Radial Quantization of Conformal Field Theory on the Lattice.

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Radial Quantization

$$H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$$

$$ds^2 = dx^\mu dx_\mu = e^{2\tau}[d\tau^2 + d\Omega^2]$$

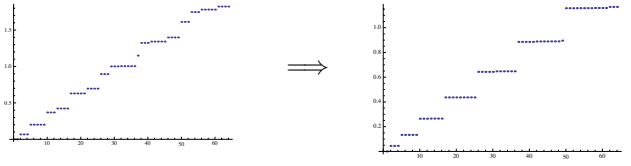
Conformal Map : $\mathcal{R}^d \to \mathcal{R} \times \mathcal{S}^{d-1}$

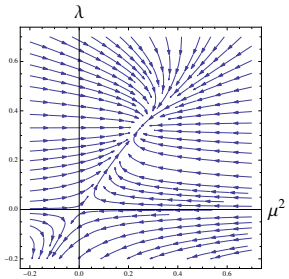
"time" $\tau = log(r)$, *"mass"* $\Delta = d/2 - 1 + \eta$

$a < \Delta r < L$ vs $a < \Delta log(r) < L$

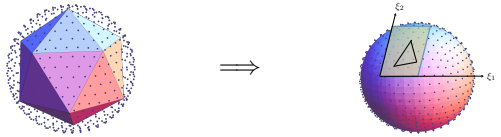
Finite Element/Regge Geometry

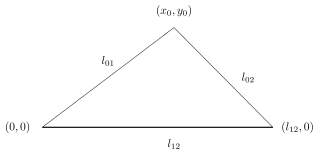
$$S = \int d^D x \sqrt{-g}[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \lambda(\phi^2 - \frac{\mu^2}{2\lambda})^2] \ .$$





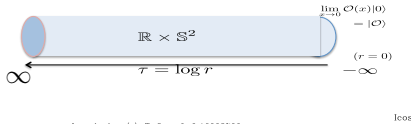
FEM improved Laplace spectra with F = 20*s*s = 1280 triangles





$$\int_{A_{012}} dxdy \partial_\mu \phi(x,y) \partial_\mu \phi(x,y) = \frac{1}{2A_{012}}[(l_{01}^2 + l_{20}^2 - l_{12}^2)(\phi_1 - \phi_2)^2 + \text{cyclic}]$$

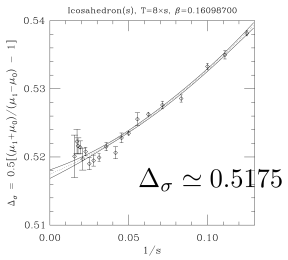
3D Ising on Tessellated Icosahedron Cylinder

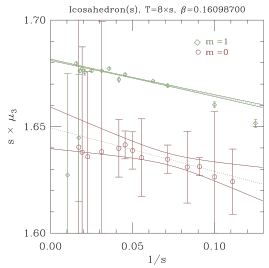


state-op corr

$\beta_{crit} = 0.16098703(3)$

$\Delta_\sigma = 1/2 + \eta/2$

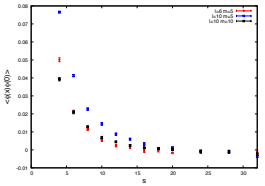




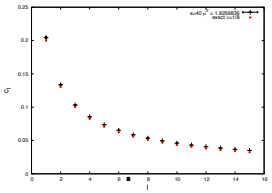
I = 0 (A),1 (T1) , 2 (H) are irreducible 120 Iscosahedral subgroup of O(3)

2D test on Conformal Projection to Riemann Sphere

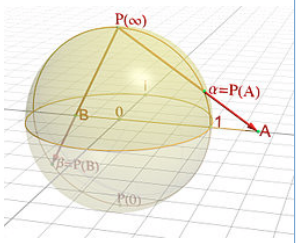
$$\langle \phi(x_1)\phi(x_2) \rangle = \frac{1}{|x_1 - x_2|^{2\Delta_\sigma}} \to \frac{1}{|1 - cos\theta_{12}|^{\Delta_\sigma}}$$



FEM convergence to spherical symmetry



Computed vs Exact Correlator



$\Delta_\sigma = \eta/2 = 1/8 \simeq 0.128$

References: R. Brower, G. Fleming and H. Neuberger Physics Letters B 721 (2013) 299–305
R. Brower, Michael Cheng, G. Flemng Lattice 2013 proceedings

Future? Broken 4D CFT for BSM physics, Twisted Graphene sheets , Ads dual ?