On the Diversity-Multiplexing Region of Broadcast Channels with Partial Channel State Information

Ivana Stojanovic, Masoud Sharif, and Prakash Ishwar

***Abstract*— In this paper, we study the reliability (error probability) and rate tradeoffs among the receivers in single-antenna block Rayleigh-fading Gaussian broadcast channels. The rate and reliability tradeoffs are quantified through the notions of average multiplexing and diversity gains in the high signal to noise ratio (SNR) regime. Diversity and multiplexing gains that can be simultaneously achieved and the associated tradeoffs are well-understood in multi-antenna (MIMO) point-to-point systems. However, broadcast channels are different from point-to-point links in two important aspects (i) the availability of additional degrees of freedom in terms of flexible diversity tradeoffs among different users for the same set of multiplexing rates, and (ii) the opportunity to exploit the inherent multiuser diversity of the channel to improve the reliability. We study the diversity-multiplexing *region* defined as the set of $2K$ tuples of diversity and average multiplexing gains corresponding to all K receivers that are simultaneously achievable. We show that the diversity-multiplexing region can be significantly enlarged compared to the case where the transmitter has no channel state information (CSI) by exploiting only partial CSI in the form of the ordering of channel fading gains of users at the transmitter. This is done by proposing a time-division opportunistic scheduling scheme that can adjust to and support the average rate requirements of all receivers while exploiting the multiuser diversity of the channel. In particular, it is shown that when the average multiplexing gain of a receiver is small, its diversity gain is increased by a factor K (the number of receivers) compared to the case where no CSI is available.**

I. Introduction and Motivation

Broadcast channels model the communication scenario where a single transmitter sends independent information through a shared medium to uncoordinated receivers. An information-theoretic study of such a channel is mainly motivated by its implications for downlink communication in cellular systems. The capacity region of a broadcast channel is the largest set of admissible user rate-tuples for which the error probability of all receivers can be made arbitrarily small for all sufficiently large coding blocklengths. The capacity region provides a first-order characterization of the fundamental performance limits of downlink communication in terms of the sustainable user rates with potentially large coding delays. However, in a cellular network users typically request various services such as voice, video, or data, which have not only different rate requirements but also different reliability (error probability) requirements for various end-user applications that may not be delay-tolerant. This motivates the need for information-theoretic studies that help clarify the underlying fundamental limits and tradeoffs between rate and reliability among receivers for a finite coding blocklengths.

The transmission reliabilities in a wireless channel are hampered by different short-term and long-term random behavior of the channel, e.g., shadowing and fading. In point-to-point channels, this random behavior of the channel can be mitigated in a number of different ways, e.g., by exploiting spatial diversity and employing multiple antennas at the receiver and/or the transmitter. In multiuser channels, such as broadcast channels, there is one additional degree of freedom that can be leveraged: the very fluctuations in the channel fading gains can be exploited via opportunistic transmissions to improve the reliability of the transmissions. This can be realized by utilizing transmitter *side information* about the channel qualities of all receivers and avoiding transmissions to the receivers who are in deep fades.

A fundamental tradeoff between rate and reliability in multi-antenna point-to-point links was studied in [1] in the regime of high SNRs. In [1], the notions of diversity and multiplexing were introduced to quantify the behavior of the error probability (reliability) and rate, respectively, at high SNRs. This tradeoff has also been explored in other topologies such as relay channels [2], [3]. In broadcast channels, the tradeoff corresponds to the diversity-multiplexing region defined as the set of simultaneously achievable $2K$ tuples of the diversities and rates of all K receivers and was first addressed in [4]–[7] where inner and outer bounds for the diversity-multiplexing region were characterized in the absence of CSI at the transmitter.

While these results provide a lot of insight into performance tradeoffs in wireless channels, they do not exploit the inherent multiuser diversity of the channel [8], [9]. In this paper, we show that using only partial CSI at the transmitter of a single antenna broadcast channel the diversity-multiplexing region can be significantly improved via an opportunistic time-sharing scheduling. This scheduling is able to adjust to and support the average rate requirement of the receivers while improving the error probabilities by exploiting the multiuser diversity of the channel. For instance, when the multiplexing gain of a receiver is small, its diversity gain is increased by a factor K (number of receivers) compared to the case where no CSI is available. Our coding scheme is, loosely speaking, robust to channel phase estimation errors and has significantly reduced complexity over techniques such as dirty-paper and superposition coding.

The rest of the paper is organized as follows. The channel

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model, background, and key ideas are outlined in Section II. For clarity and simplicity of exposition, the proposed encoding, scheduling, and decoding schemes, and the theoretical results, are first developed in the context of the two-user broadcast channel in Section III. Section IV then generalizes these results to the K-user case. Concluding remarks and ongoing and future research directions are discussed in Section V.

II. Channel Model, Background, and Key Ideas

Channel model: A K-user block Rayleigh-fading discrete-time Gaussian broadcast channel with single antennas at the transmitter and each receiver is described by following per-channel-use input-output relationship:

$$\mathbf{Y}_k = h_k \sqrt{\rho}\mathbf{X} + \mathbf{Z}_k, \qquad k \in \{1, \ldots, K\} \tag{2.1}$$

where h_k represents the k-th user's (normalized) fading coefficient distributed according to a circularly symmetric, zero-mean, unit-variance, complex Gaussian distribution $CN(0,1)$. The fading coefficients across all users are independent and identically distributed (iid), remain constant over a block of l channel uses, where l denotes the discrete-time channel coherence interval, and change to a new iid realization in the next coherence interval. For simplicity of exposition, we assume a homogeneous network in which all users have the same average SNR ρ. Without loss of generality (wlog) we assume that the channel input signal vector $\mathbf{X} \in \mathbb{C}^{l\times 1}$ is normalized such that the average transmit power in each fading block is unity. The $\mathbf{Z}_k$'s are complex additive white Gaussian noise (AWGN) vectors, i.e., vectors with iid circularly symmetric complex Gaussian elements $CN(0,1)$. These correspond to the independent in-phase and quadrature modulation components of the receiver thermal noise and are independent of the channel fading coefficients. The channel output at receiver-k is given by $\mathbf{Y}_k \in \mathbb{C}^{l\times 1}$.

Remarks: It is important to note that the channel model considered here is ergodic when one considers channel input-symbols as blocks of channel input signals occupying one coherence interval and when message codewords are allowed to span multiple coherence intervals. This enables the exploitation of the ergodicity of channel fades by averaging across many independent realizations of channel gains over time. However, in the limit of long coherence intervals (slow fading), when message codewords are confined to one coherence interval, and in the absence of transmitter CSI, the channel is more accurately modeled as a non-ergodic compound channel where the notion of *outage* [1], [10] naturally comes into prominence. In the ergodic formulation, it should also be noted that the broadcast channel considered here is degraded [11] in two scenarios (i) when the transmitter has no CSI – in which case the fading gains need to be treated as channel outputs rather than as channel states – and also (ii) when the transmitter has complete knowledge of all fading gains. However, here we are interested in the intermediate scenario where the transmitter either has access to only the relative fading gain orderings of all users or perhaps chooses to only use such conservative information due to robustness and complexity considerations. Our results indicate that the performance gains can be significant even with such limited transmitter CSI.

Background–*Error exponents:* Rate versus reliability tradeoffs have been well studied through the notion of the reliability function in the theory of error-exponents for point-to-point AWGN [12] and MAC [13] channels. In most commonly encountered communication scenarios, e.g., the point-to-point AWGN channel, the probability of message decoding error is a function of the rate R, the codeword blocklength (number of symbols) N, and the SNR ρ, and decays exponentially in N as $e^{-NE(R,\rho)+o(N)}$ for good channel codes. Loosely speaking, the largest rate of exponential decay (the exponent of the best codes) $E(R,\rho)$, whenever it is exists, is the channel reliability function. Error exponents are nondecreasing functions of R for fixed ρ reflecting the somewhat intuitive observation that it should be harder to drive the error probability to zero as one approaches capacity (which is fixed by ρ). Several lower and upper bounds on the optimal error exponents have been developed in the literature [12], [14]–[16]. The two most significant of these are the random coding and sphere packing bounds which respectively provide lower and upper bounds on the optimal error exponents. These bounds are actually nonasymptotic and they coincide in the limit of large blocklengths at rates that are above the so-called critical rate [12], [14]–[16].

Diversity and multiplexing: In contrast to the large blocklength asymptotics addressed by the traditional error-exponent theory, Zheng and Tse studied the (fixed blocklength) high-SNR asymptotics of the decoding error probability for the point-to-point block Rayleigh-fading discrete-time complex AWGN channel with multiple transmit and receive antennas [1]. The ergodic channel capacity scales logarithmically with SNR [17]. It is therefore meaningful to consider sustaining a reliable coding rate $R(\rho)$ which also grows logarithmically with SNR, i.e., $R(\rho) = r\log\rho$ bits per channel use, and study the resulting asymptotic increase of the error exponent $E(R(\rho),\rho)$ with SNR. It turns out that in this setting, the error exponent also scales logarithmically with SNR (the scaling behavior is exact) as $d(r)\log\rho$. Zheng and Tse referred to r and $d(r)$ respectively as the multiplexing gain and diversity gain of the communication system and characterized the exact high-SNR asymptotic relationship between r and $d(r)$. Although practical wireless communication systems typically operate in low SNR regimes, the diversity-multiplexing high-SNR asymptotics serve to *isolate and highlight* tradeoffs that are purely due to fading effects while suppressing the effects of thermal noise. It is shown that the exact high-SNR asymptotics of the error probability is dominated by the so-called outage error event: the event of having a sequence of atypically poor channel fading gains over the duration of a codeword[1]. For single transmit and receive antennas with coding over L coherence blocks, the outage event is given by

$$\left\{ \frac{1}{L} \sum_{i=1}^{L} \log\left(1+\rho|h_i|^2\right) \le r\log\rho \right\} \tag{2.2}$$

where the h_i's are the channel fading gains over the L coherence blocks (see [1], [9]). It turns out that the probability of this outage event behaves as $e^{-L(1-r)\log\rho}$, $0 \le r < 1$, so that $d(r) = L(1-r)$.

Multiuser reliability profile: A new tradeoff comes into picture when the number of users in a system is more than one: it is possible to trade off the diversity among many users while supporting simultaneously a fixed vector of transmission (multiplexing) rates of the users. This was recently uncovered in [4]–[7] for Rayleigh-fading Gaussian MIMO Broadcast and MAC channels and provides a more complete picture of the capacity of multiuser communication systems. However, these results were derived for encoding schemes that operate with receiver CSI only, i.e., in the absence of CSI at the transmitter.

Key insights for partial transmitter CSI: Multiuser diversity gains with pure receiver CSI are bounded by the single user point-to-point diversity gain of one (for single antenna systems) [4]–[7]. Knowledge of CSI at the transmitter through receiver feedback, opens the possibility of opportunistically scheduling transmissions to good users, suppressing transmissions to users in deep fades, and potentially improving the multiuser diversity region.

Opportunistic time division multiplexing (OTDM): In this work we confine ourselves to a class of coding and scheduling schemes that make use of only partial CSI in the form of the ordering of receiver fading gains in each coherence block. We allow for coding across multiple (L) coherence blocks with transmission to at most one user at full power[2] within each coherence block. An OTDM policy, schedules users by time-sharing system resources. However, unlike time division multiplexing (TDM) in receiver-CSI-only systems that allocate transmission slots to users regardless of their channel states, OTDM is channel-aware. With knowledge of the instantaneous relative channel fading gains and rate requirement of all users in each coherence block, an OTDM system transmits to at most one user, preferably to one with a better channel state while satisfying their rate requirements. With user channels fading independently, it is unlikely that all users will simultaneously experience bad channel conditions. The opportunistic scheduler can improve the link statistics and reliability by selective transmission that avoids bad channel states. As discussed below, in a K-user system, such a scheduling policy can give a K-fold increase in the maximum diversity gain, i.e., in the asymptotic decay-rate of the error-probability with increasing SNR.

This conservative use of CSI at the transmitter is motivated by robustness and complexity considerations. While dirty-paper/superposition coding schemes can achieve the capacity of general Rayleigh-fading Gaussian MIMO broadcast channels, they require the transmitter to have access to the complete CSI. In particular, receivers need to estimate and feedback the phases of the fading gains. Phase estimates are typically less accurate and, loosely speaking, sophisticated dirty-paper-coding like capacity-achieving strategies are sensitive to phase errors. Even with the limited transmitter CSI that we allow ourselves, it is possible to realize a K-fold increase in the multiuser diversity in a K-user system when the multiplexing demands are "small". To gain insight into why this is so, we consider a point-to-point link in which the channel fading statistics are given by the distribution of the maximum of K iid Rayleigh-distributed random variables (instead of having a simple Rayleigh distribution as in (2.2)). Following the approach in [1] to derive lower and upper bounds for the error probability of the best codes using Fano's inequality and a random coding argument respectively, it can be shown that at high SNR, the error probability is dominated by the probability of an outage event given by

$$\mathcal{E}_{out} := \left\{ \frac{1}{L} \sum_{i=1}^{L} \log\left(1+\rho \max_{1\le k\le K} |h_{ik}|^2\right) \le r\log\rho \right\} \tag{2.3}$$

The outage probability can be upper-bounded as

$$\begin{aligned} &\mathbb{P}(\mathcal{E}_{out}) \le \\ &\quad \mathbb{P}\left(\max_{1\le k\le K} \left\{ \frac{1}{L}\sum_{i=1}^{L} \log\left(1+\rho|h_{ik}|^2\right) \right\} \le r\log\rho \right) = \\ &\quad \left(\mathbb{P}\left(\frac{1}{L}\sum_{i=1}^{L} \log\left(1+\rho|h_{i1}|^2\right) \le r\log\rho \right) \right)^K \end{aligned}$$

so that $\mathbb{P}(\mathcal{E}_{out}) \lesssim e^{-KL(1-r)\log\rho}$ in the high-SNR limit. Here the second equality is because h_{ik}'s are iid across k. In a similar fashion we have

$$\begin{aligned} &\mathbb{P}(\mathcal{E}_{out}) \ge \\ &\quad \mathbb{P}\left(\max_{1\le i\le L} \max_{1\le k\le K} \left\{ \log\left(1+\rho|h_{ik}|^2\right) \right\} \le r\log\rho \right) = \\ &\quad \left(\mathbb{P}\left(\log\left(1+\rho|h_{11}|^2\right) \le r\log\rho \right) \right)^{KL} \end{aligned}$$

so that $\mathbb{P}(\mathcal{E}_{out}) \gtrsim e^{-KL(1-r)\log\rho}$ in the high-SNR limit. This shows that $d(r) = KL(1-r)$: a K-fold increase in the diversity. These arguments when extended to the most general coding schemes that respect the OTDM constraints discussed above yield the entire K-user diversity-multiplexing region (see Section IV).

However, it is worth mentioning that this gain comes at a price: by transmitting to selected at-a-time good users only, the opportunistic network cannot make deterministic decoding latency guarantees. In [6], a portion of the network resource (time, power, frequency) is deterministically

[1] It must be remarked that the notion of outage arises more naturally in a nonergodic compound channel formulation. In [1] its main role was to help characterize the high-SNR asymptotic error probability. The SNR-asymptotics can potentially be derived by studying the high-SNR behavior of the random-coding and sphere-packing bounds which hold for finite blocklengths. However, this approach may not be technically straightforward.

[2] Power-control during transmissions is disallowed but one can choose not to transmit to any user.

allocated to each user in every fading block. As a result, the codeword decoding delay for any user is between L and KL. In our set-up, the hard decoding deadline constraint is relaxed and the non-deterministic delay "penalty" is leveraged to increase the reliability region that users can simultaneously achieve. In fact, it is easy to see that for the coding constraints considered here, the diversity-multiplexing region of any scheme that guarantees a finite deterministic decoding deadline will revert to that of coding schemes without transmitter CSI as in [6]. This is because for every user there will always be an SNR-independent nonzero probability that all L transmissions to the user occurs in the coherence blocks when the user's channel is the worst.

III. Two-user OTDM system

This section details a system operating under an OTDM policy for the exemplary case of two users. We now describe the main system components and results.

Encoder: Each user-k, $k = 1, 2$, has a codebook $\mathcal{C}_k$ with $|\mathcal{C}_k| = 2^{lLR_k}$ codewords $\{X_k(1), \ldots, X_k(|\mathcal{C}_k|)\}$, with each codeword spanning L coherence blocks where one coherence block corresponds to l channel uses. R_k denotes a "raw" transmission rate in bits/code-letter. Upon receiving the message for each user, the encoder looks up the corresponding codewords in each user's codebook, fragments each user's codeword into L segments, and notifies the scheduler. The scheduler decides when to transmit each segment (in nonoverlapping coherence blocks) according to a very general probabilistic schedule described next.

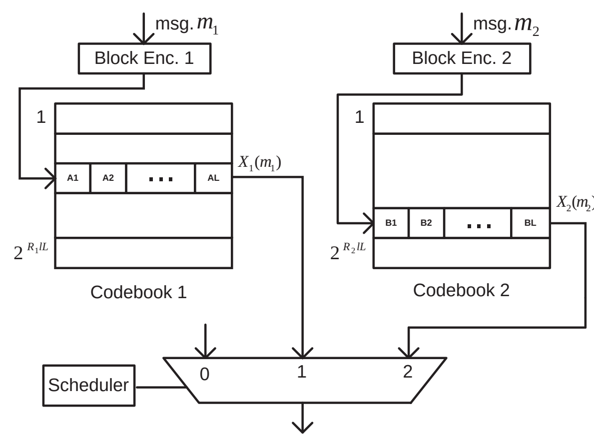
OTDM Scheduler: There are 2 distinct orderings of the fading gains of two users in each coherence block. It is convenient to represent this channel-induced relative ordering through a permutation π on the set $\{1, 2\}$, that is, a bijection $\pi : \{1, 2\} \longrightarrow \{1, 2\}$ on $\{1, 2\}$. We will interpret $\pi(k) = g$ to mean that the k-th user's channel is g-th strongest (best). Thus, $\pi(1) = 1$ means that user-1 has the best channel. We will denote the set of all possible channel-induced orderings by Π. For the two-user case of this section, Π has only two elements π_1, the identity permutation corresponding to the event $|h_1| \geq |h_2|$, and π_2 corresponding to the event $|h_1| \leq |h_2|$. The most general policy which schedules a transmission to at most one user at full power in each coherence block is described in the following definition.

Definition 3.1: (OTDM scheduling policy) Given an instantaneous channel gain ordering $\pi \in \Pi = \{\pi_1, \pi_2\}$ in each coherence block, independently draw $X_\pi \in \{0, 1, 2\}$ according to a probability mass function (pmf) $(1 - p_{1|\pi} - p_{2|\pi}, p_{1|\pi}, p_{2|\pi})$. If $X_\pi = 0$, do not transmit. If $X_\pi = k$ with $1 \leq k \leq K$ then transmit to user-k.

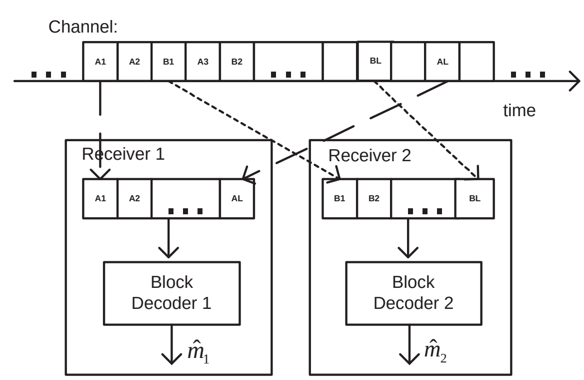
By changing the conditional channel-allocation probabilities $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$ of each individual user under each possible channel-gain ordering, the scheduler can sweep through the entire region of simultaneously realizable *average* multiplexing rate vectors of all users. In particular, note that setting $p_{1|\pi_2} = 0$ ensures that transmissions to user-1 will never occur when it is the worse of the two users. Similarly setting $p_{2|\pi_1} = 0$ ensures that transmissions to user-2 can only occur when it is the better of the two users.

Decoder: Each receiver listens to the channel and learns the schedule through either a short preamble or a pilot tone. Each receiver keeps accumulating the received "noisy" codeword segments over the coherence blocks in which transmissions were scheduled to it. Upon assembling L segments, it attempts to make a hard maximum likelihood decision about the transmitted message using its codebook.

The actions of the encoder, scheduler, and the decoder are illustrated in Fig.1.



(a)



(b)

Fig. 1. **Scheduling policy, encoding, and decoding for a $K = 2$ user channel.** *(a) The transmitter encodes messages into codewords for transmission over L coherence blocks. (b) The codewords are interleaved in the channel and decoded at the receivers at random times.*

Average rate: Receivers decode codewords at random times whose statistics are governed by those of the channel and the scheduler. Consequently, this system cannot guarantee the raw deterministic rates R_1, R_2. However, the system does guarantee *average* rates $\tau_k R_k$ bits/channel-use, where τ_k represents the average fraction of the total transmissions (in coherence-blocks) allocated to user-k. As noted earlier, schemes that impose finite deterministic decoding times, cannot improve the diversity gain over schemes that use receiver-only CSI. Let $N_k \geq L$ be a random variable which denotes the *total* number of transmissions needed to complete L channel allocations to user-k. The average transmission rate in bits per channel use is given by

$$\mathbb{E}\left[\frac{\log 2^{LlR_k}}{lN_k}\right] = R_k \cdot \mathbb{E}\left[\frac{L}{N_k}\right],\ k = 1, 2,$$

where it should be noted that $\mathbb{E}[N_k] = \mu_k(L)$ depends on both k and L. The channel acquisition probabilities p_1, p_2, for users $1, 2$ respectively, depend on both the channel and scheduler statistics. Since the channel fades are iid (exchangeable) across users and coherence blocks, all channel-gain orderings are equally likely and iid across coherence blocks, i.e., $\mathbb{P}\{\pi_1\} = \mathbb{P}\{\pi_2\} = 1/2$. Hence,

$$p_1 = \frac{1}{2}(p_{1|\pi_1} + p_{1|\pi_2}),$$

$$p_2 = \frac{1}{2}(p_{2|\pi_1} + p_{2|\pi_2}), \tag{3.1}$$

where it should be noted that $p_1 + p_2 \leq 1$, i.e., p_1 and p_2 need not add up to one because "no-transmission" is also permitted. The average rate for each user is then given by

$$R_k \cdot \mathbb{E}\left[\frac{L}{\mathrm{N_k}}\right] \overset{(a)}{\geq} \frac{L}{\mathbb{E}[\mathrm{N_k}]} R_k \overset{(b)}{=} p_k R_k,$$

where (a) is due to Jensen's inequality and (b) is an easy consequence of the following intuitive recursion formula for $\mathbb{E}[N_k] = \mu_k(L)$ which depends on L. For $\forall j > 0$ and $k = 1, 2$:

$$\begin{aligned} \mu_k(0) &= 0, \\ \mu_k(j) &= p_k(\mu_k(j-1)+1) + (1-p_k)(\mu_k(j)+1), \\ \Rightarrow \mu_k(L) &= \left(\frac{L}{p_k}\right). \end{aligned}$$

Henceforth, we will treat $\bar{R}_k := p_k R_k$ to be the average rate although, strictly speaking, it is a lower-bound on the true average rate. Here, p_k is the probabilistic counterpart of the fraction of the total transmission time allocated to user-k in regular TDM systems.

Following the approach in [4]–[7], we now formalize the notions of diversity-gain tuple, average multiplexing-gain tuple, and the diversity-average-multiplexing gain region.

Definition 3.2: (Diversity-gain, average multiplexing-gain, and the diversity-multiplexing gain region (DMGR)) The average multiplexing gain of user-k, $k \in \{1, 2\}$, operating at the raw rate $R_k(\rho)$ over L fading blocks, is defined as

$$\bar{r}_k := \lim_{\rho\to\infty} \frac{\bar{R}_k(\rho)}{\log\rho} = p_k \cdot \lim_{\rho\to\infty} \frac{R_k(\rho)}{\log\rho} = p_k r_k, \tag{3.2}$$

where r_k is the "raw" multiplexing gain of user-k. The associated diversity-gain is defined as

$$d_k := \lim_{\rho\to\infty} -\frac{\log P_{ek}\left(L, l, p_1, p_2, r_1, r_2, \rho\right)}{\log\rho}, \tag{3.3}$$

where P_{ek} denotes the minimum probability of error of user-k that can be attained by any OTDM scheme that codes over L coherence blocks, each of length l, at SNR ρ. Both $\bar{r}_k$ and d_k depend on the scheduling parameters $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$. The collection of all 4-tuples $(\bar{r}_1, \bar{r}_2, d_1, d_2)$ of simultaneously achievable user diversity gains and user average multiplexing gains as one sweeps through the set of all scheduling parameters $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$ is called the diversity-multiplexing gain region for OTDM scheduling.

In particular, given the multiplexing vector $(\bar{r}_1, \bar{r}_2)$, the scheduler first finds the scheduling parameters $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$ such that the rate constraints can be satisfied, i.e., $\bar{r}_1 \leq \frac{1}{2}(p_{1|\pi_1} + p_{1|\pi_2})$ and $\bar{r}_2 \leq \frac{1}{2}(p_{2|\pi_1} + p_{2|\pi_2})$. Secondly, the scheduler obtains the largest achievable diversity gain region by optimizing over $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$. It is worth mentioning that if the scheduler decides on opportunistically transmitting to any user, i.e., $p_{1|\pi_2} = 0$ and/or $p_{2|\pi_1} = 0$, its diversity gain will be changed as its channel distribution is no longer Rayleigh and it is rather the maximum of two Rayleigh distributed random variables (see Equation (2.3)). The following Theorem describes the DMGR for a two-user broadcast channel.

Theorem 3.1: (Two-user DMGR) For given values of L, l, the two-user diversity-multiplexing region is the collection of all 4 tuples $(\bar{r}_1, \bar{r}_2, d_1, d_2)$ that can be simultaneously realized by sweeping through the set of scheduling parameters $\{p_{k|\pi}\}_{k\in\{1,2\},\pi\in\Pi}$. This is given by the following set of parametric equations, for $k = 1, 2$, as

$$p_k = \frac{1}{2}(p_{k|\pi_1} + p_{k|\pi_2}), \tag{3.4}$$

$$0 \leq \bar{r}_k \leq p_k, \tag{3.5}$$

$$d_k = LG_k\left(1 - \frac{\bar{r}_k}{p_k}\right), \tag{3.6}$$

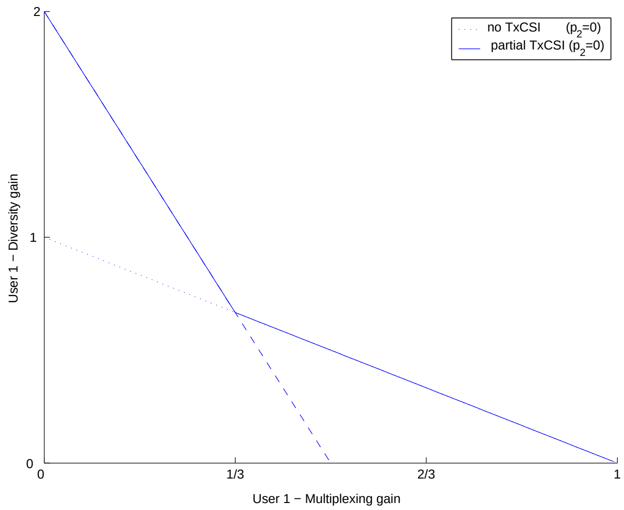
where G_1 (and similarly G_2) is defined as,

$$G_1 = \begin{cases} 2, & \mathrm{i}f \quad p_{1|\pi_2} = 0 \\ 1, & \mathrm{o}therwise. \end{cases}$$

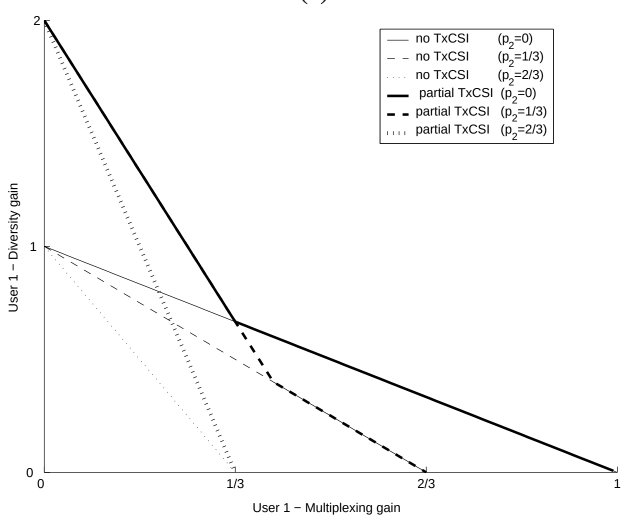
Theorem 3.1 implies that a user requesting an average rate below $\frac{1}{2}$ will have a better diversity gain (in fact up to two fold increase) using OTDM. Furthermore, since one of the two users will certainly have a multiplexing gain less than $\frac{1}{2}$, the diversity gain region is strictly improved over a system without CSI at the transmitter.

In Figures 2 and 3, we have compared the DMGR for the two cases where the transmitter has partial or no CSI. Fig. 2 shows the trade off between diversity and multiplexing gain of user-1 while user-2 has different multiplexing gains.

In particular, for the case where $\bar{r}_2 = 0$, the gain in the trade off is achieved only when $\bar{r}_1 < \frac{1}{3}$ via transmitting less frequently over the times that the channel has favorable conditions (i.e., it is the best user). As the transmitter starts allocating transmission time to the second user, the trade off between d_1 and $\bar{r}_1$ will be changed as shown in Fig. 2(b). The (d_1, d_2) region is shown in Fig. 3 for different pairs of



(a)



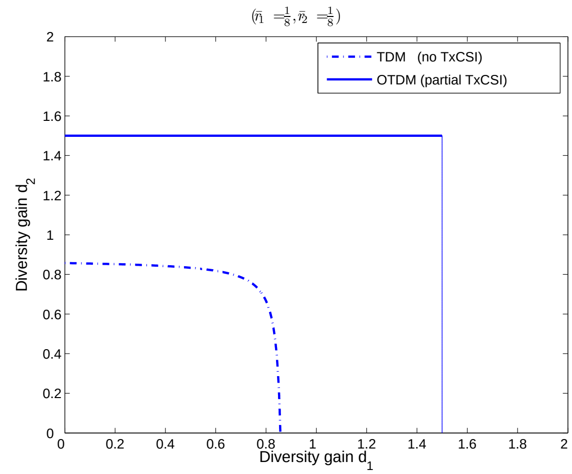
(b)

Fig. 2. **OTDM.** *(a) User-1's DMT for inactive second user* ($p_2 = 0, \bar{r}_2 = 0$)*, (b) user-1's DMT with both users active* ($p_2 > 0, \bar{r}_2 \neq 0$).

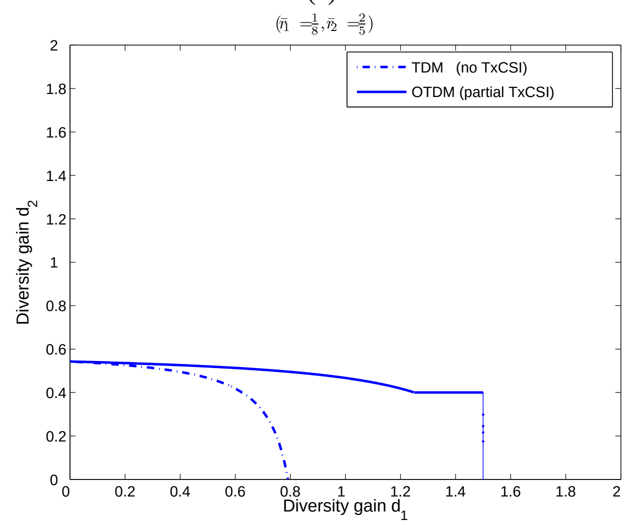
multiplexing vectors $(\bar{r}_1, \bar{r}_2)$. For small multiplexing gains, e.g., $(\bar{r}_1, \bar{r}_2) = (\frac{1}{8}, \frac{1}{8})$, OTDM nearly doubles the achievable diversity region. For larger multiplexing gains, the region is strictly larger than then the region achieved with no CSI as shown in Fig. 3(b).

IV. K-USER OTDM SYSTEM

There are $K!$ distinct equally likely orderings of K user channels in each coherence time. Denote permutations of the ordered K-tuple $(1, \ldots, K)$ by π so that π :



(a)



(b)

Fig. 3. **Two-user DMGR with partial transmit CSI:** *(a) at* $(\bar{r}_1, \bar{r}_2) = (\frac{1}{8}, \frac{1}{8})$ *OTDM improves DMGR almost* 2 *times with opportunistic transmission to both users; (b) at* $(\bar{r}_1, \bar{r}_2) = (\frac{1}{8}, \frac{2}{5})$ *optimum transmission strategy for user-*2 *depends on a desired diversity* d_1*. User-*2 *achieves the best diversity by non-opportunistic transmission if* $d_1 < 1.2$ *or by opportunistic transmission if* $d_1 > 1.2$.

$\{1, 2, \ldots, K\} \longrightarrow \{1, 2, \ldots K\}$ is a bijection on the set $\{1, \ldots, K\}$ and $\pi(k) = g$ to be interpreted as the k-th user channel is g-th strongest.

Definition 4.1: (OTDM scheduling policy) Given an instantaneous channel gain ordering $\pi \in \Pi := \{\pi_1, \ldots, \pi_{K!}\}$ in each coherence block, independently draw $X_\pi \in \{0, \ldots, K\}$ according to a pmf $\left(1 - \sum_{k=1}^{K} p_{k|\pi}, p_{1|\pi}, ..., p_{K|\pi}\right)$. If $X_\pi = 0$, do not transmit. If $X_\pi = k$ with $1 \le k \le K$ then transmit to user-k.

Theorem 4.1: (K-user DMGR) For given values of L, l, the K-user diversity-multiplexing gain region is the collection of all $2K$ tuples $(\bar{r}_1, ..., \bar{r}_K, d_1, ..., d_K)$ that can be simultaneously realized by sweeping through the set of all valid scheduling parameters $\{p_{k|\pi}\}_{k\in\{1,...,K\},\pi\in\Pi}$ and the set of all valid raw multiplexing rates $(r_1, ..., r_K)$. This is given by the following set of parametric equations. For $1 \le k \le K$,

$$p_k := \frac{\sum_{\pi\in\Pi} p_{k|\pi}}{K!}, \tag{4.1}$$
$$0 \le \bar{r}_k \le p_k, \tag{4.2}$$
$$d_k = LG_k\left(1 - \frac{\bar{r}_k}{p_k}\right), \tag{4.3}$$

where $G_k = (K - g + 1)$ if

$$p_{k|\pi} = \begin{cases} > 0, \forall\pi : \pi(k) \le g \\ = 0, \forall\pi : \pi(k) > g \end{cases}, \quad g \in \{1, ..., K\}.$$

Note that $G_k = (K - g + 1)$ only when two conditions are simultaneously met: (i) $p_{k|\pi} > 0,\ \forall\pi : \pi(k) \le g$, that is, transmissions to user-k can occur only during channel orderings in which user-k's channel is the g-th best or better, and (ii) $p_{k|\pi} = 0,\ \forall\pi : \pi(k) > g$, that is, transmissions to user-k never happen during channel orderings in which user-k's channel is worse than g-th best. This leads to a $(K-g+1)$ fold improvement in the diversity-gain of user-k over receiver-CSI-only coding schemes. Also note that $p_k \le \left(\frac{K-G_k+1}{K}\right)$.
Theorem 4.1 implies that when all the users have "small" multiplexing gains, their diversity gain can be improved by a factor of K compared to the case where transmitter lacks any CSI. Fig. 4 also shows the diversity-multiplexing trade off for user-1 in a broadcast channel with $K = 10$ while other users have multiplexing gain of zero, i.e., their rate is constant independent of P.

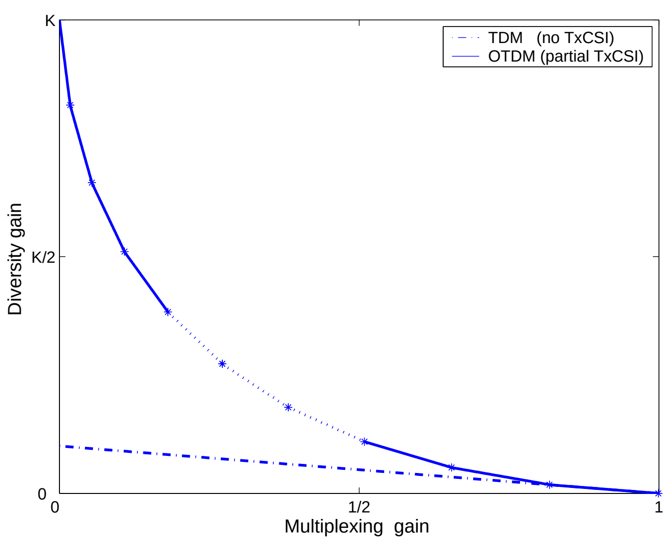


Fig. 4. *Maximum diversity multiplexing tradeoff of user-1 in a K-user broadcast channel* ($\bar{r}_1 \ne 0$ *and* $\bar{r}_k = 0, \forall k \ne 1$).

V. Summary and Ongoing Work

In this work we have derived an achievable diversity-multiplexing gain region for the K-user single antenna Rayleigh-fading Gaussian broadcast system that attempts to schedule users opportunistically during their good channel states. Users whose rate requirements can be met by scheduling transmissions only in slots when their channel states are the best, can get a K-fold improvement in their maximum diversity gain over any encoding and scheduling policy that operates with only receiver CSI. Extensions to MIMO and MAC channels, comparisons with dirty-paper coding schemes, and an analysis of the associated performance-sensitivity to channel-state estimation errors is part of our ongoing work.

Appendix A
Proof of Theorem 4.1 and 3.1

We sketch the proof of Theorem 4.1 below Theorem 3.1 is the special case of Theorem 4.1. Notation: $f(\rho) \doteq \rho^d$ implies that $\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log \rho} = d$. The symbols $\dot{\le}, \dot{\ge}$ are similarly defined.

Step 1. The probability of error for point-to-point channels with Rayleigh fading channel is lower bounded by the probability of the outage event [1]. The same approach can be used to show that the result continues to hold for channels whose fading statistics is the maximum of g independent Rayleigh fading channels. Therefore, we can obtain a lower bound for the error probability of the OTDM scheduling by the corresponding outage event.

Step 2. Consider a set of scheduling parameters $\{p_{k|\pi}\}_\pi$ for user k such that (i) transmissions to user-k occur only during channel orderings in which user-k's channel is the g-th best channel or better, and (ii) $p_{k|\pi} = 0,\ \forall\pi : \pi(k) > g$, that is, transmissions to user-k never happen during channel orderings in which user-k's channel is worse than g-th best. For this situation, there is an SNR–independent nonzero probability of the event $\mathcal{E}_{kg}$ that all the L codeword blocks of user k get transmitted during channel states in which user-k's channel is the g-th best: the worst–case channel state that satisfies conditions (i) and (ii) of the scheduling stated above. It is straightforward to show that the high–SNR asymptotics of the outage probability is dominated by the probability of outage conditional on the event $\mathcal{E}_{kg}$. This probability is given by

$$\begin{aligned} P(\mathcal{E}_{kg}) &= \mathbb{P}\{\frac{1}{L}\sum_{j=1}^{L}\log\left(1+\rho Q^j_{g:K}\right) \le r\log\rho\} \\ &\sim \mathbb{P}\{\sum_{j=1}^{L}\frac{\log Q^j_{g:K}}{\log\rho} \le L(r-1)\} \\ &= \mathbb{P}\{\sum_{j=1}^{L} z_j \le L(r-1)\} = \int_A f_g(\underline{z})d\underline{z} \end{aligned}$$

where $Q^j_{g:K}$, $j = 1, \ldots, L$ is iid with a pdf which is the g'th largest out of K independent Rayleigh–distributed random variables, $A = \{\underline{z} : \sum_{j=1}^{L} z_j \le L(r-1)\}$ and the joint pdf of L i.i.d. variables $\underline{z} = [z_1, \ldots, z_L]$ is $f_g(\underline{z}) = \prod_{j=1}^{L} f_g(z_j)$.

For Rayleigh channels, $\|h\|^2$ has exponential pdf $f(\lambda) = e^{-\lambda}$ and cdf $F(\lambda) = 1 - e^{-\lambda}$. Therefore, the marginal pdf of g'th ordered channel gain is expressed as:

$$f_g(\lambda) = K\binom{K-1}{g-1}[F(\lambda)]^{K-g}[1-F(\lambda)]^{g-1}f(\lambda)$$

and the marginal pdf of the variable $z_j := \frac{\log Q^j_{g:K}}{\log \rho}$ becomes

$$f_g(z_j) = K\binom{K-1}{g-1}[1-e^{-\rho^{z_j}}]^{K-g}[e^{-\rho^{z_j}}]^{g-1}\rho^{z_j}\ln\rho.$$

In asymptotic of $\rho \longrightarrow \infty$, one needs to consider only dominant terms [1][Section II.B] in the pdf $f_g(z_j)$. The term $e^{-\rho^{z_j}}$ drives the P_{out} exponentially fast to zero for any $z_j > 0$, thus we consider the range where $z_j < 0$ for which the term approaches 1. By Taylor expansion the term $[1-e^{-\rho^{z_j}}] \approx \rho^{z_j}$ and

$$\begin{aligned} P_{out}(r\log\rho) = \int_A f_g(\underline{z})d\underline{z} &\doteq \int_A \prod_{j=1}^{L} \rho^{z_j(K-g+1)} d\underline{z} \\ &\doteq \rho^{L(K-g+1)(r-1)} \end{aligned}$$

where the last asymptotic equality follows from Laplaces' principle. The probability of error is upper bounded by $P_e \le P_{out}(r\log\rho) + \mathbb{P}(\text{error, no outage})$. Following [1] it can be shown that $P(\text{error, no outage}) \dot{\le} \rho^{-d_{out}(r)}$ for $l \ge 1$. This establishes $P_e \doteq \rho^{-d_{out}(r)}$ and concludes the proof of Theorem 4.1.

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