Confidence, Bond Risks, and Equity Returns[☆]

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Abstract

We show that investor confidence (size of ambiguity) about future consumption growth is driven by past consumption growth and inflation. The impact of inflation on confidence has moved considerably over time and switched on average from negative to positive in 1997. Motivated by this evidence, we develop and estimate a model in which the confidence process has discrete regime shifts, and find that the time-varying impact of inflation on confidence enables the model to match bond risks over different subperiods. The model can also account for stock and bond return predictability, correlation between price-dividend ratios and inflation, among other features of the data.

Keywords: Bond Risks, Ambiguity, Confidence, Inflation

JEL Classification: G12

1. Introduction

When making decisions, investors face both risk and ambiguity. Risk refers to the situation where there is a probability measure to guide choice, while ambiguity refers to the situation where the decision maker is uncertain about the data-generating process

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itself due to cognitive or informational constraints. In this paper, investor confidence, or the size of ambiguity, is represented by a set of one-step ahead measures regarding aggregate consumption growth. In equilibrium, an ambiguity averse investor evaluates future prospects under the worst-case measure. Using forecast dispersion as a measure of confidence, we show that investor confidence is driven by past consumption growth and inflation. While the effect of past consumption growth on dispersion has been always positive, the impact of inflation has moved considerably over time and switched on average from negative to positive in 1997. This paper argues that stock and Treasury bond prices, as well as their comovements are driven by changes in investor confidence and its time-varying correlation with inflation. This specific feature of confidence allows a general equilibrium model to capture a wide range of asset pricing phenomena through the stochastic discount factor.

For the past decade and particularly during the two recessions of the early and late 2000s, stock prices plunged while Treasury bonds performed well. At the same time, inflation was low and forecast dispersion was high. With a positive impact of inflation on confidence, Treasury bonds served to hedge the risk of low consumption/output growth confidence in investor's portfolios. Consistent with this, the correlation between stocks and Treasury bonds returns was negative, the correlation between price-dividend ratios and inflation was positive, and the correlation between forecast dispersion and inflation was negative. However, this behavior of stocks/bonds and its connection with investor confidence was very different during the 1970s and particularly the 1980s, when high inflation made investors less confident about future consumption growth, and Treasuries became as risky as stocks. Treasury bond returns became positively correlated with stock returns, price-dividend ratios were negatively correlated with inflation, and forecast dispersion was positively correlated with inflation. While the risk properties of Treasury bonds relative to stocks have been documented and studied by recent papers,[2] this paper

[2]See, e.g., Campbell et al. (2014)(CPV); Baele, Bekaert, and Inghelbrecht (2010); Campbell, Sunderam, and Viceira (2016); Christiansen and Ranaldo (2007); David and Veronesi (2013); Guidolin and Timmermann (2007); Viceira (2012)

provides an equilibrium model to understand bond risks through investor confidence.

From the perspective of equilibrium asset-pricing models, another puzzling fact related to Treasury bonds is the excess bond return predictability. Against the expectations hypothesis, Fama and Bliss (1987), Campbell and Shiller (1991), Dai and Singleton (2002), and Cochrane and Piazzesi (2005) provide evidence for bond return predictability using yield spreads and forward rates as predictors. While it has been difficult to account for the bond return predictability,[3] it is much harder for an equilibrium model of bond pricing to also capture the comovements of stocks and bonds. Moreover, the moments of stock return, risk free rate, and especially the correlation between price-dividend ratio and inflation in the data pose serious challenges to equilibrium models.

This paper develops a consumption-based asset pricing model that helps to explain the preceding features of stock/bond market data. There are two main ingredients in the model. First, departing from the rational expectations hypothesis, the model assumes that all identical investors are ambiguity averse and have the recursive multiple-priors preference axiomatized by Epstein and Schneider (2003) and Hayashi (2005). This model of preference permits a three-way separation of intertemporal elasticity of substitution (IES), risk aversion, and ambiguity aversion. Investors in this economy have in mind a benchmark or reference measure of the economy's dynamics that represents the best estimate of the data-generating process. They are concerned that the reference measure is misspecified and that the true measure is actually in a set of alternative measures that are statistically close to the reference measure. The level of confidence is represented by the size of the set of alternative measures at a given time. Second, under the reference measure, consumption and dividend growth are independently and identically distributed log normal processes, with the same mean and standard deviation as in the data. The model can accommodate more complex consumption processes, including processes with predictability, conditional heteroskedasticity, and non-normality. However, those are not salient features of the consumption data. Most importantly, I want to emphasize that even

[3]Except the recent development by Bansal and Shaliastovich (2013), who show that bond risk premiums can be predicted using uncertainty in inflation and consumption growth.

with the independent and identically distributed (i.i.d.) consumption growth assumption the model generates interesting asset price behavior internally. Inflation follows a first order autoregressive moving average process ARMA(1,1) correlated with consumption growth as in the data.

Using dispersion in forecasts of future mean consumption growth from the Philadelphia Fed's Survey of Professional Forecasters (SPF) as a measure for ambiguity (confidence), we find that fluctuations in confidence are driven by past consumption growth and inflation.[4] While the effect of past consumption growth has always been positive, the impact of inflation has moved considerably over time. Specifically, over the whole sample period, low consumption growth in the past make investors more concerned about model misspecification and less confident about future consumption growth (bigger dispersion), and high consumption growth in the past made them more confident (smaller dispersion). The average effects of inflation on confidence were negative over the whole sample, however, it was moderately negative during the 1970s, strongly negative during the 1980s, and became positive during the past decade. These three subperiods correspond to three different monetary policy and inflation regimes: the pre-Volcker period, the inflation fighting period of Volcker and Greenspan, and the recent period of low inflation and increased central bank transparency. One possible interpretation is that investor confidence relied on the central bank's (CB) performance: poor performance (low consumption growth, or high/low inflation during the first two subperiods/third subperiod) makes it more difficult for investors to understand the economic environment and thus to be less confident about future growth, and good performance (high consumption growth, or low/high inflation during the first two subperiods/third subperiod) makes them more confident. Thus, following this pattern in the data, we allow the confidence and inflation processes to have two discrete regime shifts in 1979 and 1997 to capture different impacts of inflation.

[4]There are other measures of ambiguity and forecasts dispersions available, but the timing of SPF is geared to the release of the Bureau of Economic Analysis' advance report of the national income and product accounts, which makes it a perfect measure to analyze the impacts of past aggregate macro variables on dispersion.

In this paper, stock and bond price variations are driven by the joint dynamics of confidence and inflation. During the 1970s and particularly the 1980s, one of the most important tasks for the Fed was to fight high inflation. High inflation realizations made it harder for investors to understand the economic environment and thus they may have become less confident about future consumption growth. Then investors would not buy stocks and stock prices would drop, and at the same time, long-term yields would increase and bond prices would decrease because of high inflation. Because of this negative effect of inflation on confidence, the prices of stocks and Treasuries moved in the same direction, and price-dividend ratios were negatively correlated with inflation in the first two subperiods. However, in the past decade the opposite happened. Instead of fighting high inflation, the Fed faced deflationary pressures. In this case, high inflation realizations made investors feel that the economic environment was well understood and felt more confident about future consumption growth. Stock prices rose, Treasury yields increased, and bond prices decreased as the result of high inflation. Stock and bond prices moved in opposite directions and Treasuries served as a hedge in this period.

While inflation had different impacts on confidence in different subperiods, the effect of past consumption growth on confidence was always positive. The interpretation is similar to inflation except that maintaining an efficient level of consumption/output growth is always one task of the CB. Low aggregate consumption realizations make investors less confident about future consumption growth, which in turn lowers the price-dividend ratio (pro-cyclical variation of price-dividend ratios) and increases expected returns (counter-cyclical variation of expected returns). Although variations in the price-dividend ratios reflect changes in ambiguity about future expected growth, the reference mean growth rate is constant. Thus, the model will not incorrectly imply that dividend yields predict consumption and dividend growth. At the same time, the log price-dividend ratio, as a linear function of confidence, is mean reverting and, thus, dividend yields predict excess returns as in the data. Using simulation data, the model also generates similar estimation coefficients of the expectations hypothesis test and match well the bond return predictability.

Related literature

This paper is related to a number of papers that have studied the implications of ambiguity and robustness for finance and macroeconomics.[5] This paper contributes to the literature by first showing the connection between ambiguity (confidence) and inflation/consumption growth, and then using the recursive multiple-priors preference to capture the stocks/bonds comovements, stock and bond return predictability, correlation between price-dividend ratios and inflation, and many other asset-pricing puzzles.

Several studies on ambiguity are closely related to this paper. Under the recursive multiple-priors framework of Epstein and Schneider (2003), Epstein and Schneider (2007) model learning under ambiguity using a set of priors and a set of likelihoods. The set of priors is updated by a generalized Bayes rule. Epstein and Schneider (2008) analyze asset pricing implications using this learning framework. Ju and Miao (2012) propose a smooth ambiguity model with learning and study the asset return implications. Those models generate dynamics in ambiguity size, or confidence, by Bayesian learning. Ilut and Schneider (2014) show how time-varying confidence about productivity generates business cycle fluctuations and confidence follows an exogenous AR(1) process in their model. Drechsler (2013) build a model with exogenous time-varying Knightian uncertainty to explain the volatile variance premium. There are two significant differences between those models and this paper: (1) this is the first paper to show that confidence about future consumption growth is affected by past inflation and consumption growth and (2) the connection between confidence and inflation allows the model to study both stock and bond markets: stock/bond comovements, stock/bond return predictability, and the correlation between price-dividend ratios and inflation. The models mentioned above

[5]Papers that study the multiple-priors preference and its application include Cao, Wang, and Zhang (2005), Chen and Epstein (2002), Epstein and Miao (2003), Epstein and Wang (1994), Garlappi, Uppal, and Wang (2007), Ilut (2012), Leippold, Trojani, and Vanini (2008), and Routledge and Zin (2009). Epstein and Ji (2013, 2014) propose a new continuous time framework that captures ambiguity about both volatility and drift. Papers of robustness applications include Anderson, Hansen, and Sargent (2003), Cagetti, Hansen, Sargent, and Williams (2002), Hansen (2007), and Hansen and Sargent (2001, 2010). For a survey on robustness, see Backus, Routledge, and Zin (2005) and Hansen and Sargent (2008).

focus only on the real side of the economy.

This paper is related to the recent developments in bond risks, bond return predictability, and term structure. Campbell, Sunderam, and Viceira (2016) show changes in magnitude and switches in sign of the covariation between bonds and stocks, and specified a multifactor term structure model to explain this feature. CPV (2014) relate changes in bond risks to periodic regime changes in the CB's monetary policy rule and the volatilities of macroeconomic shocks. David and Veronesi (2013) propose a model of regime-switching and learning and studied the joint behavior of stocks and bonds. The current paper provides an equilibrium model to understand bond risks through investor confidence. Piazzesi and Schneider (2007) show that inflation as bad news for future consumption growth should help to generate upward sloping nominal yield curve. Bansal and Shaliastovich (2013) show that a long-run risks model with time varying volatilities of expected consumption growth and inflation can account for bond return predictability. In the model this paper presents, inflation has time varying effects on confidence, and stock/bond price variations are driven by the joint dynamics of confidence and inflation, the model also matches well bond return predictability and produces an upward sloping nominal yield curve.

This paper is also related to the distorted belief literature. In Cecchetti, Lam, and Mark (2000), the consumption growth follows a two-state Markov process and the representative agent has distorted beliefs about persistence of the state-transition probabilities. They study the asset pricing implications under a specific belief distortion. Adam, Marcet, and Nicolini (2016) examine the asset pricing implications when the agent has distorted beliefs about price in a standard consumption-based asset pricing model, and the beliefs about price change by learning from past price observations. This paper differs from distorted belief models in that I start with ambiguity-averse investors who are concerned about model misspecification. There is a set of alternative measures that are hard to distinguish from each other and investors are not sure which are the true measures. They pick the worst-case measure in equilibrium, but adapting this measure, induced by changes in confidence, as a unique prior would seem contrived. Moreover, this paper studies both stock and bond markets, and the above distorted belief papers focus only on the stock market.

The paper continues as follows. Section 2 outlines the model and solves it analytically. Section 3 discusses the results of the empirical analysis. Section 4 provides concluding comments.

2. The model

In a pure exchange economy, identical ambiguity-averse investors maximize their utility over consumption processes. Individual consumption in period t is denoted by C_t, and C_t^a is the average consumption by all individuals in the economy. In equilibrium, identical individuals choose the same level of consumption, so $C_t = C_t^a$. In Section 2.3, I will specify how each individual's confidence level responds to the history of aggregate consumption growth. Therefore, except in Section 2.3, I drop the superscripts in what follows where they are not essential for clarity. Consumption/dividend growth and inflation are given exogenously. Equilibrium prices adjust such that the agent is happy to consume the endowment.

2.1. Economy dynamics

Under reference measure P, consumption/dividend growth and inflation have the joint dynamics described as

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \sigma_c \varepsilon_{c,t+1} \\
\Delta d_{t+1} &= \zeta_d \Delta c_{t+1} + \mu_d + \sigma_d \varepsilon_{d,t+1} \\
\hat{\pi}_{t+1} &= \rho_\pi \hat{\pi}_t + \zeta_\pi \varepsilon_{c,t+1} + \sigma_\pi (\varepsilon_{\pi,t+1} + \theta_\pi \varepsilon_{\pi,t}),
\end{aligned} \tag{1}$$

where $c_t = LogC_t$, $d_t = LogD_t$, Δc_{t+1} and Δd_{t+1} are the growth rate of consumption and dividends respectively. $\hat{\pi}_t = \pi_t - \pi$ is the demeaned inflation. I assume that all shocks are i.i.d normal and orthogonal to each other. Consumption and dividend growth are i.i.d. log normal with conditional expectation μ_c and $\zeta_d \mu_c + \mu_d$, respectively. The demeaned inflation follows a quasi ARMA(1,1) process, where ρ_π is the autoregressive parameter and θ_π is the moving average parameter. ζ_π captures the correlation between consumption growth and inflation.

The literature reports several ways to model dividends and consumption separately (see, e.g., Campbell (1999); Cecchetti, Lam, and Mark (1993); Bansal and Yaron (2004); Bansal, Kiku, and Yaron (2007)), and we follow Ju and Miao (2012) in this paper. The parameter $\zeta_d > 0$ can be interpreted as the leverage ratio on expected consumption growth, as in Abel (1999); together with the parameter σ_d, this allows one to calibrate the correlation of dividend growth with consumption growth. The parameter μ_d helps match the expected growth rate of dividends.

The literature offers different inflation models. To capture the persistence of inflation, a common feature of those models is an unobserved random walk component, for example, Stock and Watson (2007) and Piazzesi and Schneider (2007). For simplicity, we assume inflation follows an ARMA(1,1) process in this paper, which also fits well higher order autocorrelation in the data.[6]

The reference measure P represents the investor's best estimate from the data. However, investors are concerned that this reference measure is misspecified and that the true measure is actually in a set of alternative measures that are statistically 'close' to the reference measure. The ambiguity-averse agent acts pessimistically and evaluates future prospects under the worst-case measure.

2.2. Ambiguity about expected consumption growth

One requirement for the alternative measures is that they must be equivalent to the reference measure P (i.e., they put positive probabilities on the same events as P). The set of alternative measures is generated by a set of different mean consumption growth rates around the reference mean value μ_c. For simplicity, this paper considers only ambiguity about consumption growth. More generally, it could also be interesting to allow for am-

[6] Piazzesi and Schneider (2007) show that the AR(1) model for inflation tends to understate higher order autocorrelation.

biguity about unleveraged dividend and inflation. Specifically, under alternative measure $p^{\tilde{\mu}}$, consumption growth follows

$$\Delta c_{t+1} = \mu_c + \tilde{\mu}_t + \sigma_c \varepsilon_{c,t+1}, \tag{2}$$

where $\tilde{\mu}_t \in A_t = [-a_t, a_t]$ with $a_t > 0$. Each trajectory of $\tilde{\mu}_t$ will yield an alternative measure $p^{\tilde{\mu}}$ for the consumption growth process. The set of measures generated by A_t is a compact set, and this set of beliefs represents the agent's confidence regarding expected consumption growth. The a_t represents the investor's confidence about future consumption growth. A larger a_t implies that the agent is more ambiguity averse, or less confident; likewise, a smaller a_t means the agent is more confident about the consumption process, or less ambiguity averse. In the following Section, we specify how a_t changes over time.

2.3. Changes in confidence

Forecasts dispersion has been used as a measure of ambiguity in the literature. We follow Anderson, Ghysels, and Juergens (2009), Ilut and Schneider (2014), and Drechsler (2013) and measure the size of ambiguity using the dispersion in forecasts of future consumption growth from the Philadelphia Federal Reserve's (Fed's) SPF. We find that fluctuations in confidence are driven by past consumption growth and inflation. Motivated by this evidence, confidence is specified in the following way,

$$a_{t+1} - a = \rho_a(a_t - a) + \kappa_c \varepsilon_{c,t+1} + \kappa_\pi \hat{\pi}_{t+1} + \sigma_a \varepsilon_{a,t+1}, \tag{3}$$

where the persistence of the shocks is captured by ρ_a and parameter a is the long-run mean confidence level. κ_c captures how confidence is affected by consumption growth, and κ_π captures how confidence is affected by demeaned inflation. $\varepsilon_{a,t+1}$ is the confidence specific shock that captures factors other than consumption and inflation, for example major economic and political shocks like the Cuban missile crisis, the assassination of JFK, and the 9/11 terrorist attacks. It could also be shocks from daily economic news.

$\varepsilon_{a,t+1}$ is i.i.d normal and orthogonal to other shocks. Investor's confidence level depends on aggregate consumption growth rather than on an individual's own past consumption growth. This specification, which is similar to Campbell and Cochrane (1999) external habit formation, simplifies the analysis. It eliminates terms in marginal utility by which extra consumption today raises confidence tomorrow.

As argued in Ilut and Schneider (2014), the reason that forecasts dispersion can be used as a measure of ambiguity is that investors sample experts' opinions and aggregate them when making decisions. Since they are ambiguity averse, stronger disagreement among experts generates lower confidence in probability assessments of the future. The dispersion is calculated as the difference between the 95th percentile and the 5th percentile of the individual forecasts in levels.[7] First, to show the overall effects of consumption growth and inflation on dispersion in the whole sample, we use the longest possible data set for forecasts dispersion, from 1968 Q4 to 2012 Q1. Since forecasts dispersion of consumption growth is only available after 1981 Q3, we use forecasts dispersion of GDP growth as its approximation before 1981 Q3.[8] Table 1 provides the OLS regression results of forecasts dispersion of different horizons on lagged dispersion, lagged demeaned consumption growth, and lagged demeaned inflation. DispT1, DispT, DispT2, and DispT3 stand for one quarter ahead, current quarter, two quarters ahead and three quarters ahead forecasts dispersion. All the results show that, within the whole sample period, past inflation has a positive (negative) effect on dispersion (confidence) and all the coefficients of inflation are significant.[9] The coefficients of consumption are positive for DispT1 and

[7]As a measure for ambiguity, forecast dispersion can be calculated differently, for example, interquartile range of individual forecasts, difference between top 10 mean forecasts and bottom 10 mean forecasts, and other quartile differences. Different calculations will generate different means and volatilities, while the autocorrelation and correlations between confidence and other variables will stay the same if these different calculations just provide different scaling factors. The changes in bond risks are mainly drived by correlation between dispersion and inflation, which are likely to be same for different measures. Actually, an earlier version of this paper shows similar results for bond risks using interquartile range of individual forecasts.

[8]For forecasts dispersion of GDP and consumption growth after 1981 Q3, the mean and standard deviation of them are very close and the correlation is 81%. This make the GDP growth forecasts dispersion a good approximation for consumption growth forecasts dispersion before 1981 Q3.

[9]According to the model's timing, the most reasonable measures are the current quarter and one quarter ahead forecasts dispersion, but to avoid the potential risk that it is not a perfect measure for

	Constant	Lag Dispersion	Lag Consumption growth	Lag Inflation	R^2
DispT1	0.97**	0.38**	-31	192**	0.58
	(0.27)	(0.10)	(34)	(49)	
DispT	0.90**	0.54**	-29	140**	0.65
	(0.27)	(0.07)	(25)	(26)	
DispT2	0.71**	0.43**	6.59	170**	0.65
	(0.24)	(0.08)	(22)	(31)	
DispT3	0.70**	0.35**	7.65	215**	0.67
	(0.24)	(0.09)	(26)	(33)	

Table 1: Predictability of confidence
This table reports the OLS regression results between dispersion and lagged macro variables. DispT1, DispT, DispT2, and DispT3 stand for one quarter ahead, current quarter, two quarters ahead and three quarters ahead forecast dispersion from the Philadelphia Fed's Survey of Professional Forecasters. The whole sample is from 1968Q4 to 2012Q1. Before 1981Q3, confidence is measured by forecast dispersion of GDP growth, and it is measured by forecast dispersion of consumption growth after 1981Q3. Quarterly U.S. real nondurable goods and services growth is from 1968Q3 to 2011Q4 from the Bureau of Economic Analysis. Quarterly U.S. inflation is calculated by GDP price deflator from 1968Q3 to 2011Q4 from the Bureau of Economic Analysis. All standard errors are heteroskedasticity-robust.

DispT, which are the most reasonable measure for confidence in our model, and the R squares are very high. Note that there are other measures of ambiguity and forecasts dispersions available, but the timing of SPF is geared to the release of the Bureau of Economic Analysis' advance report of the national income and product accounts, which makes it a perfect measure to analyze the impacts of past aggregate macro variables on dispersion.

While the overall effect of inflation on dispersion (confidence) in the whole sample was positive (negative), its magnitude has moved considerably over time and sign switched in the past decade. To provide a more intuitive understanding, we plot the average dispersion against lagged inflation in Fig. 1. The figure shows that there is a clear positive association between dispersion and lagged inflation during high inflation periods, especially for 1980s and late 1970s. Beginning in the late 1990s, dispersion and lagged inflation started to move in opposite directions. Similarly, results can be seen in a plot of the average dispersion against lagged consumption growth in Fig. 2. Different from inflation, lagged consumption growth and dispersion move in opposite directions within the whole sample period.

ambiguity, the results are shown to be similar by using other measures.The model is calibrated using statistics of the average measure.

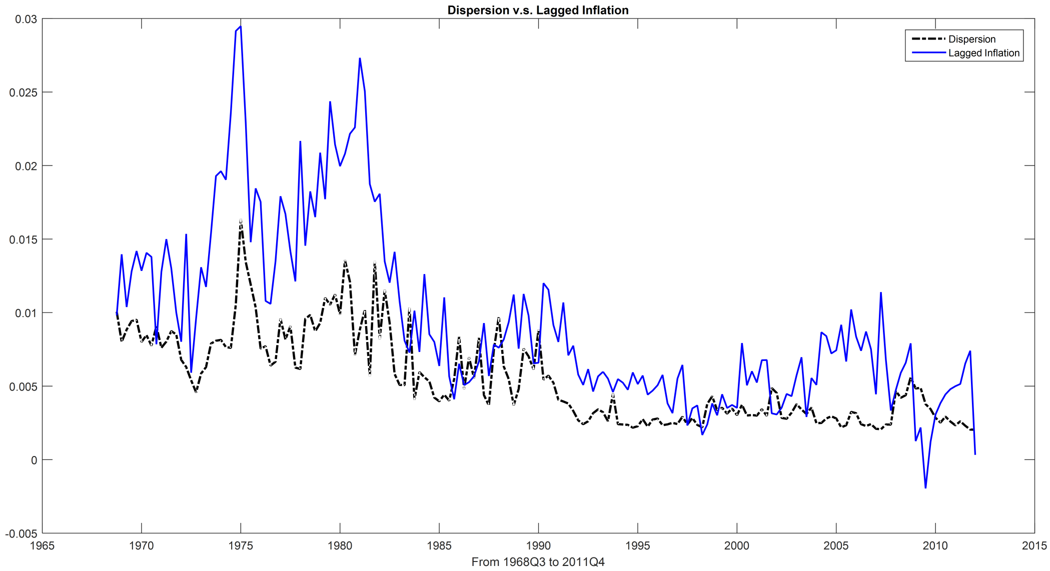

Figure 1: Time varying effect of inflation on dispersion
The dispersion is the average of current quarter, one quarter ahead, two quarters ahead and three quarters ahead forecast dispersion from the Philadelphia Fed's SPF, from 1968Q4 to 2012Q1. Lagged quarterly U.S. inflation is calculated by GDP price deflator from 1968Q3 to 2011Q4 from the Bureau of Economic Analysis.

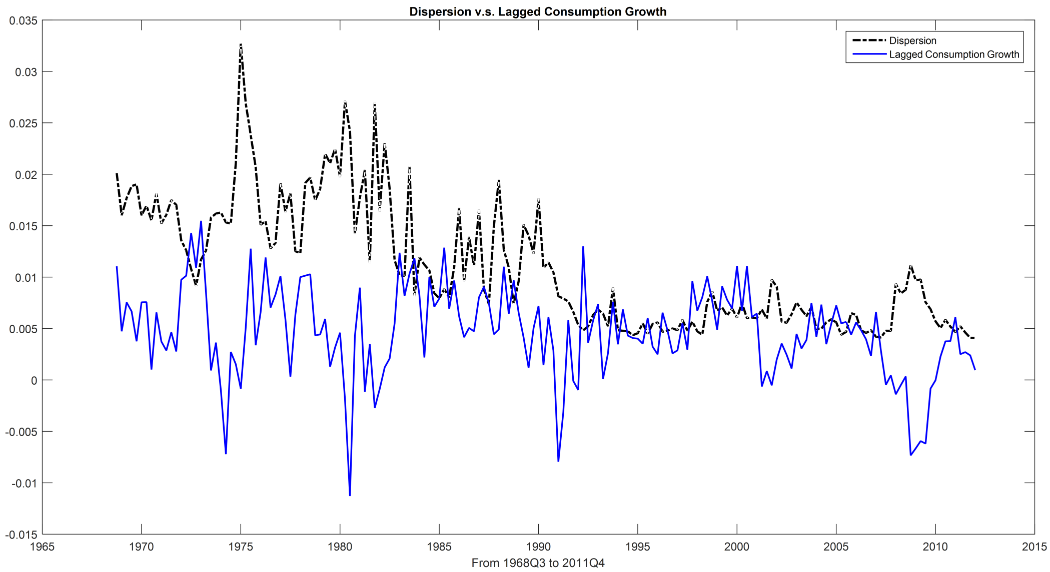

Figure 2: Effect of consumption growth on dispersion
The dispersion is the average of current quarter, one quarter ahead, two quarters ahead and three quarters ahead forecast dispersion from the Philadelphia Fed's SPF, from 1968Q4 to 2012Q1. Lagged quarterly U.S. real nondurable goods and services growth is from 1968Q3 to 2011Q4 from the Bureau of Economic Analysis.

One interpretation for Fig. 1 and Fig. 2 is that investor confidence depends on the CB's performance. Poor performance makes it more difficult for investors to understand the economic environment and thus they become less confident in probability assessments of the future. Conversely, good performance makes them more confident. Since maintaining an efficient level of consumption/output growth is always one task of the CB, this means that low consumption growth realizations should always make investors less confident about future consumption growth. It is different for inflation in that during the 1970s and particularly 1980s, one of the most important tasks for the Fed was to fight high inflation. High inflation realizations, due to the Fed's failure in this task, or due to some new factors that were not well understood, made it harder for investors to understand the economic environment and thus they became less confident about future consumption growth. However, during the past decade, instead of fighting high inflation, the Fed faced deflationary pressures, and high inflation realizations has made investors feel that the economic environment is well understood and become more confident about future consumption growth. The three subperiods in which inflation had different effects on confidence roughly correspond to three different inflation and monetary policy regimes noted earlier: the pre-Volcker period, the inflation fighting period of Volcker and Greenspan, and the recent period of low inflation and increased central bank transparency. And at the same time, CPV (2014) found that bond risks are significantly different for the three periods. To capture the time varying bond risks, which is also consistent with the confidence inflation correlation pattern in the data, we allow the confidence and inflation processes to have two discrete regime shifts in the monetary policy break dates, 1979Q2 and 1997Q1.

Table 2 shows quantitatively that the correlation between average dispersion and lagged inflation was moderately positive during the the first subperiod, strongly positive during the second subperiod, and became negative during the third subperiod. The bottom panel of Table 2 shows that the correlation between average dispersion and lagged consumption growth was always negative and stable. We will calibrate a different set of parameters for every subperiod to match the correlations between the average dispersion

	68.Q3-79.Q2	79.Q3-96.Q4	97.Q1-11.Q4
Corr(DispT, $\hat{\pi}_{-1}$)	0.45	0.64	-0.21
Corr(DispT1, $\hat{\pi}_{-1}$)	0.45	0.64	-0.20
Corr(DispT2, $\hat{\pi}_{-1}$)	0.55	0.64	-0.18
Corr(DispT3, $\hat{\pi}_{-1}$)	0.43	0.57	-0.12
Corr(Disp, $\hat{\pi}_{-1}$)	0.57	0.70	-0.22
Corr(DispT, $\triangle c_{-1}$)	-0.19	-0.29	-0.34
Corr(DispT1,$\triangle c_{-1}$)	-0.22	-0.32	-0.46
Corr(DispT2,$\triangle c_{-1}$)	-0.32	-0.16	-0.30
Corr(DispT3,$\triangle c_{-1}$)	-0.30	-0.15	-0.38
Corr(Disp,$\triangle c_{-1}$)	-0.31	-0.26	-0.44

Table 2: Confidence Inflation/Consumption growth Correlation
This table reports the correlation between dispersion and lagged macro variables. DispT, DispT1, DispT2 and DispT3 stand for one quarter ahead, current quarter, two quarters ahead and three quarters ahead forecast dispersion from the Philadelphia Fed's SPF. Disp is the average of DispT, DispT1, DispT2 and DispT3. Lagged quarterly U.S. real nondurable goods and services growth is from the Bureau of Economic Analysis. Lagged quarterly U.S. inflation is calculated by GDP price deflator from the Bureau of Economic Analysis.

and consumption/inflation. [10]

From psychological point of view, Bracha and Weber (2012) argued that, in models of risk and uncertainty, confidence results when investors believe they understand how things work. However, in unfavorable environments, the sense of predictability and perceived control is destroyed and panics are triggered. Our specification of confidence is consistent with Bracha and Weber (2012) in that when poor CB performance creates unfavorable environments, investor's sense of predictability and perceived control on consumption growth is destroyed, and, as a result, they become less confident. Likewise, good performance of the CB has positive effects on investor's sense of predictability and perceived control, rendering them less concerned about model misspecification, and, as a result, more confident.

Table 1 and 2 and Fig. 1 and Fig. 2 provide robust supporting evidence of the confidence process in Eq. (3). Since the time varying effect of inflation on confidence plays the most important role in bond risks and stock/bond pricing, we use Table 2 to

[10]For the third subperiod, the model implied correlation between dispersion and consumption growth is a little lower than in the data (40% vs 44%) due to model constraint. As shown in Table 7, consumption growth only plays a minor role in the model, this has a negligible impact.

calibrate parameters κ_c and κ_π. To avoid the potential risk that dispersion one quarter ahead is not a perfect measure for ambiguity, correlations of the average dispersion are used for calibration. All parameters in the confidence process are directly estimated from average forecasts dispersion data.

Consumption growth / inflation / confidence process fitness

As described in Sections 2.1 through 2.3, the forcing process in the model contains three variables: consumption growth, inflation, and confidence. This section describes some empirical support for our specification of the forcing process.[11]

We show in Section 2.3 that investor confidence about future consumption growth is driven by past consumption growth and inflation, although consumption growth and inflation are not driven by past confidence in our specification. To verify that this is a reasonable restriction, we estimate a first-order vector autoregression (VAR) model including these three variables and check the coefficients of past confidence on inflation and consumption growth. Table 3 provides the estimation results. As expected, lagged inflation and consumption growth play significant roles in forecasting dispersion; however, for both inflation and consumption growth predictions, the coefficients on lagged dispersion are insignificant. Furthermore, we perform a likelihood ratio test that restricts the coefficients of lagged dispersion in both the inflation and consumption growth equations to be zero simultaneously and find the p value to be 5%. We run a similar test using one-quarter-ahead dispersion calculated from the interquartile range of individual forecasts and find similar results, where the joint zero restriction hypothesis cannot be rejected at the 5% level.

The key results of the model depend on how inflation impacts ambiguity differently over different periods. From Eq. (3) in Section 2.3, ambiguity today depends on current and lagged inflation innovations. So if inflation is more persistent, a shock to inflation will affect future ambiguity more, which in turn matters for prices today. Thus it is important

[11]The size of ambiguity/confidence is measured by one-quarter-ahead forecast dispersion as in 2.3, and consumption growth and inflation are both from the Bureau of Economic Analysis. The detailed information of the data set will be discussed in Section 3.1.

	Constant	Lag Dispersion	Lag Consumption growth	Lag Inflation
Dispersion	1.17**	0.37**	-40.95	187.84**
	(0.31)	(0.07)	(31.12)	(30.91)
Consumption growth	0.0019**	0.0003	0.52**	-0.11
	(0.0007)	(0.00015)	(0.06)	(0.06)
Inflation	0.0008	0.0001	0.056	0.82**
	(0.0005)	(0.0001)	(0.05)	(0.05)

Table 3: VAR(1) Estimation

This table reports the VAR results for consumption growth, inflation, and dispersion. The dispersion is one quarter ahead forecast dispersion from the Philadelphia Fed's SPF, from 1968Q4 to 2011Q4. Same periods quarterly consumption growth and inflation are from the Bureau of Economic Analysis. The coefficients with stars are significant.

for the model to match the autocorrelation of inflation and the cross-correlation between inflation and forecast dispersion in the data. The upper panel of Table 4 reports the autocorrelation functions for inflation up to lag 5, and the lower panel provides cross correlation between dispersion and lagged inflation up to lag 5.[12] The model matches the data well for both inflation autocorrelation and the cross correlation.

2.4. Preference: Epstein-Zin preference with ambiguity aversion

Epstein and Schneider (2003) axiomatize an intertemporal model of multiple-priors utility, and Hayashi (2005) extends that model to allow for the recursive preference of Kreps and Porteus (1978) and Epstein and Zin (1989). This model of preference permits a three-way separation of intertemporal substitution, risk aversion, and ambiguity aversion. Investors' utility over consumption is represented by the following model

$$\begin{aligned} V_t(C_t) &= [(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta\{\mathcal{R}_t(V_{t+1}(C_{t+1}))\}^{\frac{1-\gamma}{\theta}}]^{\frac{\theta}{1-\gamma}} \\ \mathcal{R}_t(V_{t+1}(C_{t+1})) &= \{\min_{p_t\in\mathcal{P}_t} \mathbb{E}_{p_t}(V_{t+1}^{1-\gamma}(C_{t+1}))\}^{\frac{1}{1-\gamma}}. \end{aligned} \tag{4}$$

This is a standard Epstein-Zin preference, except that we have a "min" operator within the aggregator. $0 < \beta < 1$ reflects the agent's time preference, γ is the coefficient of

[12]The inflation autocorrelation becomes negative/very small after lag 5 for first/third subperiod. That is the reason why we only show data up to 5 period lag. Also, the parameters of inflation are calibrated to match autocorrelation of lag 1 and lag 5 for the same reason.

Autocorrelation		Lag1	Lag2	Lag3	Lag4	Lag5
68.Q3-79.Q2	Data	0.61*	0.48	0.36	0.32	0.10*
	Model	0.61*	0.43	0.29	0.18	0.10*
79.Q3-96.Q4	Data	0.87*	0.80	0.74	0.67	0.55*
	Model	0.87*	0.78	0.70	0.62	0.55*
97.Q1-11.Q4	Data	0.47*	0.40	0.32	0.24	0.23*
	Model	0.47*	0.40	0.34	0.28	0.23*
Cross_Correlation						
68.Q3-79.Q2	Data	0.57*	0.76	0.64	0.43	0.36
	Model	0.57*	0.47	0.37	0.28	0.20
79.Q3-96.Q4	Data	0.70*	0.65	0.69	0.71	0.61
	Model	0.70*	0.65	0.60	0.54	0.49
97.Q1-11.Q4	Data	-0.22*	-0.28	-0.07	-0.02	-0.14
	Model	-0.22*	-0.13	-0.07	-0.03	0.00

Table 4: Inflation/Dispersion Correlation

Quarterly U.S. inflation is calculated by GDP price deflator from the Bureau of Economic Analysis. Dispersion is the average of current quarter, one quarter ahead, two quarters ahead and three quarters ahead forecast dispersion from the Philadelphia Fed's SPF. The top part of the Table shows inflation autocorrelation, and the lower part shows cross correlation between dispersion and lagged inflation. Note the * statistics are used to calibrate model parameters.

risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the IES. Utility maximization is subject to the budget constraint,

$$W_{t+1} = (W_t - C_t) R_{c,t+1}, \tag{5}$$

where W_t is the wealth of the agent, and $R_{c,t}$ is the unobservable return on all invested wealth, or the consumption claim.

The set of one-step-ahead beliefs $\mathcal{P}_t$ consists of the measures $p_t^{\tilde{\mu}}$ generated in Section 2.2. We show in the appendix that the worst-case measure p_t^o that gives the minimum continuation value is $p_t^{-a_t}$, which is generated by likelihood with the worst mean $-a_t$ each period (See also Epstein and Wang (1994) for a proof). Thus, the "min" operator in the preference can be replaced by $p_t^o = p_t^{-a_t}$, which is generated by the worst mean.

2.5. Asset markets

2.5.1. Pricing kernel and stock price

Since the representative agent evaluates expectations under the worst-case measure when making portfolio choices, the Euler equation holds under the worst-case measure. Therefore, assets can be priced using the Euler equation under the worst-case measure. Given the worst-case measure, as in Epstein and Zin (1989), the real pricing kernel or the real stochastic discount factor can be written as,

$$M_{t,t+1} = \beta^{\theta}(\frac{C_{t+1}}{C_t})^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}, \tag{6}$$

or

$$m_{t,t+1} = \theta log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \tag{7}$$

where $m_{t,t+1} = log(M_{t,t+1})$, $r_{c,t+1} = log(R_{c,t+1})$. For any asset j with real payoff, the first-order condition yields the following asset pricing Euler condition,

$$E_{p_t^o}[exp(m_{t,t+1} + r_{j,t+1})] = 1, \tag{8}$$

where $E_{p_t^o}$ is the expectation operator for the worst-case measure, and $r_{j,t+1}$ is the log of the gross return on asset j.

To solve the model, it is assumed that the log price-consumption ratio for a consumption claim, z_t, is linear in the confidence level a_t, the demeaned inflation $\hat{\pi}_t$, and current inflation shock $\varepsilon_{\pi,t}$

$$z_t = A_0 + A_1 a_t + A_2\hat{\pi}_t + A_3\varepsilon_{\pi,t}, \tag{9}$$

and that the log price-dividend ratio for a dividend claim, $z_{t,m}$, is similarly linear

$$z_{t,m} = A_{0,m} + A_{1,m}a_t + A_{2,m}\hat{\pi}_t + A_{3,m}\varepsilon_{\pi,t}, \tag{10}$$

The log return on consumption claim is given by the Campbell and Shiller (1988b) approximation,

$$r_{c,t+1} = k_0 + k_1 z_{t+1} + \Delta c_{t+1} - z_t, \tag{11}$$

where $k's$ are log linearization constants, which are discussed in more detail below. To solve A_0, A_1, and A_2, one need to substitute (7), (9), and (11) into the Euler Eq. (8). By approximating log market return in a similar way, $A_{0,m}, A_{1,m}, A_{2,m}$ can be solved. Details of both derivations are provided in the Appendix. As noticed by previous studies (see, e.g., Campbell (1993); Campbell and Koo (1997); Bansal, Kiku, and Yaron (2007); Beeler and Campbell (2012)), the parameters A_0 and A_1 determine the mean of the price-consumption ratio,$\bar{z} = A_0(\bar{z}) + A_1(\bar{z})a$. The parameters k_0 and k_1 are nonlinear functions of $\bar{z}$, where $k_0 = log(1 + exp(\bar{z})) - \bar{z}k_1$ and $k_1 = \frac{exp(\bar{z})}{1+exp(\bar{z})}$. To get a highly accurate approximation, one needs to iterate numerically until a fixed point for $\bar{z}$ is found.

The solution coefficients for the effect of confidence a_t on the price-consumption ratio, A_1, and on the price-dividend ratio, $A_{1,m}$, respectively, are

$$A_1 = \frac{1 - \frac{1}{\psi}}{k_1\rho_a - 1},\ A_{1,m} = \frac{\zeta_d - \frac{1}{\psi}}{k_{1,m}\rho_a - 1}. \tag{12}$$

Since both $k_1$and $k_{1,m}$ are smaller than 1 under the fixed point value of $\bar{z}$, A_1 is negative if the IES, ψ, is greater than 1; and $A_{1,m}$is negative if $\zeta > \frac{1}{\psi}$. In this case, the intertemporal substitution effect dominates the wealth effect. In response to the high confidence level, or low a_t , investors buy more assets and prices increase. In addition, $|A_{1,m}| > |A_1|$ when $\zeta > 1$, which means that a confidence shock leads to a stronger reaction in the price of the dividend claim than in the price of the consumption claim. This is due to the fact that dividend ratio growth is a leveraged consumption growth with ζ as the leverage ratio. Note that an increase in persistence of confidence shocks, ρ_a, will increase the magnitude of the response of both valuation ratios to confidence fluctuations.

The solution coefficients for the effect of demeaned inflation $\hat{\pi}_t$ on the price-consumption ratio, A_2, and on the price-dividend ratio, $A_{2,m}$, respectively, are

$$A_2 = \frac{k_1\rho_\pi}{1-k_1\rho_\pi}\kappa_\pi A_1,\ A_{2,m} = \frac{k_{1,m}\rho_\pi}{1-k_{1,m}\rho_\pi}\kappa_\pi A_{1,m}. \tag{13}$$

Because both $\frac{k_1\rho_\pi}{1-k_1\rho_\pi}$ and $\frac{k_{1,m}\rho_\pi}{1-k_{1,m}\rho_\pi}$ are positive and smaller than one, the signs and magnitudes of A_2 and $A_{2,m}$ depend on κ_π. If $\kappa_\pi > 0$, A_2 and $A_{2,m}$ will have the same signs as A_1 and $A_{1,m}$, and if $\kappa_\pi < 0$, A_2 and $A_{2,m}$ will have different signs than A_1 and $A_{1,m}$.

The solution coefficients for the effect of inflation shock $\varepsilon_{\pi,t}$ on the price-consumption ratio, A_3, and on the price-dividend ratio, $A_{3,m}$, respectively, are

$$A_3 = \frac{\sigma_\pi\theta_\pi A_2}{\rho_\pi},\ A_{3,m} = \frac{\sigma_\pi\theta_\pi A_{2,m}}{\rho_\pi}. \tag{14}$$

Because ρ_π is big relative to $\sigma_\pi\theta_\pi$, the magnitudes of A_3 and A_{3m} are small relative to A_2 and $A_{2,m}$. The signs of A_3 and A_{3m} depend on θ_π.

Given the solution for the return on consumption claim, $r_{c,t+1}$, the innovation to the pricing kernel can be written as (also shown in the appendix)

$$m_{t,t+1} - E_{p_t^o}(m_{t,t+1}) = \upsilon_{mc}\varepsilon_{c,t+1} + \upsilon_{m\pi}\varepsilon_{\pi,t+1} + \upsilon_{ma}\varepsilon_{a,t+1}, \tag{15}$$

with $\upsilon_{mc} = (\theta-1)k_1A_1(\kappa_c + \kappa_\pi\zeta_\pi) - \gamma\sigma_c + (\theta-1)k_1A_2\zeta_\pi$, $\upsilon_{m\pi} = (\theta-1)k_1(A_1\kappa_\pi + A_2)\sigma_\pi + (\theta-1)k_1A_3$, and $\upsilon_{ma} = (\theta-1)k_1A_1\sigma_a$ capturing the pricing kernel's exposure to consumption shocks, inflation shocks, and exogenous confidence shocks respectively.

Although the ambiguity-averse agent acts pessimistically and evaluates asset under the worst-case measure, we are interested in expected returns under the reference model because it is supposed to be the best estimate of the data generating process based on historical data. The solution coefficients of expected market return for confidence, demeaned inflation, and inflation shock are

$$A_{1,E} = A_{1,m}(k_{1,m}\rho_a - 1),\ A_{2,E} = A_{3,E} = 0, \tag{16}$$

and if $A_{1,m} < 0$, we will have $A_{1,E} > 0$. Thus, ambiguity increases expected market return. The expected return is time-varying because a_t is time-varying. The risk-free rate can be derived by substituting $r_{c,t+1}$ into the Euler Eq. (8). As in the appendix, the coefficient on confidence a_t is negative, implying that a low confidence level (high a_t) corresponds to a low interest rate. Finally , the log nominal pricing kernel that we use to value assets with nominal payoffs is defined as

$$m^{\$}_{t,t+1} = m_{t,t+1} - \pi_{t+1}. \tag{17}$$

2.5.2. Bond prices

The time-t price of a zero-coupon bond that pays one unit of consumption n periods later is denoted $P_t^{(n)}$, and it satisfies the recursion

$$P_t^{(n)} = E_{p_t^o}[M_{t,t+1}P_{t+1}^{(n-1)}], \tag{18}$$

with the initial condition that $P_t^{(0)} = 1$. Given the linear Gaussian framework, we assume that $p_{t+1}^{(n-1)} = log(P_{t+1}^{(n-1)})$ is a linear function of confidence, demeaned inflation, and inflation shock

$$p_{t+1}^{(n-1)} = A_0^{n-1} + A_1^{n-1}a_{t+1} + A_2^{n-1}\hat{\pi}_{t+1} + A_3^{n-1}\varepsilon_{\pi,t+1}. \tag{19}$$

Then we substitute (19) in the the Euler Eq. (18), and $p_t^{(n)} = log(P_t^{(n)})$ can be solved as a linear function of time-t state variables

$$p_t^{(n)} = A_0^n + A_1^n a_t + A_2^n\hat{\pi}_t + A_3^n\varepsilon_{\pi,t}, \tag{20}$$

with $A_1^n = A_1^1 + A_1^{n-1}\rho_a$, $A_2^n = A_2^1 + A_2^{n-1}\rho_\pi + A_1^{n-1}\kappa_\pi\rho_\pi$, $A_3^n = A_3^1 + (A_2^{n-1} + A_1^{n-1}\kappa_\pi)\sigma_\pi\theta_\pi$, and A_0^n given in the appendix. Since the initial values of A_0^1 , A_1^1, A_2^1, and A_3^1 can be calculated from $P_t^{(1)} = E_{p_t^o}[M_{t,t+1}]$, all the parameter values of longer periods can be calculated recursively.

Similarly, we can get the parameter values of nominal bond price $P_t^{(n,\$)}$ using nominal pricing kernel. As shown in the appendix, the parameters for confidence and demeaned inflation satisfy the recursions $A_1^{n,\$} = A_1^{1,\$} + A_1^{n-1,\$}\rho_a$, $A_2^{n,\$} = A_2^{1,\$} + A_2^{n-1,\$}\rho_\pi + A_1^{n-1,\$}\kappa_\pi\rho_\pi$, and $A_3^{n,\$} = A_3^{1,\$} + (A_2^{n-1,\$} + A_1^{n-1,\$}\kappa_\pi)\sigma_\pi\theta_\pi$

The log holding period return from buying an n periods real bond at time t and selling it as an $n-1$ periods real bond at time $t-1$ is defined as $r_{n,t+1} = p_{t+1}^{(n-1)} - p_t^{(n)}$, and the holding period return for n periods nominal bond is defined similarly as $r^{\$}_{n,t+1} = p_{t+1}^{(n-1,\$)} - p_t^{(n,\$)}$. Given the real and nominal bond prices, we can find the shocks to real and nominal bond returns, then calculate the covariance between stock and bond returns as shown in the appendix.

3. Empirical findings

Given the analytical solutions, in this section, we simulate data by drawing shocks randomly, and show how the simulated data replicate many interesting behaviors and statistics in the data. To match our empirical finding that inflation has time varying effect on confidence, the whole sample, 1968.Q3 to 2011.Q4, is broken into three subperiods corresponding to major shifts in monetary policy. Because the earliest available data for forecasts dispersion is 1968.Q3, our sample is not able to cover the first monetary regime of Clarida, Gali, and Gertler (1999)(CGG, 1999) completely.[13] Thus our first subperiod, 1968.Q3 to 1979.Q2, covers part of the pre-Volcker period. The second subperiod, 1979.Q3 to 1996.Q4, is the same as in CGG (1999) and covers the Fed chairmanships of Paul Volcker and Alan Greenspan. The third subperiod, 1997.Q1 to 2011.Q4, as argued in CPV (2014), covers the later part of Greenspan's chairmanship and the earlier part of Ben Bernanke's chairmanship. Following CGG (1999) and CPV (2014), we assume that transitions from one regime to another are structural breaks, completely unanticipated by investors.

[13]CGG (1999) considered 1960.Q1 to 1979.Q2 as the first subperiod, which covers the Fed chairmanships of William M. Martin, Arthur Burns, and G. William Miller.

3.1. Data

We use quarterly US data on consumption, inflation, interest rates, forecasts dispersion, and aggregate stock returns from 1968.Q3 to 2011.Q4. Consumption data are based on U.S. real nondurables and services consumption per capita from the Bureau of Economic Analysis. Inflation data are calculated from GDP deflator from the Bureau of Economic Analysis via the Fed database at the St. Louis Federal Reserve.[14] The forecasts dispersion of future consumption/output growth are from the Philadelphia Fed's SPF. We use the end-of-quarter three-month T-bill from the CRSP monthly Treasury Fama risk free rates. The end-of-quarter one to five year bond yields are from the CRSP monthly Treasury Fama-Bliss discount bond yields. The daily yields for three month and one to five year bonds are from the daily dataset constructed by Gurkaynak, Sack, and Wright (GSW 2007). Stock returns are the value-weighted combined NYSE/AMEX/Nasdaq returns including dividends from CRSP, and the price-dividend ratios are measured using data for real dividends and the S&P 500 real price. All yields are continuously compounded. For the real risk-free rate, Beeler and Campbell (2012) create a proxy for the ex-ante risk-free rate by forecasting the ex-post quarterly real return on three-month Treasury bills with past one-year inflation and the most recent available three-month nominal bill yield. We estimate and use the same ex-ante risk-free rate.

3.2. Simulation and calibration

In calibration and simulations, we assume a quarterly decision interval and then generate four sets of i.i.d. standard normal random variables for each subperiod. We then use these to construct the quarterly series for consumption, dividends, inflation, and confidence. It is important to note that consumption and dividends growth is constructed under the reference measure, which offers the right dynamics to use for reporting simulation moments in that it is the one used by the econometrician and provides a good fit to the historical data. Negative realizations of ambiguity size, a_t, will be replaced with

[14] The reason that we use GDP deflator instead of CPI to calculate inflation is that the timing of SPF is geared to the release of the Bureau of Economic Analysis' advance report of the national income and product accounts, and it is natural to use the former to analyze its effect on dispersion.

zero, in which case investors fully trust the reference model. For each subperiod, we set size of ambiguity and inflation to their long-run mean values to initialize each simulation and discard the first 20 quarters before using the outputs.

For statistical inference, we report the median moments from 10,000 simulations run over the sample with numbers of observations matching the length of the actual data in each sub period. I also report the tail percentiles of the Monte-Carlo distributions (5% and 95%).

Table 5 reports the calibrated parameter values for consumption and dividend growth, inflation, and confidence processes. First, the moments of consumption and dividend growth are not significantly different for different regimes, and most importantly, they don't play important roles for the stock/bond comovements, thus we assume the parameters in the consumption and dividend growth process are time invariant and calibrated to match the moments of quarterly per capita U.S. consumption and dividend data. $\mu_c = 0.0046$ is the quarterly mean consumption growth rate from the data, and the standard deviation of the consumption is chosen to match the standard deviation in the consumption growth data $\sigma_c = 0.0044$. For the given leverage parameter, ζ_d, μ_d is chosen such that the average rate of dividend growth is equal to the mean growth rate of dividends in the data. Similarly, for the given leverage ratio, σ_d can be calibrated to match the standard deviation of dividend growth in the data. Since there is significant seasonality in quarterly dividend data, we use the implied quarterly standard deviation of dividend growth from annual data. The leverage parameter ζ_d is chosen to be 3.5 which implies that correlation of consumption and dividend growth is 0.51.

In terms of preference parameters, I use a risk aversion (RRA) of 3 and IES of 2.5. The literature examining the IES magnitude in the data leads to estimates that are both well above and below 1. Bansal and Yaron (2004) argue that if consumption volatility is time varying, IES tends to be greater than 1. Epstein, Farhi, and Strzalecki (2014) suggest that when using recursive utility and calibrating its parameters, one should make a quantitative assessment of how much temporal resolution of risk matters. They calculate a timing premium for Bansal and Yaron (2004) calibration of $RRA = 10$ and $IES = 1.5$

and found that the representative agent would give up 20% of lifetime consumption to have all risk resolved next month. The general pattern is that the timing premium is increasing with the product of RRA and IES. My specification for RRA and IES is reasonable in this sense. We set the time preference $\beta = 0.9944$ to match level of risk free rate in the whole sample.

Finally, the confidence and inflation parameters are calibrated for each subperiod. The autoregressive parameter ρ_π, moving average parameter θ_π, and mean π, are chosen to match the first-order autocorrelation, fifth-order autocorrelation, and sample mean of inflation data in each period.[15] The parameter ζ_π is chosen to match the correlation between consumption growth and inflation, and the volatility parameter σ_π is chosen to match inflation volatility in each period.[16]

All parameters in the confidence process are directly estimated from forecasts dispersion data. For each subperiod, ρ_a, is calibrated to match the first-order autocorrelation of average dispersion in the data (0.67, 0.64, and 0.65 respectively), κ_c/κ_π is chosen to match the correlation between confidence and consumption growth/inflation in Table 2, a is chosen to match the mean of average dispersion of the whole sample (0.005), σ_a is calibrated to match the volatility of average dispersion of the whole sample (0.003).[17]

3.3. Basic moments

Table 6 displays the model implications for the whole sample unconditional moments of four variables: the log consumption and dividends growth rates, log stock return, and log risk-free interest rate. Over the whole sample period, the model is calibrated to match

[15] In the data, inflation autocorrelation becomes negative/very small after lag 5 for first/third subperiod. To pin down ρ_π and θ_π, we choose to match autocorrelation of lag 1 and 5, which provides better overall fits to data.

[16] Note that inflation follows a quasi ARMA(1,1) process in this paper, the model cannot match correlation between inflation and consumption growth in the second subperiod. We choose to match the correlation between inflation and GDP growth instead. Since the correlation between inflation and consumption growth plays only a negligible role in this model, we expect model results to be almost the same.

[17] The number of survey participants for Philadelphia Fed's SPF varies from maximum of 53 in 2006 Q1 to minimum of 9 in 1990 Q2. To reduce the noise due to changes in the number of survey participants, we calibrate means and volatilities of each subperiod using whole sample mean and volatility.

Time Invariant Parameters

Preference	β	γ	ψ		
	0.9944	3	2.5		
Consumption	μ_c	σ_c			
	0.0046	0.0044			
Dividends	μ_d	ζ_d	σ_d		
	-0.0144	3.5	0.0262		

Time Varying Parameters

Inflation	ρ_π	θ_π	ζ_π	σ_π	π
68.Q3-79.Q2	0.818	-0.5	-0.00296	0.00264	0.0153
79.Q3-96.Q4	0.988	-0.18	-0.00058	0.0025	0.0097
97.Q1-11.Q4	0.948	-0.69	0.0006	0.002	0.0052
Confidence	a	ρ_a	κ_c	κ_π	σ_a
68.Q3-79.Q2	0.004	0.72	-0.00095	0.117	0.00218
79.Q3-96.Q4	0.004	0.43	-0.001	0.3	0.0021
97.Q1-11.Q4	0.004	0.755	-0.00145	-0.015	0.0022

Table 5: Configuration of model parameters.
This table reports consumption growth, dividend growth, inflation, and confidence processes parameters. All parameters are given in quarterly terms.

the first and second moments of log consumption and dividends growth rates and the level of risk-free rate, with those moments labeled with stars.

As shown in Table 6, the model matches well the key asset price moments. Specifically, the model replicates the level of the log risk-free rate over the whole sample period, model implied equity return and volatility are close to data, while the volatility of log risk-free rate is somewhat lower than the data counterpart.

The first and second moments of equity return and risk-free rate implied by different model settings are provided in Table 7. When we shut down the exogenous confidence shocks, inflation, and consumption growth shocks separately, the volatilities of the two variables are lower, the median equity return is smaller, and the median risk free rate is higher. While all three sources of confidence fluctuations have the similar effects on the stock market, the inflation channel has the biggest impact. When we set $\kappa_c = \kappa_\pi = \sigma_a = a = 0$, from the last row of Table 7, the magnitude of this change is huge. This means ambiguity as a whole plays the most important role in the first and second moments of

	Data	Model		
Moment	Estimate	Median	5%	95%
$E(\Delta c)$	1.84	1.84*	1.62	2.06
$\sigma(\Delta c)$	0.88	0.88*	0.80	0.96
$E(\Delta d)$	0.70	0.70*	-0.81	2.22
$\sigma(\Delta d)$	6.1	6.1*	5.5	6.6
$E(r_e)$	7.9	9.2	6.0	12.4
$\sigma(r_e)$	14.97	15.3	13.7	17.1
$E(r_f)$	1.72	1.72*	1.2	2.0
$\sigma(r_f)$	2.32	0.74	0.47	1.29

Table 6: Model implied moments
This table presents annualized moments (whole sample) for the model and data from the quarterly datasets. The model implied moments displayed is the median, 5%, and 95% percentiles from 10,000 finite sample simulations of equivalent length to the dataset. Column 2 display data statistics measured in real terms, with data sampled on a quarterly frequency covering the period from 1968Q3 to 2011Q4. Means and volatilities of returns and growth rates are expressed in percentage terms. Return volatility in the data is constructed following Campbell, Lettau, Malkiel, and Xu (2001). Note the * statistics are used to calibrate model parameters.

equity return and risk-free rate.

3.4. Bond beta, stock return / inflation correlation, and the "Fed Model"

Bond risks

Recent studies (Campbell, Pflueger, and Viceira (2014); Campbell, Sunderam, and Viceira (2016); David and Veronesi (2013)) have shown the facts that the covariance between stocks and Treasury bonds has switched from positive to negative in the past decade, and the magnitudes are very different over different subperiods. Using the same method as in Campbell, Sunderam, and Viceira 2016, we calculate CAPM beta for 5 year nominal bonds. Fig. 3 plots the time series history, where the beta of bonds with stocks was slightly positive in the 1970's, much higher in the 1980's, spiked in the mid-1990's, and declined to negative average values after the late 1990's.

Table 8 shows quantitatively the average CAPM betas of 5 year bonds over the three different subperiods, 68.Q3-79.Q2, 79.Q3-96.Q4, and 97.Q1-11.Q4. The beta was

	$E(r_e)$	$\sigma(r_e)$	$E(r_f)$	$\sigma(r_f)$
Data	7.9	14.97	1.72	2.32
Fully Specified Model	9.2	15.3	1.72	0.74
$\kappa_c = 0$	9.1	14.6	1.78	0.72
$\kappa_\pi = 0$	7.9	8.5	2.23	0.46
$\sigma_a = 0$	9.1	14.9	1.81	0.68
$\kappa_c = \kappa_\pi = \sigma_a = a = 0$	2.86	6.06	2.97	0.00

Table 7: Basic moments for different settings

This table presents annualized moments (whole sample) for different model settings and data from the quarterly datasets. The model implied moments displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset (from 1968Q3 to 2011Q4). Means and volatilities of returns and growth rates are expressed in percentage terms.

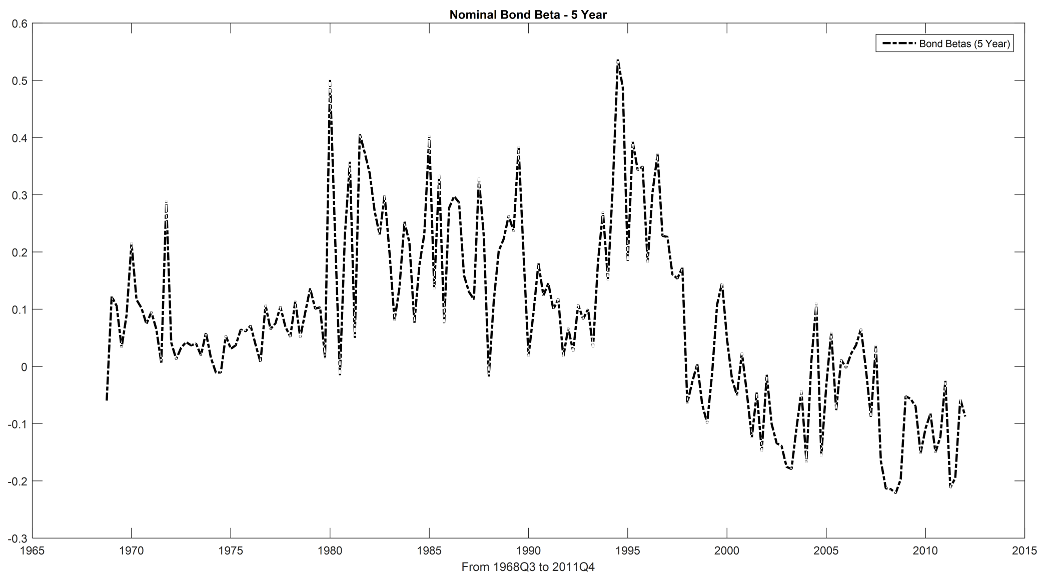

Figure 3: CAPM Beta of 5 Year Nominal Bond

History of the realized beta of 5 year nominal Treasury bonds with CRSP value-weighted stock index (the bond-stock covariance divided by the realized variance of stock returns) from 1968Q3 to 2011Q4. The quarterly betas are calculated using a rolling three-month window of daily data.

slightly positive in the first subperiod (0.07), fairly big in the second subperiod(0.21), and switched to -0.7 in the third subperiod. There is no way for one to calculate the first two subperiods CAPM beta for real bonds because no inflation-indexed bonds were available, and the third period TIPS beta was -0.08 for 5 year real bonds. Our model matches the nominal and real CAPM betas closely with the data. Note that we don't use Treasury bonds and inflation-indexed bonds in our calibration procedure and therefore the nominal and real CAPM betas provide verification of the model's out-of-sample performance.

The "Fed Model" and stock return / inflation correlation

Another related fact about Treasury bonds is the so called "Fed Model" which was the leading practitioner model of equity valuation, and this model implies a positive association between inflation and dividend yields.[18] But, for the same reason as for the nominal CAPM betas, the correlation between inflation and dividend yields has switched to negative in the past decade. As shown in Table 8, the correlation for the three subperiods are -0.64, -0.70, and 0.16 respectively. Our model lines up with the data well, but with moderately different magnitudes.

Several studies report a negative correlation between real stock returns and inflation, including Lintner (1975), Bodie (1976), Miller, Jeffrey, and Mandelker (1976), Nelson (1976), and Fama and Schwert (1977). All these papers use a data sample from before the second subperiod in our model. Our model implies that the correlation between stock return and inflation is -0.34 for the first subperiod, which is consistent with these studies.

The mechanism

The mechanism that generates the time varying risk properties of Treasury bonds relative to stocks relies on the connection between investor confidence and inflation in our model. During the first subperiod and particularly the second subperiod, one of the most important tasks for the Fed was to fight high inflation. High inflation realizations,

[18]As mentioned in John Y. Campbell (2004), the idea is that stocks and bonds compete for space in investors' portfolios. If the yield on bonds rises, then the risk-adjusted yield on stocks must also rise to maintain the competitiveness of stocks. Since the nominal bond yields were mainly influenced by inflation, thus the Fed model implies that stock yields are highly correlated with inflation.

		Nominal Bond Beta	Real Bond Beta	corr(pd ,pi)
68.Q3-79.Q2	Data	0.07	N/A	-0.64
	Model	0.11	-0.08	-0.81
79.Q3-96.Q4	Data	0.21	N/A	-0.70
	Model	0.26	-0.07	-0.99
97.Q1-11.Q4	Data	-0.06	-0.08	0.16
	Model	-0.17	-0.07	0.31

Table 8: Bond Risks

This table presents nominal and real bond betas, and price-dividend ratio/inflation correlation across different subperiods for both data and model. Note that the real bond betas in the data are taken from CPV (2014). The model implied statistics displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset.

due to the Fed's failure in this task, or due to some new factors that were not well understood, made it harder for investors to understand the economic environment and thus they became less confident about future consumption growth. As a result, investors did not buy stocks and stock prices dropped, and at the same time, long-term yields increased and bond prices decreased because of high inflation. The prices of stocks and Treasuries moved in the same direction, and price-dividend ratios/stock returns were negatively correlated with inflation in the first two subperiods because of this negative effect of inflation on confidence. However, in the past decade the opposite happened. Instead of fighting high inflation, in this period the Fed has faced deflationary pressures. In this case, high inflation realizations made investors feel that the economic environment is well understood and they became more confident about future consumption growth. Then stock prices rose, Treasury yields increased, and bond prices decreased as the result of high inflation. Stock and bond prices move in opposite directions and Treasuries served as a hedge in this period.

For real bonds, there is no inflation risk. Because confidence is persistent, any factors that cause a decrease in confidence will lower yields and raise bond prices. At the same time, low confidence implies low stock prices. Thus, stock prices and real bond prices move in opposite directions and real bonds are safe assets.

3.5. The predictability of bond returns

From the perspective of equilibrium asset-pricing models, another puzzling fact related to Treasury bonds is the excess bond return predictability. Against the expectations hypothesis, Fama and Bliss (1987), Campbell and Shiller (1991), Dai and Singleton (2002), and Cochrane and Piazzesi (2005) provide evidence for bond return predictability using yield spreads and forward rates as predictors. Bansal and Shaliastovich (2013) show that the long run risks model with time varying volatility for expected consumption growth and inflation can account for bond return predictability. In this paper, we follow the approach of Campbell and Shiller (1991) and Cochrane and Piazzesi (2005), and provide evidence for bond return predictability in our model.

Denote $y_t^{(n)} = -\frac{1}{n}p_t^{(n)}$ as the yield on the real n periods bond and $f_t^{(n)} = p_t^{(n-1)} - p_t^{(n)}$ as the real forward rate with n periods to maturity at time t. We use dollar superscript to refer to nominal quantities, e.g., $y_t^{(n,\$)}$ and $f_t^{(n,\$)}$ denotes nominal yield and forward rate. The nominal excess log return on buying an n-period bond at time t and selling it at time$t+m$ as an $n-m$ period bond is defined as $rx_{t\to t+m}^{n,\$} = ny_t^{(n,\$)} - (n-m)y_{t+m}^{(n-m,\$)} - my_t^{(m,\$)}$.

Table 9 provides bond predictability evidence of Campbell and Shiller (1991) in that the slope coefficients in expectations hypothesis projections are negative and decreasing with maturity, as in the data.[19] The slope coefficient for the return on a two-year bond is -0.58 in the model versus -0.41 in the data, and it decreases to -1.16 in the model and -1.15 in the data, respectively, for the return on a five-year bond.

To further evaluate the predictability of bond returns using yields, we run regressions using the same approach as in Cochrane and Piazzesi (2005) by first regressing the average of one-year nominal excess bond returns of two to five years to maturity on one- to five-year forward rates, and then extracting a single bond factor $\hat{rx}_t$ from this regression, finally forecasting excess bond returns at each maturity n from two to five

[19]The EH slopes, CP slopes, and CP R^2 in the data are from Bansal and Shaliastovich (2013), where they use quarterly observations of U.S. bond yields from 1969 to 2010. Their sample period is almost identical to ours.

	1Y	2Y	3Y	4Y	5Y
Data					
Yield Level	5.91	6.16	6.35	6.51	6.63
Std.Yield	3.19	3.12	3.01	2.93	2.83
EH Slope		-0.41	-0.78	-1.14	-1.15
CP Slope		0.44	0.85	1.28	1.43
CP R^2		0.15	0.17	0.20	0.17
Model					
Yield Level	5.72	5.81	5.89	5.96	6.02
Std.Yield	2.50	2.40	2.34	2.30	2.27
EH Slope		-0.58	-0.81	-1.0	-1.16
CP Slope		0.78	0.94	1.08	1.20
CP R^2		0.25	0.19	0.16	0.14

Table 9: Bond Returns Prediction
This table presents the whole sample nominal term structure, slopes in the expectations hypothesis regressions, and slopes and R^2s in Cochrane and Piazzesi (2005) single-factor bond premium regressions. The model implied statistics displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset (from 1968Q3 to 2011Q4). The end-of-quarter one to five year bond yields are from the CRSP monthly Treasury Fama-Bliss discount bond yields.

years, $rx^{n,\$}_{t\to t+1} = const + b_n \hat{rx}_t + error$. They show that the estimate b_n are positive and increasing with horizons. Table 9 shows the slopes and R^2s of the regression using quarterly observations of U.S. bond yields from 1969 to 2010 from Bansal and Shaliastovich (2013). Our model delivers a similar pattern and magnitude for the slopes and R^2s for four and five years excess bond returns. In sum, the model matches well the bond return predictability evidence from both the expectations hypothesis regressions and the single factor regression of Cochrane and Piazzesi (2005).

Table 9 also provides nominal yields and volatilities for bonds of one to five years to maturity. Our model can match the increase in yields and decrease in volatilities across maturities in the data. However, the level of volatilities in the model is somewhat smaller than in the data: it is 2.5% at one-year and 2.27% at five-year maturity, relative to 3.19% and 2.83% in the data, and our model implied yields are flatter than yields in the data.

3.6. Consumption, dividends, and return predictability

In this model, the fluctuations in price dividend ratio are driven by the joint dynamics of confidence and inflation, which create variations in equity returns. Consumption and

	Data			Model				
	$\hat{b}$	t	$\hat{R}^2$	50%	50%	50%	5%	95%
				(b)	(t)	(R^2)	(R^2)	(R^2)
Panel A: $\sum_{j=1}^{J}(r_{m,t+j} - r_{f,t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
4Q	-0.11	-2.09	0.07	-0.21	-3.10	0.16	0.04	0.30
12Q	-0.28	-4.77	0.20	-0.52	-4.04	0.36	0.06	0.61
20Q	-0.40	-8.22	0.28	-0.73	-4.69	0.44	0.05	0.74
Panel B: $\sum_{j=1}^{J}(\Delta c_{t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
4Q	-0.00	-0.59	0.01	-0.00	-0.24	0.01	0.00	0.09
12Q	-0.02	-1.24	0.07	-0.00	-0.30	0.03	0.00	0.22
20Q	-0.03	-1.89	0.16	-0.00	-0.36	0.05	0.00	0.33
Panel C: $\sum_{j=1}^{J}(\Delta d_{t+j}) = a + b(p_t - d_t) + \varepsilon_{t+j}$								
4Q	0.02	1.13	0.03	-0.00	-0.14	0.01	0.00	0.08
12Q	0.02	0.47	0.01	-0.01	-0.17	0.03	0.00	0.22
20Q	0.06	1.29	0.04	-0.02	-0.21	0.05	0.00	0.32

Table 10: Predictability of excess return, consumption, and dividend by dividend yield
Columns 2-4 of this table display coefficients, T-statistics, and R-squared statistics from predictive regressions of excess returns, consumption growth, and dividend growth on log price-dividend ratios using historical data. The data employed in the estimation are real, sampled on a quarterly frequency and cover the period from 1968Q3 to 2011Q4. Columns 7-9 present the model implied median, 5%, and 95% percentiles R-squared of the predictive regressions from 10,000 finite sample simulations of equivalent length to the dataset. Columns 5-6 display the model implied median of coefficients and T-statistics. Standard errors are Newey–West with 2*(horizon-1) lags.

dividend growth have constant expectations under the reference model; thus, theoretically, the price-dividend ratio will not predict consumption and dividend growth. At the same time, consistent with the model implications, many empirical studies have argued that the log price-dividend ratio predicts excess stock returns and not dividend growth (Campbell and Shiller (1988b); Fama and French (1988); Hodrick (1992)).

Table 10 provides the model's implied predictability results. It shows regressions for excess returns, consumption growth, and dividends growth, measured over horizons of one, three, and five years, onto the log price-dividend ratio at the start of the measurement period. The reported results include both model and data statistics. Panels B and C show that for both the data and the model there is relatively little predictability in consumption and dividend growth. All coefficients are insignificant in the data and for the model. Beeler and Campbell (2012) ran the same regression for quarterly data over the period 1947 Q2 through 2008 Q4, and found no predictability in consumption and dividend growth for all horizons.

For excess returns, the model-implied median finite-sample coefficients, t-statistics, and R^2s match the data well. Consistent with the data, the model-implied median R^2s rise with maturity, and the magnitudes of both coefficients and t-statistics increase with maturity.

3.7. Price-dividend ratio and excess return

In addition to the preceding facts, many empirical studies have shown puzzling links between aggregate asset markets and macroeconomics: Price–dividend ratios move pro-cyclically (Fama and French 1989) and conditional expected equity premiums move counter-cyclically (Campbell and Shiller (1988a,b); Fama and French (1989)).

In this model, the log price-dividend ratio is a linear function of confidence and inflation, with a negative coefficient on size of ambiguity (confidence), $A_{1,m}$; a drop in consumption growth lowers the confidence level, increases the size of ambiguity, a_t, and thus lowers the price-dividend ratio. The model implies a pro-cyclical price-dividend ratio variation. Because aggregate consumption growth and output growth are positively correlated, I use consumption growth as a pro-cyclical economic indicator. As shown in Section 2.5, the coefficient on confidence, or the size of ambiguity for the expected return, $A_{1,E}$, is positive. A high consumption growth realization increases investors' confidence level, decreases a_t, and results in low expected return. Therefore, the model implies that conditional expected equity returns move counter-cyclically.

4. Robustness

4.1. Different regime breaks

We follow CPV (2014) and choose the second regime break in 1996Q4. They argue that the third subperiod was a period of very significant monetary policy shifts toward transparency and gradualism. However, we do not model monetary policy in this paper and bond risks are mainly determined by the confidence/inflation correlation. We chose

		Nominal Bond Beta
68.Q3-87.Q2	Data	0.13
	Model	0.33
87.Q3-06.Q1	Data	0.09
	Model	0.06

Table 11: Bond Risks

This table presents nominal and real bond betas across different subperiods for both data and model. The model implied statistics displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset.

1996Q4 as the second regime break point because bond risks come to switch signs around 1996Q4 and, at the same time, the effect of inflation on confidence also changes from negative to positive (see Fig. 1 and Fig. 3).

From a monetary policy point of view, 1996 is not a clear regime break point in the literature because this is right in the middle of the Greenspan term. For the purpose of robustness, in this section, we consider alternative regimes and split the whole sample into two subperiods using a more obviously defined break point (beginning of the Greenspan era), 68Q3 - 87Q2 and 87Q3 - 06Q1. The second period corresponds to the Great Moderation or whole Greenspan term. Confidence and inflation parameters are estimated to match moments of dispersion data for each subperiod, and all other parameters are kept the same as before. The bond risks results are shown in Table 11. The model matches well the bond risks pattern in the data, showing that the bond betas in both periods are positive but bigger for the first period. This is because in the data, the inflation/dispersion correlation is positive for both periods and the first period is bigger (0.62 vs. 0.42). The results show that our model is robust to alternative regime breaks.

4.2. GDP vs consumption

In this paper, we use GDP forecasts dispersion before 1981Q3 to approximate consumption forecasts dispersion when the latter are not available, and we use consumption growth as the endowment growth for the whole sample. To make sure that our results are not driven by differences between consumption and output dynamics, in this section we try two alternative models with (i) actual GDP growth before 1981Q3 where GDP

		Nominal Bond Beta	Real Bond Beta	corr(pd ,pi)
	Data	0.07	N/A	-0.64
68.Q3-79.Q2	Benchmark	0.11	-0.08	-0.81
	Case I	0.11	-0.08	-0.85
	Data	0.21	N/A	-0.70
79.Q3-96.Q4	Benchmark	0.26	-0.07	-0.99
	Case I	0.26	-0.07	-0.99
	Data	-0.06	-0.08	0.16
97.Q1-11.Q4	Benchmark	-0.17	-0.07	0.31
	Model	-0.18	-0.07	0.30

Table 12: Bond Risks - Case I

This table presents nominal and real bond betas, and price-dividend ratio/inflation correlation across different subperiods for both data and model. Note that the real bond betas in the data are taken from CPV (2014). The model implied statistics displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset.

growth forecasts are used and (ii) GDP growth (actual + forecasts) for the subsample (after 1981Q3) where consumption growth forecasts are available. For both cases, we use the same calibration method as in the paper.

In case I, we replace consumption growth with GDP growth before 1981Q3 and the bond risks results are shown in Table 12. As one can see, the results are almost the same as for the benchmark model, which is because the confidence/inflation correlation is the same as before. The slight changes in the confidence/growth correlation, inflation/growth correlation, and moments (mean, volatility) of the endowment process do not play significant roles in bond risks. The results for other dimensions (e.g., basic moments, equity/bond returns prediction) are similar.

Since the results show almost no change in case I, we use GDP growth (actual + forecasts) for the whole sample for case II. Table 13 reports the model's performance on bond risks. Again, the model implies that moments are very close to the benchmark model. This is because the correlation patterns between GDP growth dispersion and inflation/growth are similar to the consumption growth dispersion used previously. However, the correlation between price dividend ratios and inflation is somewhat bigger in the thrid subperiod, which is due to a increase in the correlation between dispersion and

		Nominal Bond Beta	Real Bond Beta	corr(pd ,pi)
	Data	0.07	N/A	-0.64
68.Q3-79.Q2	Benchmark	0.11	-0.08	-0.81
	Case II	0.11	-0.08	-0.85
	Data	0.21	N/A	-0.70
79.Q3-96.Q4	Benchmark	0.26	-0.07	-0.99
	Case II	0.22	-0.08	-0.99
	Data	-0.06	-0.08	0.16
97.Q1-11.Q4	Benchmark	-0.17	-0.07	0.31
	Case II	-0.16	-0.08	0.81

Table 13: Bond Risks - Case II

This table presents nominal and real bond betas, and price-dividend ratio/inflation correlation across different subperiods for both data and model. Note that the real bond betas in the data are taken from CPV (2014). The model implied statistics displayed are the median values from 10,000 finite sample simulations of equivalent length to the dataset.

inflation for this subperiod. The model's performance on all other dimensions is similar to that of the benchmark model.

The reports shown in Tables 12 and 13 confirm that the results from our benchmark model are not driven by the difference between consumption and output dynamics.

4.3. Inflation ambiguity

In this paper, we assume there is only ambiguity about consumption growth, and the results are driven by covariance between realized inflation and growth dispersion; thus, inflation ambiguity is not essential to our results. However, one can relax this assumption and allow for inflation ambiguity as well. We discuss intuitively how our results will change for two cases of inflation ambiguity (i) constant size of ambiguity about mean inflation and (ii) time-varying ambiguity about mean inflation.

For case I, since inflation is negatively correlated with confidence in the first and second subperiods, an ambiguity-averse investor will choose an upper bound inflation level as the worst case in equilibrium. However, the equilibrium inflation level will be the lower bound for the third subperiod because of the positive correlation between inflation and confidence. Therefore, with a constant size of inflation ambiguity, the only impact to our results involves the changes in the level of nominal bond yields, higher for the first two subperiods and lower for the last subperiod. Note that mean inflation does not enter into the confidence equation and thus has no real effects.

For case II, if we assume that the stochastic process for the size of inflation ambiguity is exogenous, the equilibrium inflation level will still be the upper bound for the first two periods and the lower bound for the third period. Not only the level of nominal bond yields, but also the yield curve will become steeper if the ambiguity process is persistent.

For other cases of inflation ambiguity, one needs to look at the connections among the driving forces; for example, Ulrich (2013) argues that inflation ambiguity can generate an upward-sloping term premium for nominal bond yields in a setting in which this is otherwise not possible.

4.4. Magnitude of ambiguity

Given that our confidence process parameters are estimated directly using forecasts dispersion data, one natural question is whether the size of ambiguity is reasonable. I use the error detection probability approach suggested by Anderson, Hansen, and Sargent (2003) to provide a sense of magnitude for the size of ambiguity.

This approach quantifies the statistical closeness of two measures by calculating the average error probability in a Bayesian likelihood ratio test of two competing models. Intuitively, measures that are statistically close will be associated with large error probabilities, but measures that are easy to distinguish imply low error probabilities. Formally, let l be the log likelihood function of the worst-case measure relative to the reference measure and P^a be the alternative worst-case measure. Then, the average probability of a model detection error in the corresponding likelihood ratio test is $\epsilon = 0.5 \cdot P(l > 0) + 0.5 \cdot P^a(l < 0)$, where ϵ is just a simple equally weighted average of the probability of rejecting the reference model when it is true ($P(l > 0)$) and the probability of accepting the reference model when the worst case model is true ($P^a(l < 0)$). To obtain a closed-form solution, assume the size of ambiguity is a constant a; it follows that $\epsilon = \Phi(-\frac{a}{2}\sqrt{\frac{N}{\sigma^2}})$, where N is length of consumption data, Φ is the cumulative distribution function of the standard normal, and ϵ is decreasing in N and a and increasing in σ.[20]

In general, a closed-form expression for the detection error probability is not available because the size of ambiguity is not constant over time. However, the linear Gaussian framework for the consumption growth, inflation and confidence processes in this paper allows one to calculate the exact likelihood function values using the Kalman filter. The state space representation encompasses exactly these processes: consumption growth and inflation as the measurement equation and confidence process as the state transition equation. The reference measure is i.i.d normal and, thus, the likelihood function is simple; the likelihood function value of the worst-case measure is obtained recursively using the Kalman filter given the simulated data. Then, the error probability is calculated using simulated data. In this paper, parameters are estimated from data and the detection-error probability is about 5% on average and 10% for the second subperiod, which is considered to be reasonable and implies that investors would not be able to identify the correct models about 10% of the time.

5. Conclusion

Alternative asset pricing models generally are able to account for the equity premium, volatility and risk-free rate puzzles. However, the stock/bond comovements and the predictability of stock/bond return in the data pose a serious challenge to many models.

First, the CAPM betas of Treasury bonds was slightly positive during 1970s, high during 1980s, and switched to negative in the past decade, and at the same time, consistent with “the Fed” model and early studies on stock return inflation correlation, dividend yields are positively associated with inflation in the first two subperiods, and, contrary to “the Fed” model, negatively associated with inflation in the third subperiod. To capture this time varying feature of bond risks, the stochastic discount factors need to have a time varying connection with inflation. Second, the excess bond returns of Treasury bonds are

[20] This is very intuitive in that, when time period N is long enough, or when the worst case model is different enough (big a), agent can use the consumption data to statistically separate the reference model from the worst case belief. When the variance is big, it is hard for agent to distinguish the reference model from alternative models.

predictable by yield spreads and forward rates. While it is difficult enough to account for bond return predictability, it is much harder for equilibrium models of bond pricing to capture also the stocks/bonds comovements. Finally, the moments of stock return, risk free rate, and especially the behavior of the price-dividend ratios in the data also pose serious challenges to equilibrium models.

In this paper, departing from the rational expectation hypothesis that there is a single objective probability (coinciding with the investor's subjective belief) measure governing the state process, we assume the investor is ambiguity averse. Investors' confidence, or the size of ambiguity, is represented by a set of one-step-ahead measures regarding the consumption growth rate. Changes in confidence correspond to changes in the set of expected consumption growth rates.

We find that stock and bond price variations are driven by the joint dynamics of confidence and inflation. During the 1970s and particularly the 1980s, one of the most important tasks for the Fed was to fight high inflation. High inflation realizations, due to the Fed's failure in this task, or due to some other factors that were not well understood, made it harder for investors to understand the economic environment and consequently less confident about future consumption growth. As a result investors would not buy stocks and stock prices dropped, and at the same time, long-term yields increased and bond prices decreased because of high inflation. The prices of stocks and Treasuries moved in the same direction, and price-dividend ratios were negatively correlated with inflation in the first two subperiods because of this negative effect of inflation on confidence. However, in the past decade the opposite happened. Instead of fighting high inflation, now the Fed faced deflationary pressures. In this case, high inflation realizations made investors feel that the economic environment was well understood and they became more confident about future consumption growth. Stock prices rose, Treasury yields increased, and bond prices decreased as the result of high inflation. Stock and bond prices moved in opposite directions and Treasuries served as a hedge in this period.

While inflation had different impacts on confidence in different subperiods, the effect of past consumption growth was always positive. The interpretation is similar to inflation except that maintaining an efficient level of consumption/output growth is always one task of the CB. Low aggregate consumption realizations make investors less confident about future consumption growth, which in turn lowers the price-dividend ratio (pro-cyclical variation of price-dividend ratios) and increases expected returns (counter-cyclical variation of expected returns). Although variations in the price-dividend ratios reflect changes in ambiguity about future expected growth, the reference mean growth rate is constant. Thus, the model will not incorrectly imply that dividend yields predict consumption and dividend growth. At the same time, log price-dividend ratio (as a linear function of confidence) is mean reverting and, thus, dividend yields predict excess returns as in the data. The calibrated model can also match the first and second moments of market return and risk-free rate observed in the data. Using simulation data, the model generates similar estimation coefficients of the expectations hypothesis test and match well the bond return predictability.

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AppendixA.

AppendixA.1. Forcing process

The economic dynamics follow

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + \sigma_c \varepsilon_{c,t+1} \\
\Delta d_{t+1} &= \zeta_d \Delta c_{t+1} + \mu_d + \sigma_d \varepsilon_{d,t+1} && \text{(A.1)} \\
\hat{\pi}_{t+1} &= \rho_\pi \hat{\pi}_t + \zeta_\pi \varepsilon_{c,t+1} + \sigma_\pi (\varepsilon_{\pi,t+1} + \theta_\pi \varepsilon_{\pi,t}) && \text{(A.2)} \\
a_{t+1} - a &= \rho_a (a_t - a) + \kappa_c \varepsilon_{c,t+1} + \kappa_\pi \hat{\pi}_{t+1} + \sigma_a \varepsilon_{a,t+1}, && \text{(A.3)}
\end{aligned}$$

with $\varepsilon_{c,t+1}$, $\varepsilon_{d,t+1}$, and $\varepsilon_{a,t+1} \sim i.i.d.\ N(0,1)$.

AppendixA.2. The worst-case belief

First, I prove the worst-case belief is the one with lowest expected growth rate, $-a_t$. Given the endowment and confidence process, rewrite the utility over consumption as

$$\frac{V_t}{C_t} = \left(1 - \beta + \beta \left\{ \min_{p_t \in \mathcal{P}_t} \mathbb{E}_{p_t} \left[\left(\frac{V_{t+1}}{C_{t+1}} \right)^{1-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}} . \quad \text{(A.4)}$$

Let $v_t = \frac{V_t}{C_t}$, and the state variable is a_t. Now the utility can be rewritten as

$$v_t(a_t) = \left(1 - \beta + \beta \left\{ \min_{\tilde{\mu}_t \in [-a_t, a_t]} \mathbb{E}_{\tilde{\mu}_t} \left[(v_{t+1}(a_{t+1}))^{1-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1}{\theta}} \right)^{\frac{\theta}{1-\gamma}}, \tag{A.5}$$

Since the process for a_t is independent of the choice of $\tilde{\mu}_t$, thus choice of $\tilde{\mu}_t$ has no effect on a_{t+1} . Plus $v_t(a_t)$ is increasing function of $\tilde{\mu}_t$, therefore the worst-case belief is the one with lowest expected growth rate, $\tilde{\mu}_t = -a_t$. Then the min operator can be replaced with the worst-case measure.

AppendixA.3. Solving the model

The Euler equation for the economy is evaluate under the worst-case measure

$$\mathbb{E}_{-a_t} \left[exp \left(\theta log\beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} + r_{i,t+1} \right) \right] = 1, \tag{A.6}$$

where $r_{c,t+1}$ is the log return on the consumption claim and $r_{i,t+1}$ is the log return on any asset.

First, conjecture that the log price-consumption ratio, z_t, and the log price-dividend ratio for a dividend claim, $z_{t,m}$, follow

$$z_t = A_0 + A_1 a_t + A_2 \hat{\pi}_t + A_3 \varepsilon_{\pi,t}, \tag{A.7}$$

$$z_{t,m} = A_{0,m} + A_{1,m} a_t + A_{2,m} \hat{\pi}_t + A_{3,m} \varepsilon_{\pi,t}. \tag{A.8}$$

The log return on consumption claim and log return on dividend claim are given by the Campbell and Shiller (1988b) approximation

$$r_{c,t+1} = k_0 + k_1 z_{t+1} + \Delta c_{t+1} - z_t \tag{A.9}$$

$$r_{m,t+1} = k_{0,m} + k_{1,m} z_{t+1,m} + \Delta d_{t+1} - z_{t,m}. \tag{A.10}$$

AppendixA.3.1. Return on consumption claim

To solve A_0, A_1 and A_2, I substitute (A.2), (A.3), (A.7), and (A.9) into Euler Eq. (A.6). z_t can be found by the method of undetermined coefficients, using the fact that the Euler equation must hold for all values of state variables a_t and $\hat{\pi}_t$. Collecting all terms involving a_t, it follows that $A_1 = \frac{1-\frac{1}{\psi}}{k_1\rho_a - 1}$; Collecting all terms involving$\hat{\pi}_t$, it follows that $A_2 = \frac{k_1\rho_\pi}{1-k_1\rho_\pi}\kappa_\pi A_1$; Collecting all terms involving $\varepsilon_{\pi,t}$, it follows that $A_3 = \frac{\sigma_\pi\theta_\pi A_2}{\rho_\pi}$; And collect all terms involving constant implies that

$$A_0 = \frac{\begin{array}{c} log\beta + (1-\frac{1}{\psi})\mu_c + k_0 + k_1A_1(1-\rho_a)a \\ +0.5\theta[(1-\frac{1}{\psi})\sigma_c + k_1A_1(\kappa_c + \kappa_\pi\zeta_\pi) + k_1A_2\zeta_\pi]^2 \\ +0.5\theta(k_1A_1\sigma_a)^2 + 0.5\theta[(k_1A_1\kappa_\pi + k_1A_2)\sigma_\pi + k_1A_3]^2 \end{array}}{1-k_1}.$$

AppendixA.3.2. Pricing kernel / Intertemporal Marginal Rates of Substitution (IMRS)

The log real pricing kernel is

$$m_{t,t+1} = logM_{t,t+1} = \theta log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1}. \tag{A.11}$$

And given the solution for return on consumption claim, $r_{c,t+1}$, the innovation to pricing kernel can be written as

$$m_{t,t+1} - \mathbb{E}_{-a_t}(m_{t,t+1}) = v_{mc}\varepsilon_{c,t+1} + v_{m\pi}\varepsilon_{\pi,t+1} + v_{ma}\varepsilon_{a,t+1}, \tag{A.12}$$

with $v_{mc} = (\theta-1)k_1A_1(\kappa_c + \kappa_\pi\zeta_\pi) - \gamma\sigma_c + (\theta-1)k_1A_2\zeta_\pi$, $v_{m\pi} = (\theta-1)k_1(A_1\kappa_\pi + A_2)\sigma_\pi + (\theta-1)k_1A_3$, and $v_{ma} = (\theta-1)k_1A_1\sigma_a$ capturing the pricing kernel's exposure to consumption shocks, inflation shocks, and exogenous confidence shocks respectively. The log nominal pricing kernel that we use to value assets with nominal payoffs is defined as

$$m^{\$}_{t,t+1} = m_{t,t+1} - \pi_{t+1}. \tag{A.13}$$

AppendixA.3.3. Risk-free rate

Given $r_{c,t+1}$ solved above, the log pricing kernel, can be used to solve the risk-free rate, $r_{f,t} = log(\frac{1}{\mathbb{E}_{-a_t}(M_{t,t+1})})$. The solution for risk-free rate is $r_{f,t} = A_{0,f} + A_{1,f}a_t + A_{2,f}\hat{\pi}_t + A_{3,f}\varepsilon_{\pi,t}$, with $A_{1,f} = -\frac{1}{\psi}$, $A_{2,f} = A_{3,f} = 0$, and

$$A_{0,f} = \begin{array}{c} -\theta log\beta + \gamma\mu_c - (\theta-1)k_0 - (\theta-1)k_1A_0 - (\theta-1)k_1A_1(1-\rho_a)a + (\theta-1)A_0 \\ -0.5[(\theta-1)k_1A_1\sigma_a]^2 - 0.5[(\theta-1)k_1(A_1\kappa_\pi + A_2)\sigma_\pi + (\theta-1)k_1A_3]^2 \\ -0.5[(\theta-1)k_1A_1(\kappa_c + \kappa_\pi\zeta_\pi) - \gamma\sigma_c + (\theta-1)k_1A_2\zeta_\pi]^2 \end{array}. \tag{A.14}$$

AppendixA.3.4. Return on dividend claim

Given the solution for log return on consumption claim, substitute $r_{c,t+1}$, (A.2), (A.3), (A.8), and (A.10) into Euler Eq. (A.6) and use the method of undetermined coefficients, $A_{0,m}$, $A_{1,m}$, $A_{2,m}$ and $A_{3,m}$ can by found in a similar way

$$A_{1,m} = \frac{\zeta - \frac{1}{\psi}}{k_{1,m}\rho_a - 1} \tag{A.15}$$

$$A_{2,m} = \frac{k_{1,m}\rho_\pi}{1 - k_{1,m}\rho_\pi}\kappa_\pi A_{1,m}$$

$$A_{3,m} = \frac{\sigma_\pi\theta_\pi A_{2,m}}{\rho_\pi} \tag{A.16}$$

$$A_{0,m} = \frac{\begin{array}{c} \theta log\beta + (\zeta_d - \gamma)\mu_c + \mu_d + (\theta-1)(k_0 + A_0(k_1-1)) + k_{0,m} \\ +(\theta-1)k_1A_1(1-\rho_a)a + k_{1,m}A_{1,m}(1-\rho_a)a \\ +0.5[(\kappa_c+\kappa_\pi\zeta_\pi)((\theta-1)k_1A_1 + k_{1,m}A_{1,m}) + (\zeta_d-\gamma)\sigma_c \\ +\zeta_\pi((\theta-1)k_1A_2 + k_{1,m}A_{2,m})]^2] \\ +0.5\theta[(\theta-1)k_1(A_1\kappa_\pi + A_2)\sigma_\pi + k_{1,m}(A_{1,m}\kappa_\pi + A_{2,m})\sigma_\pi \\ +(\theta-1)k_1A_3 + k_{1,m}A_{3,m}]^2 \\ +0.5[(\theta-1)k_1A_1 + k_{1,m}A_{1,m}]^2\sigma_a^2 + 0.5\sigma_d^2 \end{array}}{1 - k_{1,m}} \tag{A.17}$$

The expected equity return are calculated under the reference measure that fit the data best. Given $A_{0,m}$, $A_{1,m}$, $A_{2,m}$ and $A_{3,m}$, it is straight forward to find the expected market return under reference measure, $\mathbb{E}_t(r_{m,t+1}) = A_{0,E} + A_{1,E}a_t + A_{2,E}\hat{\pi}_t + A_{3,E}\varepsilon_{\pi,t}$, with

$$A_{1,E} = A_{1,m}(k_{1,m}\rho_a - 1), \quad (A.18)$$

$$A_{2,E} = 0, \quad (A.19)$$

$$A_{3,E} = 0, \quad (A.20)$$

$$A_{0,E} = k_{0,m} + (k_{1,m} - 1)A_{0,m} + k_{1,m}A_{1,m}(1 - \rho_a)a + \zeta_d\mu_c + \mu_d \quad (A.21)$$

The innovation in market return is $r_{m,t+1} - \mathbb{E}_t(r_{m,t+1}) = v_{ec}\varepsilon_{c,t+1} + v_{e\pi}\varepsilon_{\pi,t+1} + v_{ea}\varepsilon_{a,t+1} + v_{ed}\varepsilon_{d,t+1}$, where $v_{ec} = k_{1,m}[A_{1,m}(\kappa_c + \kappa_\pi\zeta_\pi) + A_{2,m}\zeta_\pi] + \zeta_d\sigma_c$, $v_{e\pi} = k_{1,m}[(A_{1,m}\kappa_\pi + A_{2,m})\sigma_\pi + A_{3,m}]$, $v_{ea} = k_{1,m}A_{1,m}$, and $v_{ed} = \sigma_d$ which capture the stock returns' exposure to consumption shocks, inflation shocks, exogenous confidence shocks, and dividend growth shocks respectively. The conditional variance of market return is $v_{ec}^2 + v_{e\pi}^2 + v_{ea}^2 + v_{ed}^2$, and the equity premium is given by $\mathbb{E}_t(r_{m,t+1} - r_{f,t}) + 0.5Var_t(r_{m,t+1})$.

AppendixA.3.5. Bond prices

The time-t price of a zero-coupon bond that pays one unit of consumption n periods later is denoted $P_t^{(n)}$, and it satisfies the recursion $P_t^{(n)} = E_{p_t^o}[M_{t,t+1}P_{t+1}^{(n-1)}]$. We assume that $p_{t+1}^{(n-1)} = log(P_{t+1}^{(n-1)})$ is a linear function of confidence, demeaned inflation, , and inflation shock

$$p_{t+1}^{(n-1)} = A_0^{n-1} + A_1^{n-1}a_{t+1} + A_2^{n-1}\hat{\pi}_{t+1} + A_3^{n-1}\varepsilon_{\pi,t+1}. \quad (A.22)$$

Then we substitute (A.22) into the the Euler equation for the price recursion, and $p_t^{(n)} = log(P_t^{(n)})$ can be solved as a linear function of time-t state variables

$$p_t^{(n)} = A_0^n + A_1^n a_t + A_2^n\hat{\pi}_t + A_3^n\varepsilon_{\pi,t}, \quad (A.23)$$

with $A_1^n = A_1^1 + A_1^{n-1}\rho_a$, $A_2^n = A_2^1 + A_2^{n-1}\rho_\pi + A_1^{n-1}\kappa_\pi\rho_\pi$, $A_3^n = A_3^1 + (A_2^{n-1} + A_1^{n-1}\kappa_\pi)\sigma_\pi\theta_\pi$, and $A_0^n = A_0^1 + 0.5Var_t(p_{t+1}^{(n-1)}) + Cov_t(m_{t,t+1},\, p_{t+1}^{(n-1)}) + A_0^{n-1} + A_1^{n-1}(1 - \rho_a)a$, where

$$
\begin{aligned}
Var_t(p_{t+1}^{(n-1)}) &= v_{pc}^2 + v_{p\pi}^2 + v_{pa}^2 && \text{(A.24)}\\
v_{pc} &= A_2^{n-1}\zeta_\pi + A_1^{n-1}(\kappa_c + \kappa_\pi\zeta_\pi) && \text{(A.25)}\\
v_{p\pi} &= (A_2^{n-1} + A_1^{n-1}\kappa_\pi)\sigma_\pi + A_3^{n-1} && \text{(A.26)}\\
v_{pa} &= A_1^{n-1}\sigma_a && \text{(A.27)}\\
Cov_t(m_{t,t+1},\, p_{t+1}^{(n-1)}) &= v_{mc}v_{pc} + v_{m\pi}v_{p\pi} + v_{ma}v_{pa} && \text{(A.28)}
\end{aligned}
$$

Since the initial values of $A_0^1 = -A_{0,f}$, $A_1^1 = -A_{1,f}$, $A_2^1 = -A_{2,f} = 0$, and $A_3^1 = -A_{3,f} = 0$ can be calculated from $P_t^{(1)} = E_{p_t^\circ}[M_{t,t+1}]$, all the parameter values of longer horizons can be calculated recursively. Similarly, assume $p_t^{(n,\$)} = A_0^{n,\$} + A_1^{n,\$}a_t + A_2^{n,\$}\hat{\pi}_t + A_3^{n,\$}\varepsilon_{\pi,t}$, we can get the parameter values of nominal bond price $P_t^{(n,\$)}$ using nominal pricing kernel $m_{t,t+1}^\$$ and nominal price recursion $P_t^{(n,\$)} = E_{p_t^\circ}[M_{t,t+1}^\$ P_{t+1}^{(n-1,\$)}]$. The parameters satisfy the recursions

$$
\begin{aligned}
A_1^{n,\$} &= A_1^{1,\$} + A_1^{n-1,\$}\rho_a && \text{(A.29)}\\
A_2^{n,\$} &= A_2^{1,\$} + A_2^{n-1,\$}\rho_\pi + A_1^{n-1,\$}\kappa_\pi\rho_\pi && \text{(A.30)}\\
A_3^{n,\$} &= A_3^{1,\$} + (A_2^{n-1,\$} + A_1^{n-1,\$}\kappa_\pi)\sigma_\pi\theta_\pi && \text{(A.31)}\\
A_0^{n,\$} &= A_0^{1,\$} + 0.5Var_t(p_{t+1}^{(n-1,\$)}) + Cov_t(m_{t,t+1}^\$,\, p_{t+1}^{(n-1,\$)}) + A_0^{n-1,\$} + A_1^{n-1,\$}(1-\rho_a)\bar{a}, && \text{(A.32)}
\end{aligned}
$$

where

$$
\begin{aligned}
Var_t(p_{t+1}^{(n-1,\$)}) &= v_{p\$c}^2 + v_{p\$\pi}^2 + v_{p\$a}^2 && \text{(A.33)}\\
v_{p\$c} &= A_2^{n-1,\$}\zeta_\pi + A_1^{n-1,\$}(\kappa_c + \kappa_\pi\zeta_\pi) && \text{(A.34)}\\
v_{p\$\pi} &= (A_2^{n-1,\$} + A_1^{n-1,\$}\kappa_\pi)\sigma_\pi + A_3^{n-1,\$} && \text{(A.35)}\\
v_{p\$a} &= A_1^{n-1,\$}\sigma_a && \text{(A.36)}\\
Cov_t(m_{t,t+1}^\$,\, p_{t+1}^{(n-1,\$)}) &= Cov_t(m_{t,t+1}^\$,\, p_{t+1}^{(n-1,\$)}) && \text{(A.37)}\\
Cov_t(m_{t,t+1},\, p_{t+1}^{(n-1,\$)}) &= v_{mc}v_{p\$c} + v_{m\pi}v_{p\$\pi} + v_{ma}v_{p\$a} && \text{(A.38)}\\
Cov_t(\pi_{t+1},\, p_{t+1}^{(n-1,\$)}) &= \zeta_\pi v_{p\$c} + \sigma_\pi v_{p\$\pi}. && \text{(A.39)}
\end{aligned}
$$

The log holding period return from buying an n periods real bond at time t and selling it as an $n-1$ periods real bond at time $t-1$ is defined as $r_{n,t+1} = p_{t+1}^{(n-1)} - p_t^{(n)}$, and the holding period return for n periods nominal bond is defined similarly as $r_{n,t+1}^{\$} = p_{t+1}^{(n-1,\$)} - p_t^{(n,\$)}$. Given the real and nominal bond prices, we can find the shocks to real and nominal bond returns

$$
\begin{aligned}
r_{n,t+1} - \mathbb{E}_t(r_{n,t+1}) &= v_{rnc}\varepsilon_{c,t+1} + v_{rn\pi}\varepsilon_{\pi,t+1} + v_{rna}\varepsilon_{a,t+1} &\text{(A.40)}\\
v_{rnc} &= v_{pc} &\text{(A.41)}\\
v_{rn\pi} &= v_{p\pi} &\text{(A.42)}\\
v_{rna} &= v_{pa}, &\text{(A.43)}
\end{aligned}
$$

and

$$
\begin{aligned}
r_{n,t+1}^{\$} - \mathbb{E}_t(r_{n,t+1}^{\$}) &= v_{rn\$c}\varepsilon_{c,t+1} + v_{rn\$\pi}\varepsilon_{\pi,t+1} + v_{rn\$a}\varepsilon_{a,t+1} &\text{(A.44)}\\
v_{rn\$c} &= v_{p\$c} &\text{(A.45)}\\
v_{rn\$\pi} &= v_{p\$\pi} &\text{(A.46)}\\
v_{rn\$a} &= v_{p\$a}. &\text{(A.47)}
\end{aligned}
$$

Then the covariance between stock and bond returns are

$$
\begin{aligned}
Cov_t(r_{m,t+1},\ r_{n,t+1}) &= v_{ec}v_{rnc} + v_{e\pi}v_{rn\pi} + v_{ea}v_{rna} &\text{(A.48)}\\
Cov_t(r_{m,t+1},\ r_{n,t+1}^{\$}) &= v_{ec}v_{rn\$c} + v_{e\pi}v_{rn\$\pi} + v_{ea}v_{rn\$a}. &\text{(A.49)}
\end{aligned}
$$