ of my model, I make use of the impulse response function of business lending capacity. I thus construct my financial shocks as changes in the supply of bank lending which arise due to innovations in the risk-bearing capacity of the financial sector, which are orthogonal to macroeconomic conditions. Figure 6 plots the time series of this shock.

Thus far, this procedure produced estimates of a financial shock which is common to all industries. Yet shocks to credit availability may affect industries differentially depending on their dependence on external finance. To allow for this, I load the financial shocks $\tilde{B}_t$ onto each industry based on a measure EFD_i of the the industry's external finance dependence, which I construct according to Rajan and Zingales (1995).[41] Since the quantitative results do not significantly change, the result reported hereafter are for financial shocks which load

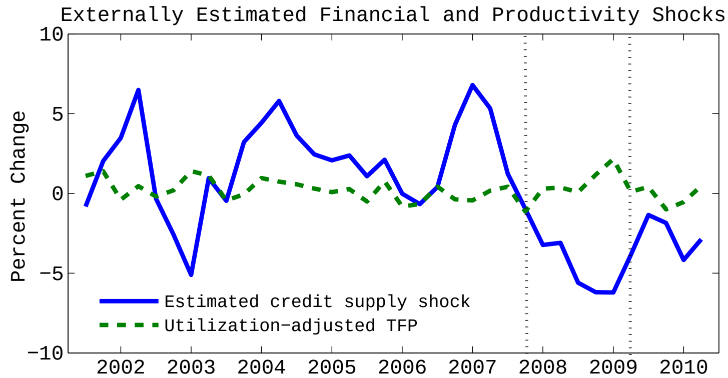
[38]See Gilchrist and Zakrajsek (2011) for evidence.

[39]To see why changes in this measure of business lending capacity mostly reflect supply-side changes, consider the following example. Suppose that a business draws down an existing line of credit it has with its bank. This is recorded as a fall in unused commitments, but reflects an increase in demand for credit rather than a contraction in the supply of credit. However, the loan is now recorded as an on-balance sheet commercial or industrial loan. Therefore, the fall in unused commitments is exactly offset by the increase in commercial and industrial loans outstanding, leaving bank lending capacity unchanged. So this measure of business lending capacity is largely unresponsive to firms drawing down their lines of credit.

[40]I thank Simon Gilchrist for kindly sharing the excess bond premium data.

[41]In this manner, I obtain a time-varying, industry-specific financial shock $\tilde{B}_{it}$ which can be fed into the model.Although they varies across industries in any given quarter, these shocks to each industry are perfectly correlated across time, and so should not be interpreted as idiosyncratic shocks.

Figure 6:



Notes: This figure shows the series of quarter-to-quarter growth in utilization-adjusted TFP measure of Fernald (2012) and the credit supply shocks, estimated as changes in the business lending capacity of the financial sector which are due to orthogonalized innovations to the excess bond premium. Financial shocks were estimated using an identified VAR.. TFP data was obtained from the San Francisco Fed database.

equall onto all industries. Let $\hat{B}_t$ denote the M-by-1 vector of these shocks.

B. Estimating productivity shocks

The Federal Reserve Bank of San Francisco produces a quarterly series on TFP for the US business sector, adjusted for variations in factor utilization, according to Fernald (2012). As such, this series is readily mapped into my model as an aggregate productivity shock $\tilde{z}_t$. Figure 6 plots time series for this productivity shock. Let $\hat{z}_t \equiv \tilde{z}_t \vec{1}$ denote the M-by-1 vector of these shocks.

C. Decomposing Observed Fluctuations in Industrial Production

With the estimated shocks at hand, I use log-linearized expression (28) to decompose observed cyclical fluctuations in industrial production into components coming from the financial shocks, productivity shocks, and a residual.

$$\hat{X}_t = G_X \hat{B}_t + H_X \hat{z}_t + \varepsilon_t \tag{29}$$

Components $G_X \hat{B}_t$ and $H_X \hat{z}_t$ reflect how much of observed, industry-level fluctuations in industrial output come from financial and productivity shocks, respectively, when I feed the estimated financial and productivity shocks $\hat{B}_t$ and $\hat{z}_t$ through the model. ε_t is the component of these fluctuations which is unexplained by either of these shocks.

I then feed these shocks into the model and perform a variance decomposition of aggregate industrial production. To do this, note that the assumption $X_t = G_X B_t + H_X z_t + \varepsilon_t$ implies

Table 1: Variance Decomposition of IP: 2001Q4:2007Q3

	Share of Aggregate Volatility
Productivity Shocks	0.205
Financial Shocks	0.279
Residual	0.516

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the pre-recessionary period 2001 Q4 - 2007 Q3. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Financial shocks were estimated using an identified VAR, and capture quarterly credit supply shocks to the productive sector. Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database. The residual is the component of aggregate industrial production which is unexplained after these shocks are fed through the log-linearized model.

that output volatility can be decomposed in the following way, where Σ_{XX}, Σ_{BB},Σ_{zz}, and $\Sigma_{\varepsilon\varepsilon}$ denote the variance-covariance matrices of X_t, B_t, z_t, and ε_t, respectively.[42]

$$\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X' + \Sigma_{\varepsilon\varepsilon} \tag{30}$$

In addition, letting $\bar{s}$ denote the M-by-1 vector of industry shares of aggregate output during the median year of my sample, 2005, the volatility of aggregate industrial output - henceforth *aggregate volatility* - can be approximated by $\sigma \equiv \bar{s}'\Sigma_{XX}\bar{s}$.[43] Then the fraction of observed aggregate volatility generated by financial shocks, for example, is given by $\left(\bar{s}'G_X\Sigma_{BB}G_X'\bar{s}\right)/\sigma^2$. This is derived in further detail in Appendix A4.

The variance decomposition of output before 2007 is given in Table 1. In the period 2001 - 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production. The remaining half is unaccounted for by either type of shock.

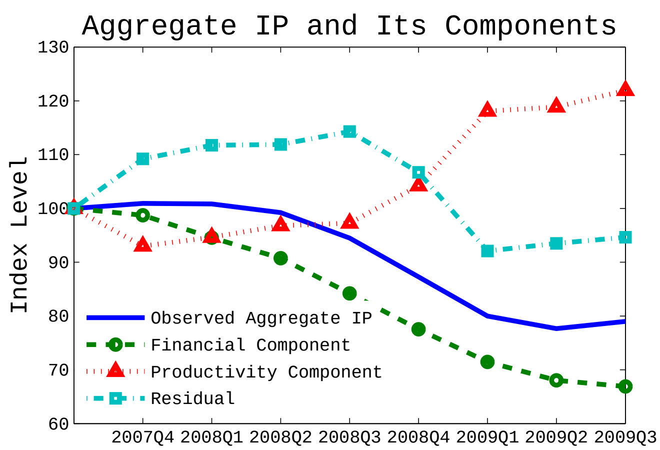
However, the story is different for the Great Recession. Figure 7 plots the time series of aggregate industrial production during the Great Recession, as well as a simulation for each of its components.[44] These counterfactual series are constructed by feeding each of the estimated components through the model one at a time, and thus represents how aggregate industrial production would have evolved in the absence of other shocks, beginning in 2007

[42] Implicit in this decomposition is the assumption that B_t,z_t, and ε_t are orthogonal to each other.

[43] I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19%. This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.

[44] The time series for observed aggregate IP is constructed from the cyclical component of IP growth. It is constructed as an aggregate index of the observed industry-level growth rates.

Figure 7:



Notes: This figure shows the time series of aggregate industrial production and its components. Observed aggregate industrial production is an index constructed from the de-trended, seasonally-adjusted industry-level quarter-to-quarter growth rates in the output of the 23 industrial production industries at the three-digit NAICS level, obtained from FRB IP Indexes. Each of the other series depict counterfactual indexes constructed from the respective components of the observed series, beginning in 2007 Q3, and represent how aggregate IP would have evolved in the absence of other shocks. Financial shocks were estimated using an identified VAR Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database.

Q3.

During the recession, productivity shocks had virtually no adverse effects on industrial production - in fact, they actually *mitigated* the downturn. Rather, financial shocks are the main culprit, accounting for two-thirds of the peak-to-trough drop in aggregate industrial production during the recession. The remaining one-third is not accounted for by either shock. Furthermore, the credit network of these industries played a quantitatively significant role during this period, amplifying the effects of the financial shocks by about 15% (i.e. adding 3.98 percentage points to the peak-to-trough drop in the financial component of aggregate industrial production).

Second Method: Structural Factor Analysis

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster, Sarte, and Watson (2011)[45], using data on the output and employment growth of US IP industries. The procedure involves two steps. I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use dynamic factor methods to decompose each of these shocks into an aggregate component and an idiosyncratic component.

[45] Foerster et al. (2011) allow only for productivity shocks in driving observed fluctuations.

In what follows, the identification of shocks relies on imposing the structure of the model on the data. In addition, all observed fluctuations are assumed to be driven entirely by financial and productivity shocks.[46]

A. Step 1: Structural Estimation of Shocks

Recall from (28) that I observe the cyclical fluctuations in the output and employment of each industrial production industry, denoted $\hat{X}_t$ and $\hat{N}_t$ respectively. Looking at this data through the lense of the model, a fluctuation in the output of an industry i at quarter t is comprised of four components: fluctuations due to financial and productivity shocks directly to industry i, and fluctuations due to financial and productivity shocks to other industries, which are then transmitted to industry i via network effects. I use the model to filter these observed fluctuations for network effects, thereby backing-out the industry-level financial and productivity shocks to each industry. I use the model to filter out these network effects from observed fluctuations, thereby revealing the industry-level financial and productivity shocks to each industry.

To do this, recall that from (28) I have an exactly identified system of equations. Given the observations $\hat{X}_t$ and $\hat{N}_t$, I then invert the system to back-out industry-level each quarter over my sample period 1997 Q1 to 2013 Q4. Denote by $\check{B}_t$ and $\check{z}_t$ the M-by-1 vectors of financial and productivity shocks estimated with this procedure in quarter t. And let $Q \equiv H_X - G_X G_N^{-1} H_N$.

$$\check{B}_t = G_N^{-1}\left(\hat{N}_t - H_N \check{z}_t\right) \tag{31}$$

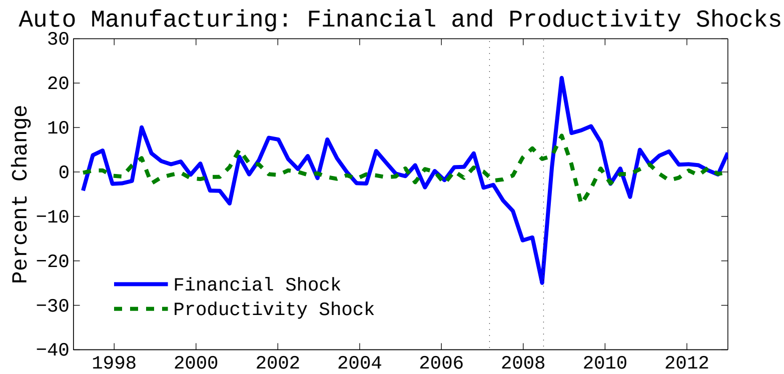
$$\check{z}_t = Q^{-1}\hat{X}_t - Q^{-1} G_X G_N^{-1} \hat{N}_t \tag{32}$$

The model is able to separately identify these shocks because each type of shock has quantitatively differential effects on an industry's output and employment. Namely, productivity shocks affect an industry's output relative to its employment through Cobb-Douglas production functions. On the other hand, financial shocks do not affect production functions, but tightent the cash-in-advance constraints.

Figure 8 shows the time series of the estimated financial and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

[46] In Foerster et al. (2011) and Acemoglu et al. (2012), fluctuations are assumed to be driven either entirely by productivity shocks; in Bigio and La'O (2016), they are driven entirely by financial shocks. I make a weaker assumption and allow for both types of shocks, and utilize output and employment data to identify them.

Figure 8:



Notes: This figure shows the quarterly time series of the productivity and financial shocks to the auto manufacturing industry over the sample period. Financial shocks are capttured by percent changes in parameters B_i in the model, and thus represent exogenous tightnening in the cash-in-advance constraint of an industry. Productivity shocks are changes in TFP. These shocks were estimated using the log-linearized model, and quarterly data on the employment and output growth of IP industries, obtain from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively.

Between 2007 and 2009, the output and employment of IP industries took a sharp drop for a number of quarters. As illustrated in the figure, this contraction shows up in the model as an acute tightening in the financial constraints of these firms, reaching up to a 25 percent decline in a single quarter. Note also that the TFP of the auto manufacturing did not fluctuate greatly over this recessionary period; in fact, it increased slightly. These features broadly hold across most industries in industrial production.

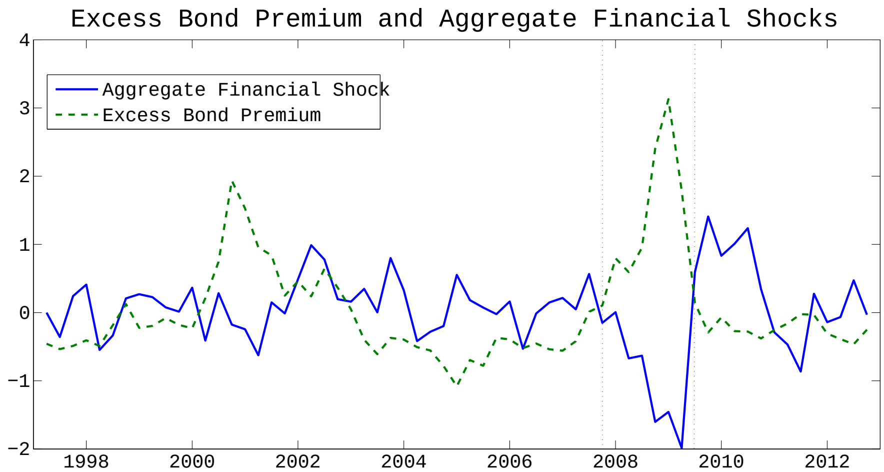
B. Step 2: Dynamic Factor Analysis

Next, I decompose the financial and productivity shocks, $\check{B}_t$ and $\check{z}_t$, into an aggregate and industry-level shock. I assume that each may be described by a common component and a residual idiosyncratic component.

$$\check{B}_t = \Lambda_B F_t^B + u_t \ , \qquad\qquad \check{z}_t = \Lambda_z F_t^z + v_t \tag{33}$$

Here, F_t^B and F_t^z are scalars denoting the common factors affecting the output and employment growth of each industry at quarter t. I interpret these factors as aggregate financial and productivity shocks, respectively. The M-by-1 vectors Λ_B and Λ_z denote the factor loadings, and map the aggregate shocks into each industry's financial and productivity shocks. Together, $\Lambda_B F_t^B$ and $\Lambda_z F_t^z$ comprise the aggregate components of $\check{B}_t$ and $\check{z}_t$. Furthermore, I assumethat the common factors each follow an AR(1) process. The residual components, u_t and v_t, unexplained by the common factors, are the idiosyncratic or industry-level shocks

Figure 9:



Notes: This figure shows the time series of the excess bond premium of Gilchrist and Zakrajsek (2012) and the aggregate component of the financial shocks to industrial produciton industries estimated using the structural factor analysis of these industries' quarterly output and employment growth. The excess bond premium is a component of corporate credit spreads designed to capture changes in the risk-beary capacity of the financial sector. This measure was obtain from Gilchrist and Zakrajsek (2012).

affecting each industry's financial and productivity growth. (The dynamic factor model and its assumptions are described in more detail in Appendix A4.) Hence, I estimate two dynamic factor models; one for the financial shocks $\check{B}_t$ and one for the productivity shocks $\check{z}_t$.[47] The aggregate component explain most of the financial shocks suffered by auto manufacturing. These features are fairly representative of those in other industries.

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks to the excess bond premium.[48] estimated using the model with a measure of the risk-bearing capacity of the US financial sector - namely, the excess bond premium of Gilchrist and Zakrajsek (2012). Figure 9 plots the two series together. The negative correlation (-0.51) between the two indicates that the sharp drop in the supply of credit from the financial sector is picked up by model as a dramatic tightening in the financial constraints of firms in IP. Hence the large aggregate financial shocks estimated by the structural factor analysis is broadly reflective of the severe credit crunch that occurred during this period.

[47] I use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more infortmation in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

[48] Recall the excess bond premium of Gilchrist and Zakrajsek (2012) is a measure of the risk-bearing capacity of the financial sector.

Table 2: Pre-Recession Composition of Agg. Vol.: 1997Q1:2006Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	0.365
Agg. Component	0.133
Idios. Component	0.232
Financial Shocks	0.635
Agg. Component	0.45
Idios. Component	0.185

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the period 1997 Q1 - 2006 Q4. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Shocks to industrial produciton industries were estimated using the structural factor analysis of these industries' quarterly output and employment growth, obtained from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively. The aggregate and idiosycnratic components were estimated by dynamic factor analysis of the industry-level financial shocks, where the common components are assumed to follow an AR(1) process.

C. Decomposing Observed Fluctuations in Industrial Productions

I next perform a variance decomposition of observed industrial production from 1997 Q1 to 2013 Q4. The factor model (33) implies that output volatility Σ_{XX} can be decomposed into components coming from Σ^B_{FF}, Σ^z_{FF} , Σ_{uu} , and Σ_{vv} the variance-covariance matrices of the factors F^B_t, F^z_t, and the idiosyncratic shocks u_t, v_t, respectively.

$$\Sigma_{XX} = G_X \Sigma_{BB} G'_X + H_X \Sigma_{zz} H'_X \tag{34}$$

$$\Sigma_{BB} = \Lambda_B \Sigma^B_{FF} \Lambda'_B + \Sigma_{uu} \qquad \Sigma_{zz} = \Lambda_z \Sigma^z_{FF} \Lambda'_z + \Sigma_{vv} \tag{35}$$

I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19%.[49] The results are summarized in Table 2.

Before the Great Recession, aggregate volatility was driven primarily by aggregate financial shocks and idiosyncratic productivity shocks; aggregate financial shocks account for nearly a half of aggregate volatility. Nevertheless, idiosyncratic productivity shocks account for a quarter of aggregate volatility.

Furthermore, the credit network of industrial production industries amplified these shocks, accounting for nearly one-fifth of observed aggregate volatility. Put differently, in the absence of the credit linkage channel of propagation, aggregate volatility from 1997-2006 would be 17 percent lower.

Aggregate financial shocks were the primary driver of the Great Recession. I perform an

[49]This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.

accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production over 2007Q4: 2009Q2 can be explained by each type of shock. For each quarter during this period, I use the estimated shocks to decompose the drop in aggregate IP into components arising from each type of shock. For each quarter, this yields a breakdown of the quarterly decline in aggregate IP across each shock. I then take a weighted sum of these breakdowns across quarters, with weights reflecting the share of the total fall in aggregate IP that occurred each quarter. I thus measure how much of the total peak-to-trough decline in aggregate IP during the Great Recession can be accounted for by each type of shock.

I find that both aggregate and idiosyncratic productivity shocks were on average slightly positive during this period. As such, changes in productivity did not contribute to the decline in aggregate IP during the recession. On the contrary, the observed movements in aggregate IP can be accounted for by financial shocks. I find that 73 percent of the drop in aggregate IP is due to an adverse aggregate financial shock. This is natural given the financial crisis that occurred during the beginning of the recession.

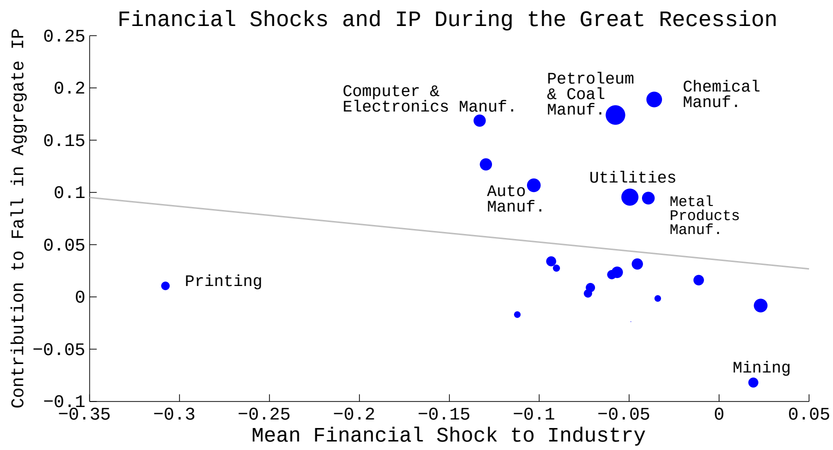
Financial shocks explain all of the downward pressure on industrial production. Indeed, 73 percent of the drop in aggregate IP is due to aggregate financial shocks. Of the remaining 27 percent not explained by the aggregate financial shock, idiosyncratic financial shocks to the three most systemically important industries can account for a sizable fraction.

Figure 10 depicts the relationship between industry-level financial shocks and an industry's contribution to aggregate output, for industrial production industries during the Great Recession. The horizontal axis plots the mean financial shock (including both the aggregate and idiosyncratic components) to each IP industry over the recessionary period; the vertical axis the share of the peak-to-trough drop in aggregate output attributable to each industry. The plot is weighted by the measure of each industry's systemic importance produced by my quantitative analysis in section IV.B, and are augmented with a least-squares line.

Large financial shocks to a few systemically important industries can explain the bulk of the decline in aggregate industrial production during the Great Recession. In fact, idiosyncratic shocks to the oil and coal products manufacturing, chemical products manufacturing, and auto manufacturing industries account for about 9 percent of the decline (or one-third of the decline unaccounted for by aggregate shocks), despite comprising only about 25 percent of aggregate IP. This suggests that idiosynratic financial shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

In contrast, both the aggregate and idiosyncratic components of productivity shocks were slightly positive during this period on average. As such, changes in productivity did not contribute to the decline in aggregate IP during the recession.

Figure 10:



Notes: This figure shows a scatter plot of industrial production industries by the mean quarterly financial shock to each industry during the recessionary period 2007 Q4 - 2009 Q2, and by each industry's contribution to the peak-to-trough fall in aggregate industrial production observed over this period. Each industry's spot is weighted by a measure of the indsutry's systemic importance to the US economy, computed using numerical simulations. The figure includes a least-squares line. Shocks were estimated using a structural factor approach and quarterly data on the employment and output growth of IP industries, obtained from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively. The shocks consist of both the aggregate and idiosyncratic components. An industry's contribution to the peak-to-trough drop in aggregate industrial production is computed by simulating the path of an index of each indsutry-level component of aggregate IP (i.e. how aggregate IP would evolve with shocks to exactly one industry) and computing the peak-to-trough change between 2007 Q4 and 2009 Q2.

Take-Aways from the Two Empirical Analyses

The broad picture which emerges from these empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession. While much of the previous literature has relied on shocks to aggregate TFP drive the business cycle, the dearth of direct evidence for such shocks has raised concerns about their empirical viability. On the other hand, importance of the financial sector for real activity, and standard interpretations of the causes of the Great Recession, point to disturbances in financial markets as a crucial player in the business cycle.

I have argued that the credit and input-output interlinkages of firms can create a powerful mechanism by which a shock to one firm's financial constraint propagates across the economy. The confluence of my empirical results suggest that once we account for these interlinkages, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

Conclusion

In this paper, I showed that inter-firm lending plays an important role in business cycle fluctuations. First, I introduced supplier credit into a network model of the economy. In this model, a shock to one firm's liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects. Thus, the endogenous response in cash-in-advance constraints to financial shocks is crucial for how the economy behaves in response to financial shock.

To evaluate the model quantitatively, I constructed a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data. Counterfactual exercises reveal that these interlinkages can produce a powerful amplification mechanism of industry-level shocks.

Finally, I used the model to investigate which shocks drive the US business cycle when we account for the linkages between industries. To do so, I took two approaches to identifying shocks. In the first, I identified shocks without the use of my model, by estimating an identified VAR. In the second, I constructed shocks using my model, and used factor methods to decompose them into aggregate and idiosyncratic components. Feeding these shocks though the model showed financial shocks to be a key driver of aggregate fluctuations, particularly during the Great Recession, and productivity shocks to play only a minor role. Thus, accounting for the role that credit and input-output interlinkages play helps to capture the empirical importance of financial shocks in US business cycle fluctuations.

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Appendix

A1. Demand for Trade Credit

Recall that the tightness of firm i's cash-in-advance constraint is given by

$$\chi_i \equiv \underbrace{\frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i}}_{\textit{debt/revenue ratio}} + \underbrace{1 - \frac{\tau_i}{p_i x_i}}_{\textit{cash/revenue ratio}}$$

Firm i's problem is to choose its input purchases and trade credit borrowing to maximize its profits, subject to its cash-in-advance constraint. Recall that competition amongst suppliers forces each firm to offer the maximum trade credit allowed by the borrowing constraint. Therefore, firm i takes τ_i as given.

$$\begin{aligned}\max_{n_i,\, x_{i-1},\, \tau_{i-1}} \quad & p_i x_i - w n_i - p_{i-1} x_{i-1} \\ s.t. \;\; w n_i + p_i x_{i-1} \quad & \leq \chi_i(\tau_{i-1}) p_i x_i \\ \\ \tau_{i-1} & \leq \theta_i p_i x_i\end{aligned}$$

Notice that in general, there is a tradeoff to taking more trade credit (i.e. to increasing τ_{i-1}). A higher τ_{i-1} relaxes firm i's cash-in-advance constraint, allowing it to purchase more inputs *ceteris paribus*. But a higher τ_{i-1} may also tighten its supplier's cash in advance constraint, causing the price of its intermediate good p_{i-1} to increase.

Let τ^*_{i-1} denote the optimal amount of trade credit borrowing. We can solve for optimal τ^*_{i-1} separately from n_i and x_{i-1}. In particular, there are three relevant cases.

Case 1) If both i and $i-1$ are unconstrained in equilibrium, then there is no tradeoff to firm i taking marginally more τ_{i-1}. So there is a continuum of τ_{i-1} between which firm i is indifferent: the set of all τ_{i-1} such that both firm i and firm $i-1$ are unconstrained in equilibrium, i.e. $\chi_i, \chi_{i-1} < 1$. Without loss of generality, we can take $\tau^*_{i-1} = \min \tau_{i-1} | \;\; \chi_i, \chi_{i-1} < 1$.

Case 2) If i is unconstrained in equilibrium, but $i-1$ is constrained in equilibrium, then the tradeoff mentioned above applies. The optimal τ_{i-1} will be the minimum such that i's cash-in-advance constraint is not binding. Any $\tau_{i-1} > \tau^*_{i-1}$ will further constrain supplier $i-1$, and therefore i will face a higher input price p_{i-1}. And any $\tau_{i-1} < \tau^*_{i-1}$ will mean that firm i will be constrained in equilibrium and will have to reduce production.

Case 3) If firm i is constrained in equilibrium, τ^*_{i-1} is the maximum allowable by the trade credit borrowing constraint: $\tau^*_{i-1} = \theta_i p_i x_i$. To see this, first recall that firm i actually consists of a continuum of identical firms with CRS production. Being constrained, each individual firm has an incentive to take the maximum amount of trade credit. They do not internalize the fact that, when all firms do this, they may increase the price p_{i-1} of inputs that they face. Therefore, even if there is a $\tilde{\tau}_{i-1} < \theta_i p_i x_i$ such that industry-wide profits will be higher (taking into account tradeoff of lower input price p_{i-1}), these firms are unable to coordinate on that $\tilde{\tau}_{i-1}$. Thus, in any equilibrium in which firm i is constrained (i.e. its cash-in-advance constraint is binding), the trade credit borrowing constraints bind and $\tau_{i-1} = \theta_i p_i x_i$.

Given its choice of τ^*_{i-1}, firm i then chooses its inputs to solve:

$$\begin{aligned}\max_{n_i,\, x_{i-1}} \quad & p_i x_i - w n_i - p_{i-1} x_{i-1} \\ s.t. \;\; w n_i + p_i x_{i-1} \quad & \leq \chi_i(\tau^*_{i-1}) p_i x_i\end{aligned}$$

A2. Proof of Amplification

From the definitions of χ_i and ϕ_i, we have

Table 3:

α	$P(\hat{\alpha} \leq \alpha)$	% Change in GDP	Credit Network Amplification
0	0.18	4.04%	77.2%
0.1	0.32	3.26%	43.0%
0.2	**0.5**	**2.92%**	**28.1%**
0.4	0.66	2.59%	13.6%
0.5	0.75	2.50%	9.6%
1	0.97	2.28%	0%

Notes: This table reports the results of the sensitivity analysis. Recall that α is the fraction of accounts receivable that banks can collateralize to borrow from the bank, and controls the substitutability of cash and bank credit for firms in the model. The first column indicates the value of α used. The second column yields the fraction of Italian firms which collateralizes less than α of their receivables to borrow from banks, as estimated by Omiccioli (2005). The third column lists the total percentage change in GDP in response to a 1 percent financial shock to all US industries. The fourth column lists by how much the credit network effects amplify the drop in GDP in response to the shock. The bold row indicates the baseline calibration.

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \theta_{i,i-1} - \theta_{i+1,i}\frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})}\right)\right\}$$

Here, $r_i = 1$ denotes firm i's returns-to-scale. It follows that

$$\frac{d\,\phi_{i-1}}{d\,B_i} = \begin{cases} \frac{1}{r_i}\frac{\alpha\theta_{i,i-1}}{\phi_i\omega_{i,i-1}(1-\eta_i)} > 0 & if\ \phi_{i-1} < 1 \\ 0 & otherwise \end{cases}$$

$$\frac{d\,\phi_j}{d\,B_i} = 0\ \forall\, j > i\ \ and\ \ \frac{d\,\phi_j}{d\,B_i} = \frac{1}{r_i} > 0\ for\ j = i$$

Putting these cases together, we can write $\frac{d\,log\,\phi_j}{d\,B_i}$ for any j.

$$\frac{d\,log\,\phi_j}{d\,B_i} = \begin{cases} \frac{1}{r_i} > 0 & if\ j = i \\ \frac{1}{\phi_j}\frac{1}{r_j}\frac{\theta_{kj}}{\phi_k\omega_{kj}(1-\eta_k)}\frac{d\,\phi_k}{d\,B_i} \geq 0\ \forall\, k & if\ j < i \\ 0 & otherwise \end{cases}$$

It follows that $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ and $\frac{d}{d\,\theta_{ij}}\left(\frac{d\,log\,\phi_j}{d\,B_i}\right) \geq 0$.

A3. Sensitivity Analysis

In section IV.A, I computed the change in GDP to a counterfactual one percet aggregate financial shock. Table 3 reports these results for different values of α.

Recall that a higher α mitigates the transmission of financial shocks, as firms are better able to offset a lower cash/revenue ratio with a higher loan from the bank. When $\alpha = 1$, the two are perfect substitutes and financial shocks have no credit network effects. While the multiplier effect of the credit network indeed falls as α approaches 1, credit network effects are quantitatively significant for reasonable values of α.

A4. Structural Factor Analysis: Aggregate Volatility

Recall that the growth in industry output can be written as a function of the industry financial and productivity shocks. Recall that X_t is a vector of the percentage change $\tilde{x}_{it}$ in each industry's output at time t.

$$X_t = G_X B_t + H_X z_t$$

And the shocks B_t and z_t, in turn, are composed of an aggregate and idiosyncratic components.

$$B_t = \Lambda_B F_t^B + u_t \qquad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B$$

$$z_t = \Lambda_z F_t^z + v_t \qquad F_t^z = \gamma_z F_{t-1}^z + \iota_t^z$$

Then letting Σ_{XX} denote the variance-covariance matrix of X_t (and similarly for the other variables), we have

$$\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X'$$

$$\Sigma_{BB} = \Lambda_B \Sigma_{FF}^B \Lambda_B' + \Sigma_{uu} \qquad \Sigma_{zz} = \Lambda_z \Sigma_{FF}^z \Lambda_z' + \Sigma_{vv}$$

where Σ_{uu} and Σ_{vv} are diagonal matrices.

Let $\bar{s}$ denote a time-invariant vector of industry shares of aggregate output. Then the variance of aggregate output, i.e. *aggregate volatility* in the economy, is approximately given by

$$\sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s} = \bar{s}' G_X \Sigma_{BB} G_X' \bar{s} + \bar{s}' H_X \Sigma_{zz} H_X' \bar{s}$$

Then the contribution of aggregate financial shocks to aggregate volatility is given by

$$\frac{\bar{s}' G_X \left(\Lambda_B \Sigma_{FF}^B \Lambda_B' \right) G_X' \bar{s}}{\sigma^2}$$

A5. Solution for Sytlized Model

I solve in closed form for aggregate output in the stylized (vertical) economy. I proceed recursively, beginning with the final firm in the chain, firm M.

Firm M

Recall that firm M collects none of its sales from the household up front (does not give the household any trade credit, $\tau_M = 0$). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from $M-1$, to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

$$\max_{n_M, x_{M-1}, b_M, \tau_{M-1}} \quad p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t.\ \ wn_M + p_{M-1}x_{M-1} \leq b_M + \tau_{M-1} + p_M x_M - \tau_M$$
$$b_M \leq B_M p_M x_M + \alpha \tau_M$$
$$\tau_{M-1} p_{M-1} x_{M-1} \leq \theta_M p_M x_M$$

We can combine the constraints to re-write the problem.

$$\max_{n_M, x_{M-1}, b_M, \tau_{M-1}} \ p_M x_M - wn_M - p_{M-1}x_{M-1}$$
$$s.t.\ \ wn_M + p_{M-1}x_{M-1} \leq \chi_M p_M x_M$$

where $\chi_M = \frac{\tau^*_{M-1}}{p_M x_M} + B_M$. Here, τ^*_{M-1} denotes firm M's choice of trade credit borrowing, based on the arguments given in Appendix A1. (Notice that when firm M is constrained, $\chi_M = \theta_{M,M-1} + B_M$).

If firm M is unconstrained in equilibrium, then the optimality conditions equate the marginal cost of each type of input with the marginal revenue.

$$w = \eta_M \frac{p_M x_M}{n_M} \qquad p_{M-1} = (1 - \eta_M) \frac{p_M x_M}{x_{M-1}} \tag{36}$$

Firm M's expenditure in inputs is then

$$wn_M + p_{M-1}x_{M-1} = (\eta_M + (1 - \eta_M))\, p_M x_M \tag{37}$$

Therefore, firm M is then unconstrained in equilibrium if and only if its expenditure at its unconstrained optimum is less than its liquidity at this optimum.

$$p_M x_M < \chi_M p_M x_M \qquad i.e.\ \ \chi_M > 1 \tag{38}$$

If firm M is constrained in equilibrium, then its binding cash-in-advance pins down its level of output. The only choice left to make is how much labor to hire n_M versus how much intermediate goods x_{M-1} to purchase, given its level of output x_M. Because χ_M is independent of M's choice of n_M and x_{M-1}, the problem of maximizing profits subject to the binding cash-in-advance is equivalent to minimizing its expenditure $n_M + x_{M-1}$ subject to producing x_M. Thus, it solves the following cost-minimization problem.

$$min_{n_M,\, x_{M-1}}\, wn_M + p_{M-1}x_{M-1}$$
$$s.t.\ x_M = z_M n_M^{\eta_M} x_{M-1}^{(1-\eta_M)}$$

Then firm M's optimality condition equates the ratio of expenditure on each input with the ratio of each input's share in production.

$$\frac{wn_M}{p_{M-1}x_{M-1}} = \frac{\eta_M}{(1 - \eta_M)} \tag{39}$$

Using this, we can rewrite M's binding cash-in-advance as

$$wn_M \left(1 + \frac{(1 - \eta_M)}{\eta_M}\right) = \chi_M p_M x_M \tag{40}$$

Rearranging yields

$$w = \eta_M \chi_M \frac{p_M x_M}{n_M} \tag{41}$$

Combining (41) with its analog (36) in the unconstrained case, we can see that

- if $\chi_M > r_M$ (i.e. if firm i is unconstrained in equilibrium)

$$w = \eta_M \frac{p_M x_M}{n_M}$$

- otherwise

$$w = \eta_M \chi_M \frac{p_M x_M}{n_M}$$

These two cases imply that we can write the optimality condition as

$$w = \phi_M \eta_M \frac{p_M x_M}{n_M} \tag{42}$$

$\phi_M \equiv min\{1,\ \chi_M\}$ represents the distortion in firm M's optimal labor usage due to its cash-in-advance. Financial frictions introduce wedge between firm's marginal benefit and cost of production. The wedge between these two objects is increasing in the tightness χ_M of M's constraint, and decreasing in the returns-to-scale of firm M's production function.

Firm M-1

Given firm M's solution, we can proceed to firm M-1's problem.

$$max_{n_{M-1}, x_{M-2}}\ p_{M-1}x_{M-1} - wn_{M-1} - p_{M-2}x_{M-2}$$

$$s.t.\ \ wn_{M-1} + p_{M-2}x_{M-2} \leq \chi_{M-1}p_{M-1}x_{M-1}$$

where

$$\chi_{M-1} = \frac{\tau^*_{M-2}}{p_{M-1}x_{M-1}} + B_{M-1} + 1 - (1-\alpha)\frac{\tau_M}{p_{M-1}x_{M-1}}$$

Here, τ^*_{M-2} denotes firm M-1's optimal choice of trade credit borrowing. Again, if the firm's trade credit borrowing constraint binds in equilibrium, then $\chi_{M-1} = \theta_{M-1} + B_{M-1} + 1 - (1-\alpha)\frac{\tau_M}{p_{M-1}x_{M-1}}$. If firms' trade credit borrowing constraints bind in equilibrium, then

$$\chi_{M-1} = \theta_{M-1} + B_{M-1} + 1 - (1-\alpha)\frac{\theta_M p_M x_M}{p_{M-1}x_{M-1}}$$

And (??) and (??) imply that $\frac{p_M x_M}{p_{M-1}x_{M-1}} = \frac{1}{\phi_M \omega_{M,M-1}(1-\eta_M)}$. Therefore,

$$\chi_{M-1} = \theta_{M-1} + B_{M-1} + 1 - \alpha\frac{\theta_M}{\phi_M(1-\eta_M)}$$

Since ϕ_M is given by (??), this is a closed-form expression for χ_{M-1}. Note that, since ϕ_M depends on χ_M, χ_{M-1} is an increasing function of χ_M; this interdependence of cash-in-advances comes from the trade credit relationship between M and M-1.

Given χ_{M-1}, the solution to firm M-1's problem takes the same form as that of firm M. (Note that χ_{M-1} does not depend directly on M-1's choice of n_{M-1} versus x_{M-2}. Therefore, when constrained in equilibrium, M-1 will solve the analogous cost-minimization problem as M to maximize profits.) The cash-in-advance places a wedge ϕ_{M-1} between the marginal benefit of hiring labor and the marginal cost

$$w = \phi_{M-1}\eta_{M-1}\frac{p_{M-1}x_{M-1}}{n_{M-1}}$$

Given the above expressions for χ_{M-1} and χ_M, the the wedge $\phi_{M-1} = min1, \chi_{M-1}$ is a closed-form expression.

Equilibrium

Each other firm's problem is symmetric. Continuing recursively, I obtain the closed-form solution for each firm. To summarize, I have, for each firm i

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}$$

where

$$\phi_i = min\{1, \chi_i\} \quad and \quad \chi_i = B_i + \theta_i + 1 - \theta_{i+1}\frac{1}{\phi_{i+1}\omega_{i,i-1}(1-\eta_i)}$$

Market clearing conditions are given by

$$C = Y \equiv x_M \,, \qquad N = \sum_{i=1}^{M} n_i$$

Given these expressions, the task is to write each n_i as a function of aggregate output x_M, starting with firm M-1. From the firm optimality conditions, we have the following three expressions:

$$wn_{M-1} = \phi_{M-1}\eta_{M-1}p_{M-1}x_{M-1}, \quad wn_M = \phi_M\eta_M p_M x_M, \quad p_{M-1}x_{M-1} = wn_M\frac{(1-\eta_M)}{\eta_M}$$

Combining these yields n_{M-1} as a function of x_M.

$$wn_{M-1} = \phi_M\phi_{M-1}\eta_{M-1}\omega_{M,M-1}(1-\eta_M)p_M x_M$$

Continuing recursively, we can write n_i as a function of x_M, for each i.

$$wn_i = p_M x_M\left(\prod_{j=i}^{M}\phi_j\right)\left(\prod_{j=i}^{M-1}\omega_{j+1,j}(1-\eta_j)\right)\eta_i \tag{43}$$

The household's preferences and optimality conditions imply

$$w = \frac{V'(N)}{U'(x_M)} = x_M \tag{44}$$

Let good M be the numeraire. Combining (44) with (43) yields a closed-form expression for each firm's labor use.

$$n_i = \eta_i\prod_{j=i+}^{M}\omega_{j,j-1}(1-\eta_j)\phi_j \tag{45}$$

By recursively plugging in the production functions into one another, we can obtain aggregate output as a function of the labor use of each firm, where $\delta_{M-i} \equiv \prod_{j=0}^{i-1}(1-\eta_{M-j})$.

$$Y = \left(\prod_{i=0}^{M-1} z_{M-i}^{\delta_{M-i}}\right)\left(\prod_{i=0}^{M-1} n_{M-i}^{\eta_{M-i}\delta_{M-i}}\right) \tag{46}$$

Then combining (46) and (45) yields a closed-form expression for aggregate output (12).