Remarks on, CFTs, Radial Lattice Quantization* and Graphene**!

Richard Brower
Boston University, May 10, 2014

PPCM:
Field Theoretic Computer Simulations for Particle Physics and Condensed Matter

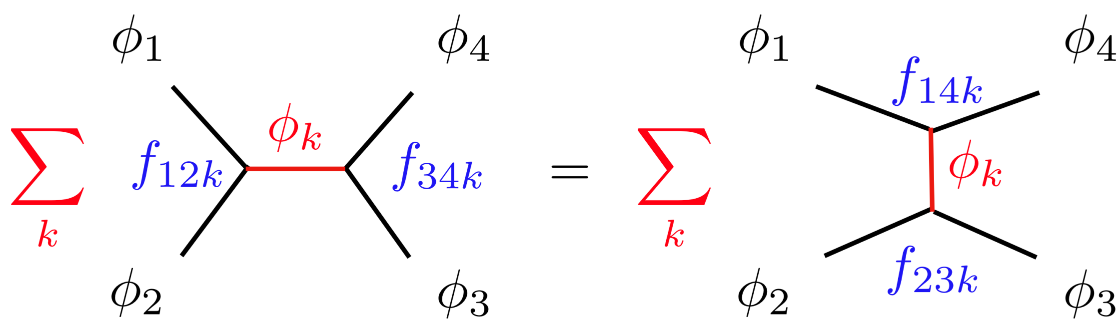
*RCB, G. Fleming, H. Neuberger & M. Cheng
**RCB, C. Rebbi, D. Schaich & M. Cheng

CFT are highly constrained

$$\langle \phi(x_1)\phi(x_2)\rangle = C\frac{1}{|x_1 - x_2|^{2\Delta}}$$

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_k \frac{f_{ijk}}{|x_1 - x_2|^{\Delta_i+\Delta_j-\Delta_k}}\mathcal{O}_k(0)$$

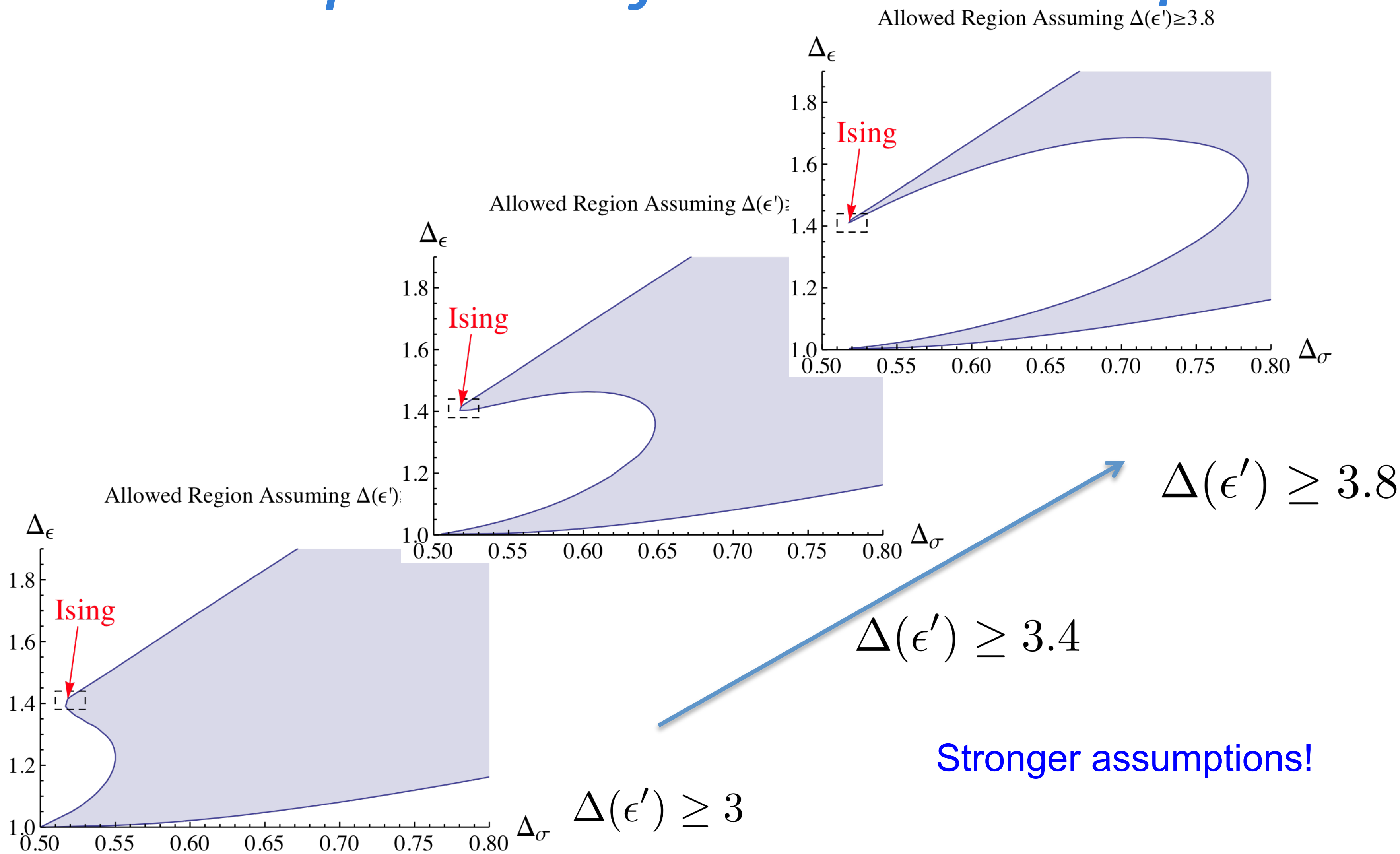
OPE & factorization completely fixed the theory*
(i.e. Data: spectral + couplings to conformal blocks)



complete sum over
the conformal blocks
"partial waves".
Only "tree" diagrams!

* "Solving the 3D Ising Model with the Conformal Bootstrap" (El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin and Vichi) arXiv:1203.6064v1v [hep-th] (2012)

*Inequalities from Bootstrap**



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Radial Quantization

Evolution: $H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$

$$ds^2 = dx^\mu dx_\mu = e^{2\tau}[d\tau^2 + d\Omega^2]$$

Can drop
Weyl factor!

$$\mathbb{R}^d \to \mathbb{R} \times \mathbb{S}^{d-1}$$

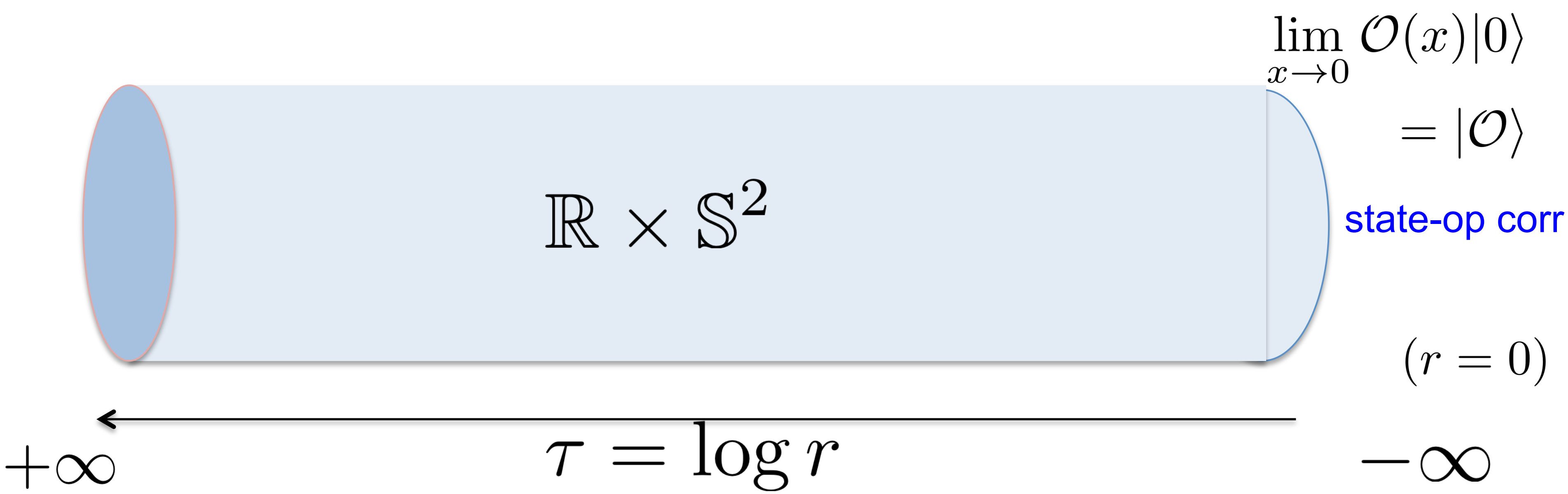
$$\text{"time"} \quad \tau = log(r) \;, \quad \text{"mass"} \quad \Delta = d/2 - 1 + \eta$$

Exponentially Large Lattice:

$$a < \Delta r < L \quad \text{vs} \quad a < \Delta log(r) < L$$

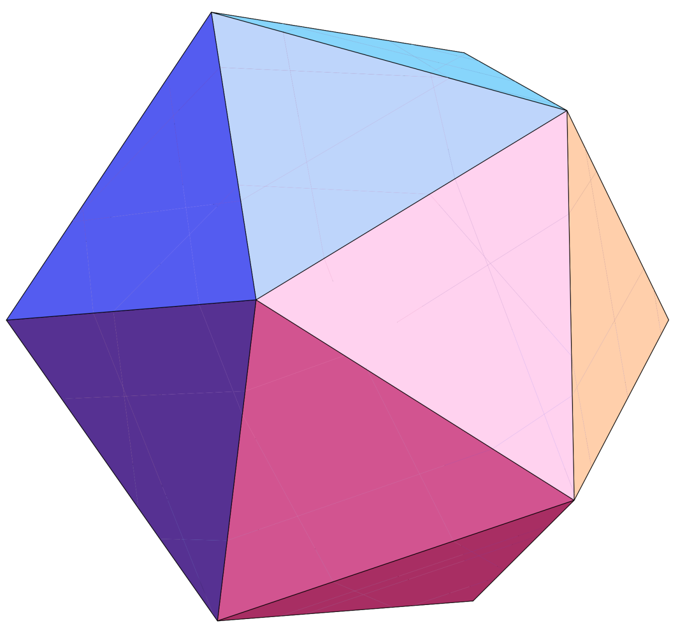
3-d Ising at Wilson-Fisher FP

$$Z_{Ising} = \sum_{\sigma(x,t)=\pm 1} e^{\beta \sum_{t,\langle x,y\rangle} \sigma(t,x)\sigma(t,y) + \beta \sum_{t,x} \sigma(t+1,x)\sigma(t,x)}$$

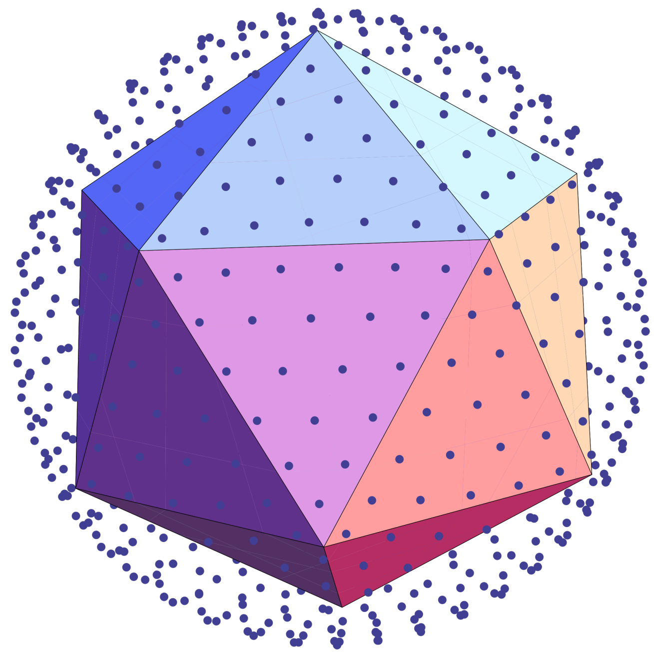


Order s Refined Triangulated Icosahedron

$s = 1$

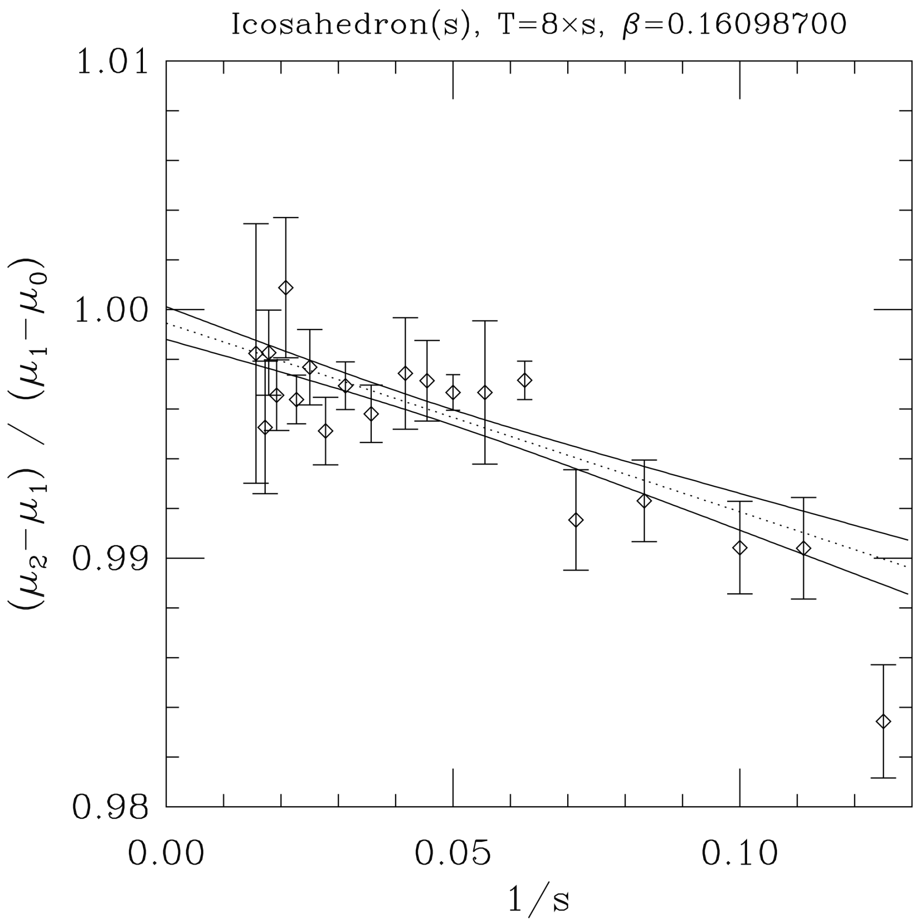


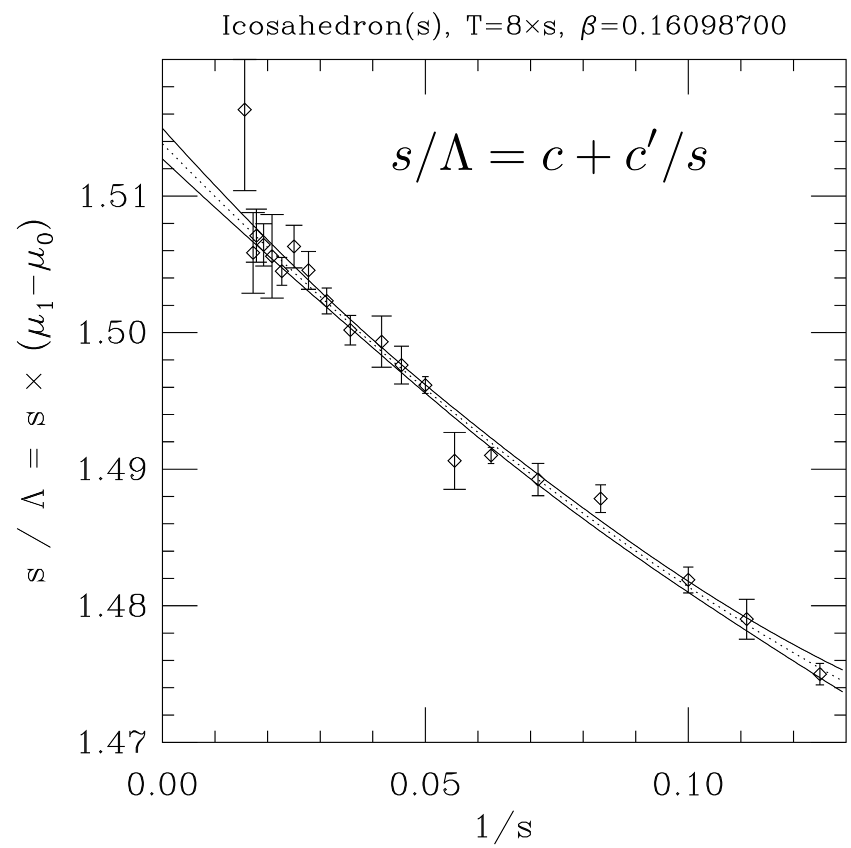
$s = 8$



l = 0 (A),1 (T1) , 2 (H) are irreducible 120
Iscosahedral subgroup of O(3)

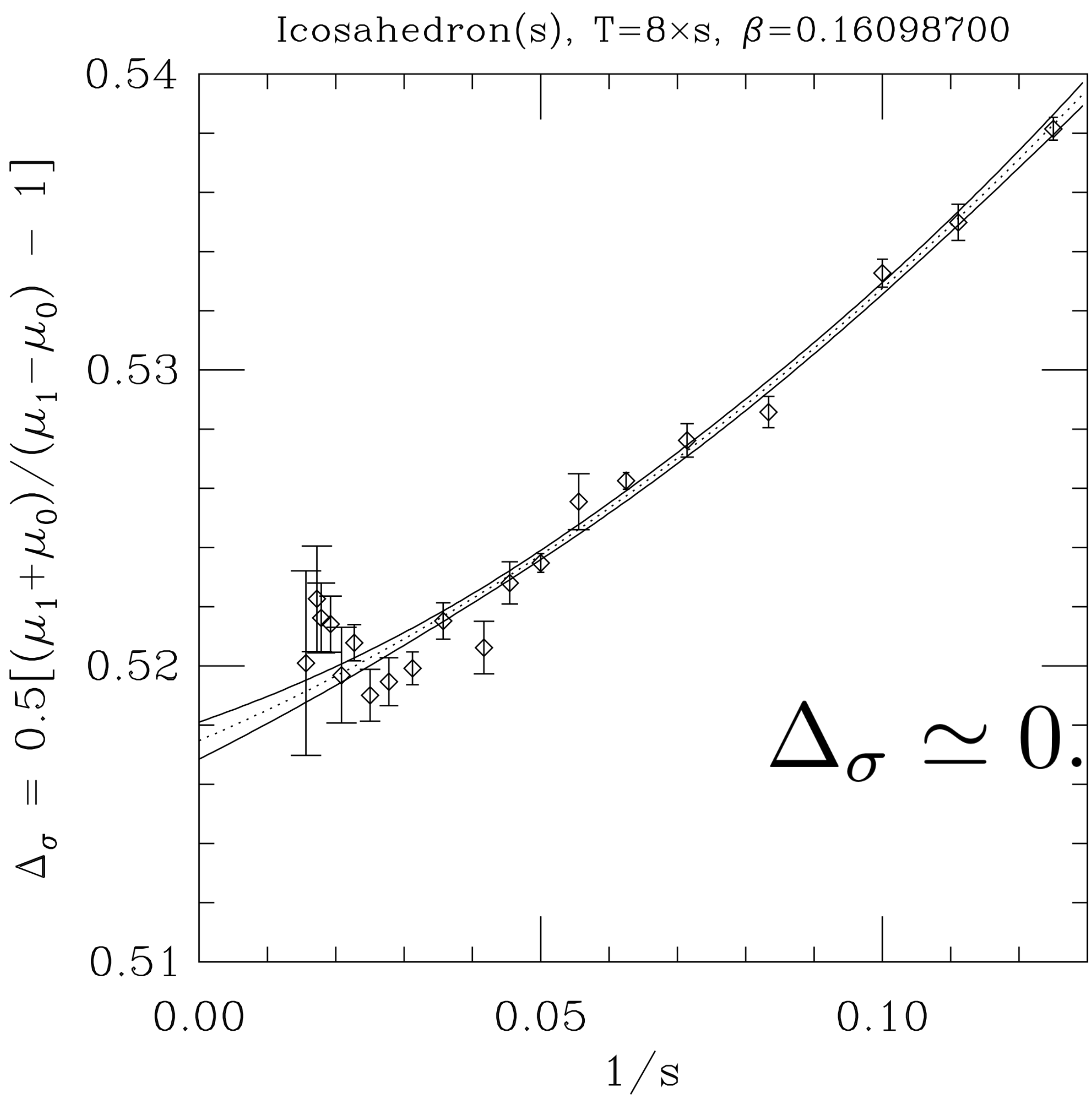
Check Descendant Relation & rescale "log(r)"





$$c = 1.5105(7)$$

Current Fit:

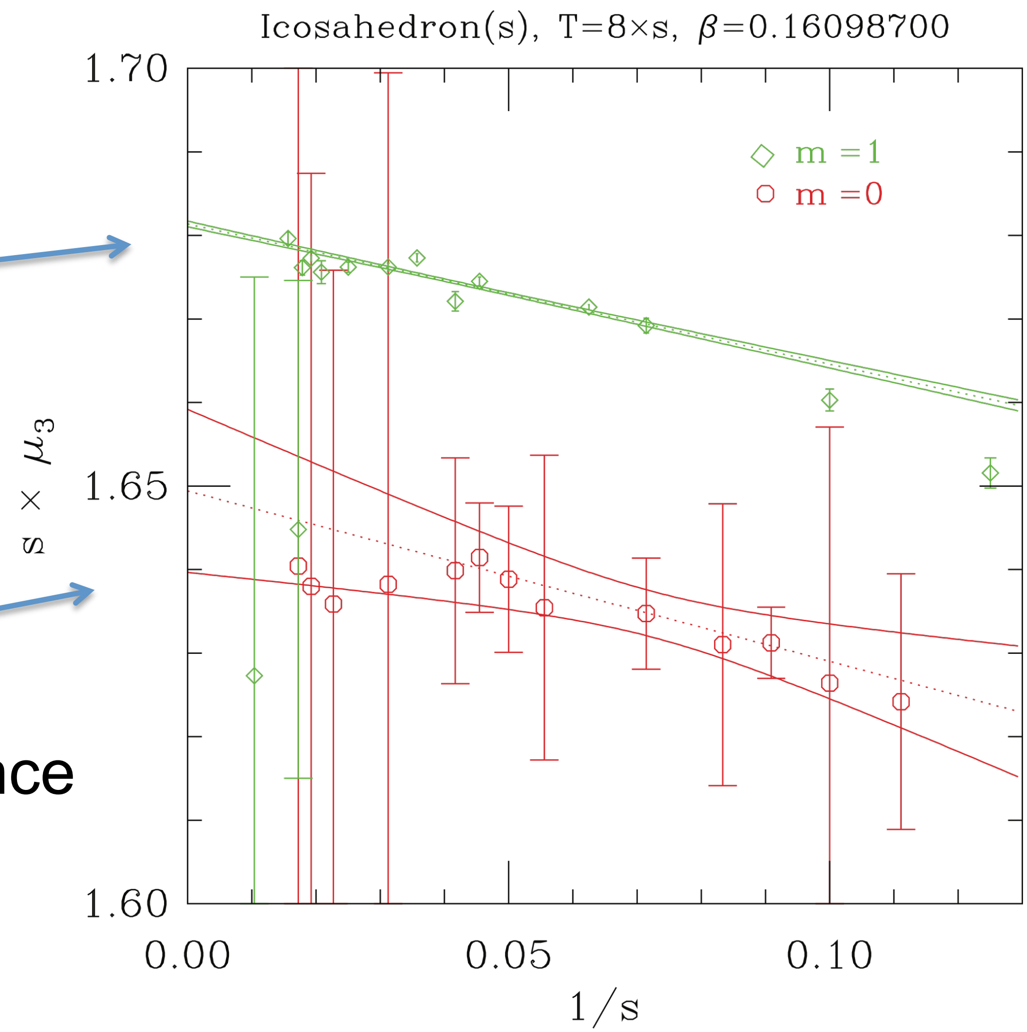


$$\Delta_\sigma \simeq 0.5175$$

Wrong Theory?
Failure to recover O(4,1) at l = 3?

G rep

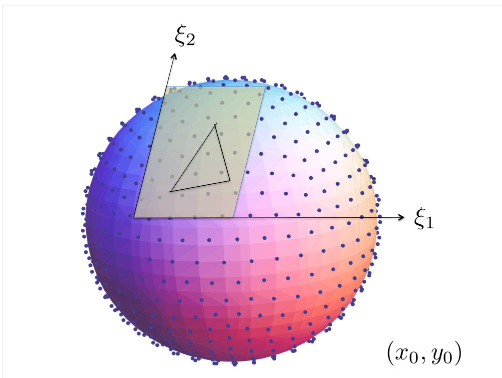
T2 rep

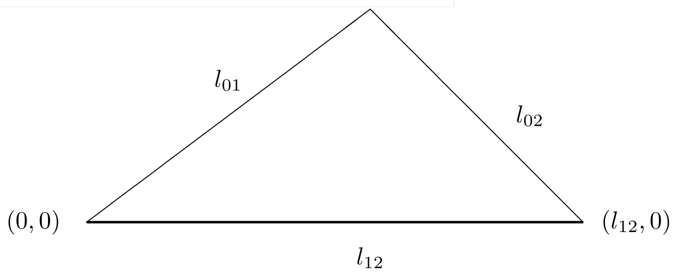


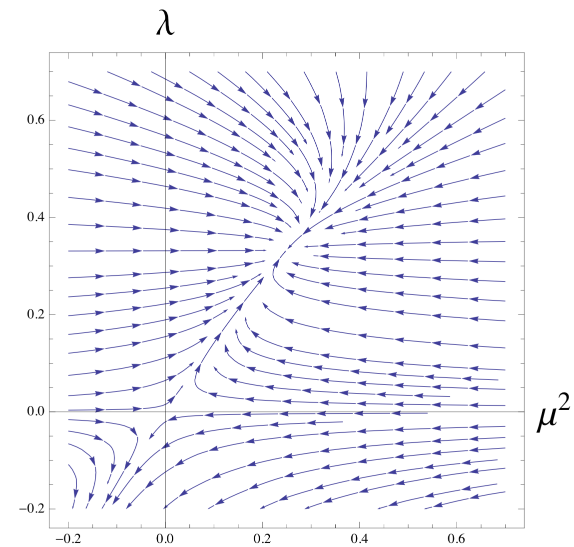
Apparent lack of convergence to a single O(3) irreducible representation for l = 3

Finite Element Method/Regge Geometry

$$S = \int d^D x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda(\phi^2 - \frac{\mu^2}{2\lambda})^2\right] .$$







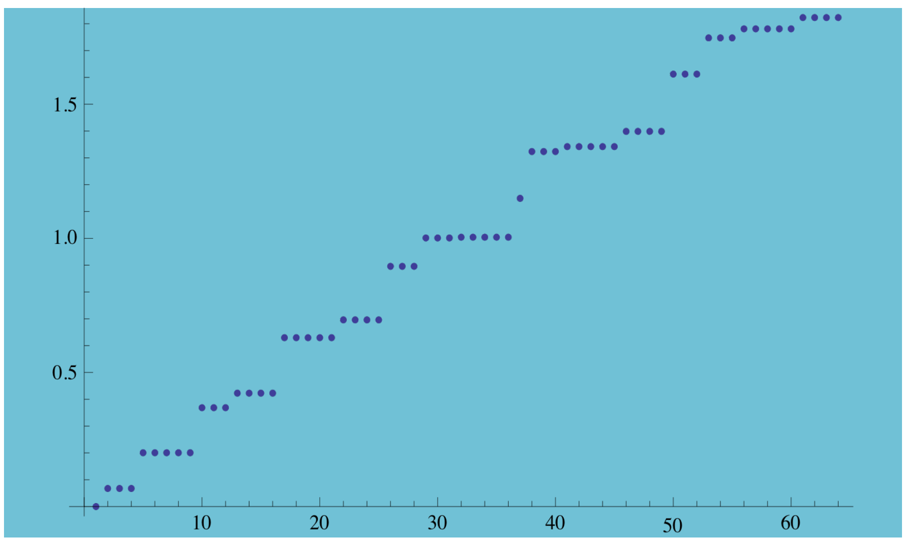
$$\int_{A_{012}} dx dy \partial_\mu\phi(x,y)\partial_\mu\phi(x,y) = \frac{1}{2A_{012}}[(l_{01}^2 + l_{20}^2 - l_{12}^2)(\phi_1 - \phi_2)^2 + \text{cyclic}]$$

See also: Christ, Friedberg, Lee on “Random Lattice” NP (1982)

FEM fixes the huge Spectral defects

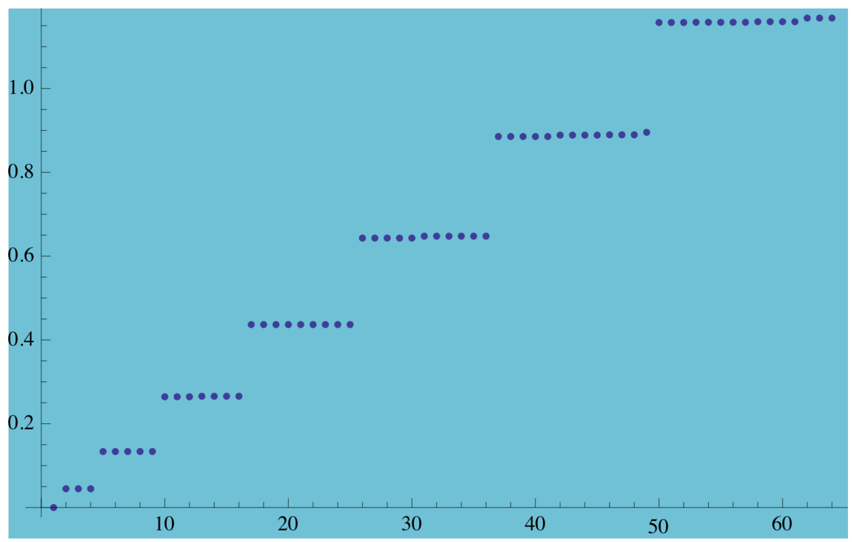
For s = 8 first (l+1)*(l +1) = 64 ev

BEFORE (K = 1)



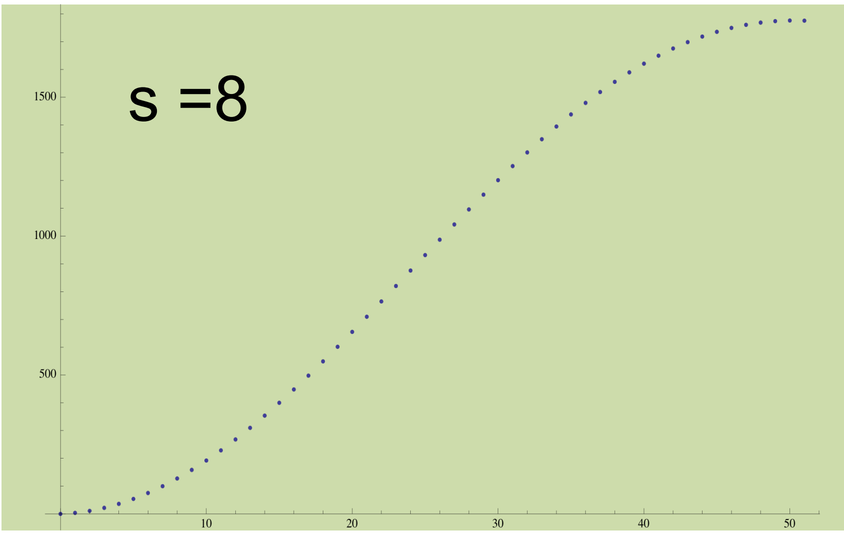
l, m

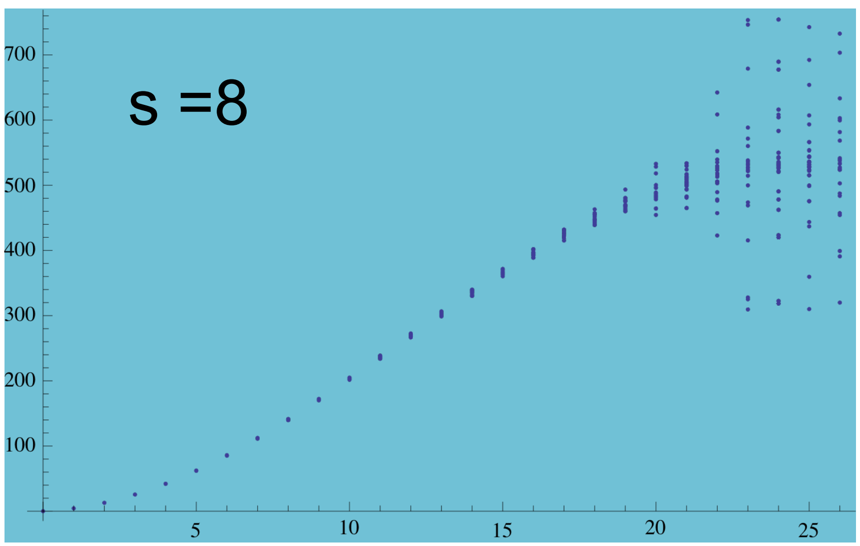
AFTER (FEM K's)

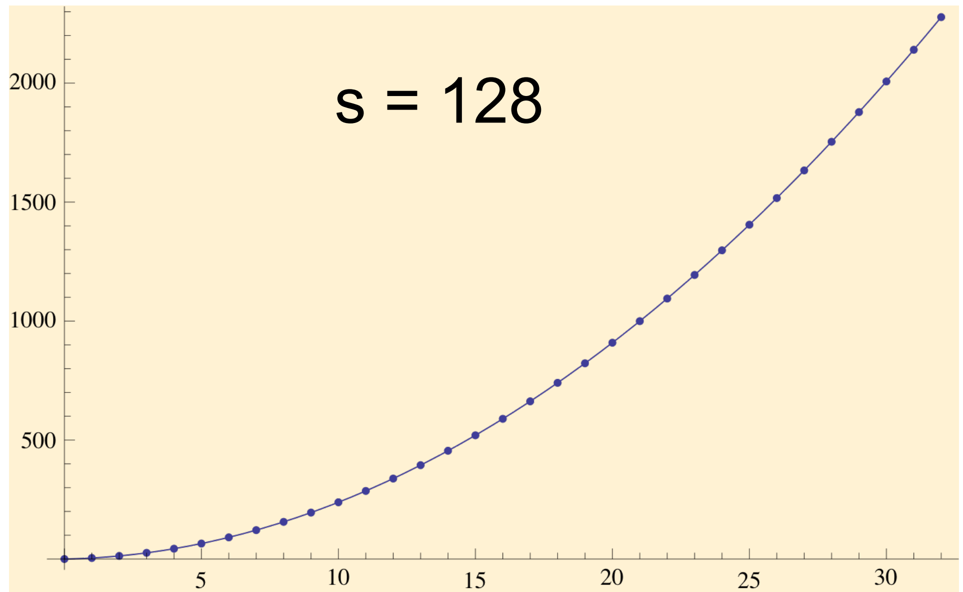


l, m

Spectrum of FE Laplacian on a sphere



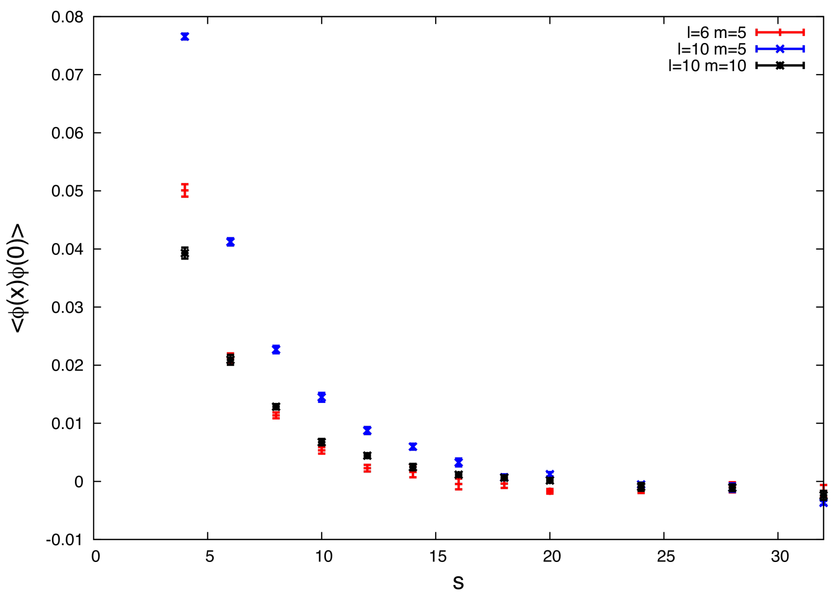


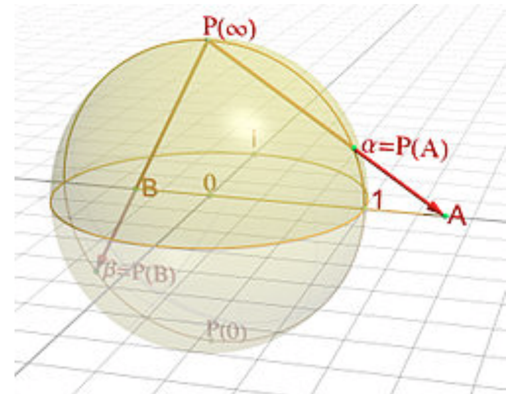


Fit

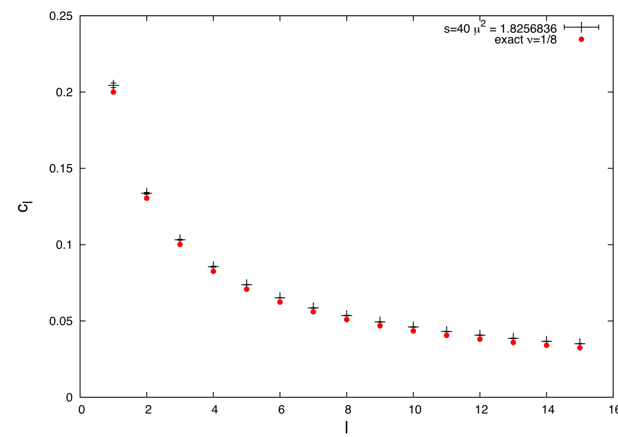
$$l + 1.00012\, l^2$$
$$- 13.428110^{-6}\, l^3 - 5.5724410^{-6}\, l^4$$

2D test on Conformal Projection to Riemann Sphere





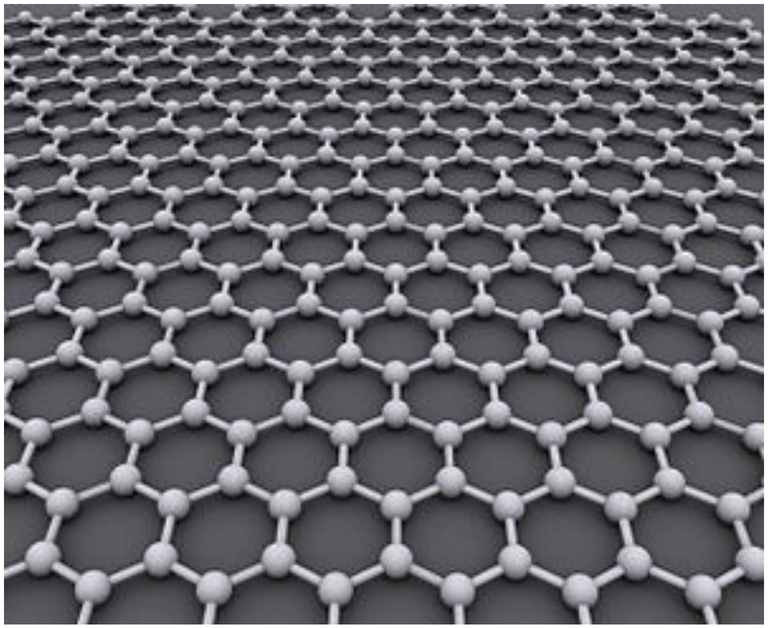
$$\Delta_\sigma = \eta/2 = 1/8 \simeq 0.128$$

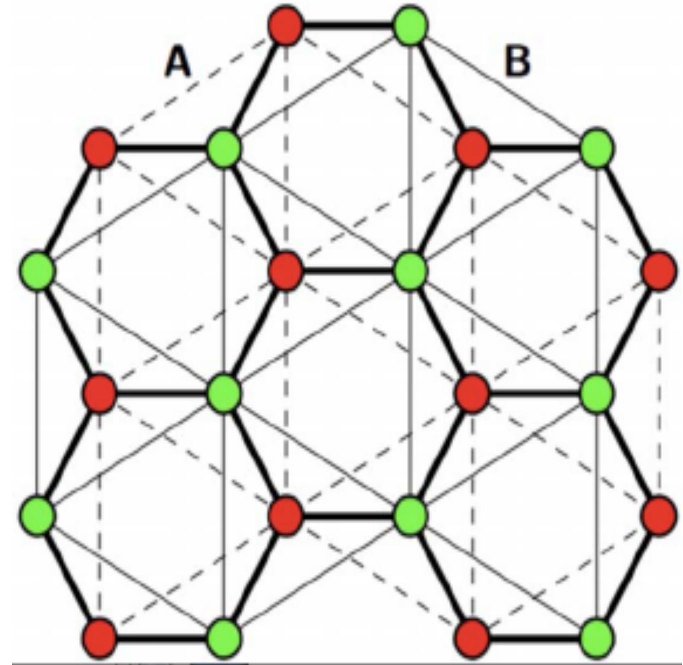


$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta}} \to \frac{1}{|1 - cos\theta_{12}|^{\Delta}}$$

Future? Broken 4D CFT for BSM physics,
Twisted Graphene sheets , Ads dual ?

Graphene





2d Carbon hexagonal lattice: A/B Bravais sub lattices $\sqrt{3}a\ ,\ a = 1.42\AA$

Effective field 2+1 relativistic theory: 4 copies of 2 component Dirac fields

Strong electrostatic fields: $\Rightarrow$ $e^2_{eff} = \frac{e^2}{\hbar v} \simeq 300 \times \frac{e^2}{\hbar c}$

Phonons act like gauge fields.

Tight Binding Hamiltonian

$$H = H_2 + H_C = a^\dagger_{xs} K_{xy} a_{ys} + q_x V_{xy} q_y$$

$$q_x = a^\dagger_{x\uparrow} a_{x\uparrow} + a^\dagger_{x\downarrow} a_{x\downarrow} - 1 \equiv a^\dagger_x a_x - b^\dagger_x b_x$$

Exact internal symmetry: U(1)x SU(2) Fermion number and Spin symmetry

Generators:

$$Q = \sum_x q_x \rightarrow Q = [a^\dagger_x a_x - b^\dagger_x b_x]$$

$$J_\pm = a^\dagger_{x,s} \sigma^{ss'}_\pm a_{x,s'} \quad \rightarrow \quad J_+ = J^\dagger_- = (-1)^{B_x} a^\dagger_x b^\dagger_x$$

$$J_3 = a^\dagger_{x,s} \sigma^{ss'}_3 a_{x,s'}/2 \quad \rightarrow \quad J_3 = [a^\dagger_x a_x + b^\dagger_x b_x]/2 - 1$$

$s = \uparrow, \downarrow$ “a = up spin elect/b =down spin holes”

Lagrangian for Tight Binding Hamiltonian

$$\mathcal{L} = \psi_x^\dagger(\partial_t + ie\phi\sigma_3 + e^2 V_{xx})\psi_x - \kappa \sum_{\langle x,y\rangle} \psi_x^\dagger \psi_y + \frac{1}{4}\phi_x(t) V_{xy}^{-1} \phi_y(t)$$

Normal Ordering Term

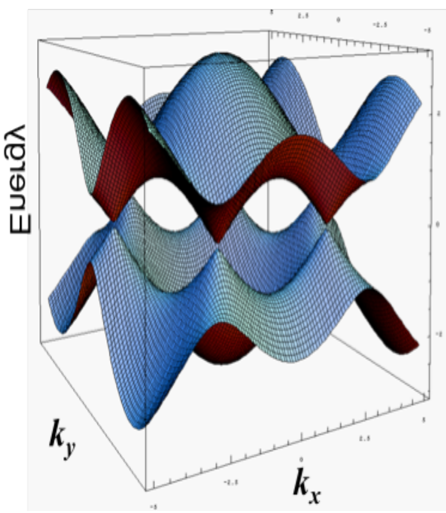
Invert and convolute by FFTs

Discretize time axis: Each spin has kinetic term (e=0)

$$D(E,k) = \begin{array}{cc} & \begin{array}{cc} \quad\quad\quad A \quad\quad\quad & \quad\quad\quad B \quad\quad\quad \end{array} \\ & \left[\begin{array}{cc} (e^{iE\delta} - 1) + m\delta & \kappa c_k \delta \\ \kappa c_k^* \delta & (e^{iE\delta} - 1) - m\delta \end{array} \right] \begin{array}{c} A \\ B \end{array} \end{array}$$

where

$$\begin{aligned} \omega_k &= \pm\sqrt{c_k^* c_k} \\ &= \pm\sqrt{3 + 2\cos(\sqrt{3}k_x a) + 4\cos(\sqrt{3}k_y a/2)\cos(3k_x a/2)} \end{aligned}$$



Recall Staggered Hypercubic Lattice

Unitary transform of Naive to Staggered Fermion

$$\sum_{\pm\mu} \bar{\psi}(x)\gamma_\mu \psi(x+\mu) = \sum a^\dagger_\mu \eta_\mu(x) a_{x+\mu}$$

where $\gamma_{-\mu} \equiv -\gamma_\mu$

Defining the Unitary matrix $S(x_1, x_2, x_3, x_4) = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$

$$\psi_y = S(y) a_y \quad , \quad \psi^\dagger_x = a^\dagger_x S^\dagger(x)$$

$$\eta_\mu(x) = S^\dagger(x)\gamma_\mu S(x+\mu) = (-1)^{x_1+\cdots+x_{\mu-1}}$$

Graphene Naive Dirac Equation

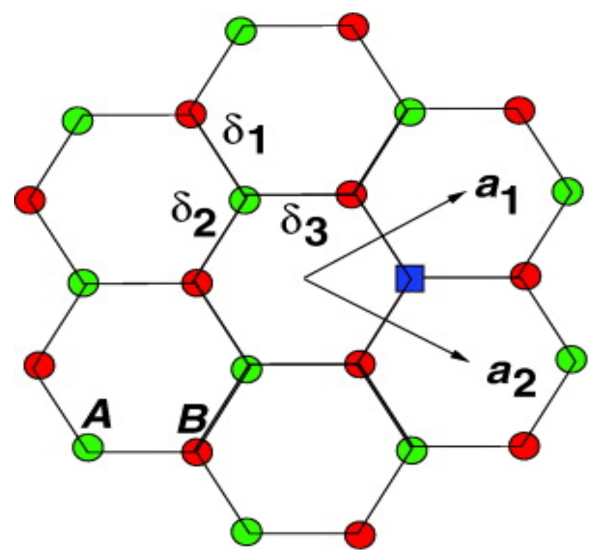
$$\hat{e}_1 + \hat{e}_2 + \hat{e}_3 = 0 \qquad \hat{e}_i \cdot \hat{e}_j = \frac{3\delta_{ij} - 1}{2}$$

Defining:

$$\sigma^{(\mu)} \equiv \hat{e}_\mu \cdot \vec{\sigma} \qquad \Longrightarrow \qquad \sigma^{(1)} + \sigma^{(2)} + \sigma^{(3)} = 0$$

Graphene Dirac Equation

$$\sum_{\mu=1,2,3} \psi^\dagger_x \, \hat{e}_\mu \cdot \vec{\sigma} \, \psi_{x+\hat{e}_\mu} + h.c.$$



With x restricted to A (green) sublattice
h.c. give B (red) sublattice contribution
NOTE: Hermiticity require all backward
link B to A also have + sign!

Graphene “Staggered” Transformation

Path ordered closed Path give +1. For example one Hexagon is

$$\sigma^{(1)}\sigma^{(2)}\sigma^{(3)}\sigma^{(1)}\sigma^{(2)}\sigma^{(3)} = -\sigma^{(1)}\sigma^{(3)} - \sigma^{(2)}\sigma^{(3)} = 1$$

This implies path independence for the path order spin product

$$S(x) = \sigma^{(\mu_k)} \; \cdots \; \sigma^{(\mu_3)}\sigma^{(\mu_2)}\sigma^{(\mu_1)}$$

so we introduce local operators $\psi_y = S(y)a_y \quad , \quad \psi_x^\dagger = a_x^\dagger S^\dagger(x)$

to show equivalence to the traditional 2 spin tight binding action

$$\sum_\mu \psi_x^\dagger \, \hat{e}_\mu \cdot \vec{\sigma} \, \psi_{x+\hat{e}_\mu} + h.c. = \sum_\mu [a^\dagger_{\uparrow,x} \, a_{\uparrow,x+\hat{e}_\mu} + a^\dagger_{\downarrow,x} \, a_{\downarrow,x+\hat{e}_i}] + h.c.$$

Improved TIME DISCRETIZATION ?

Continuum D.R. $E = \pm i\sqrt{\omega_k^2 + m^2} \quad \text{with} \quad \omega_k^2 = c_k^* c_k$

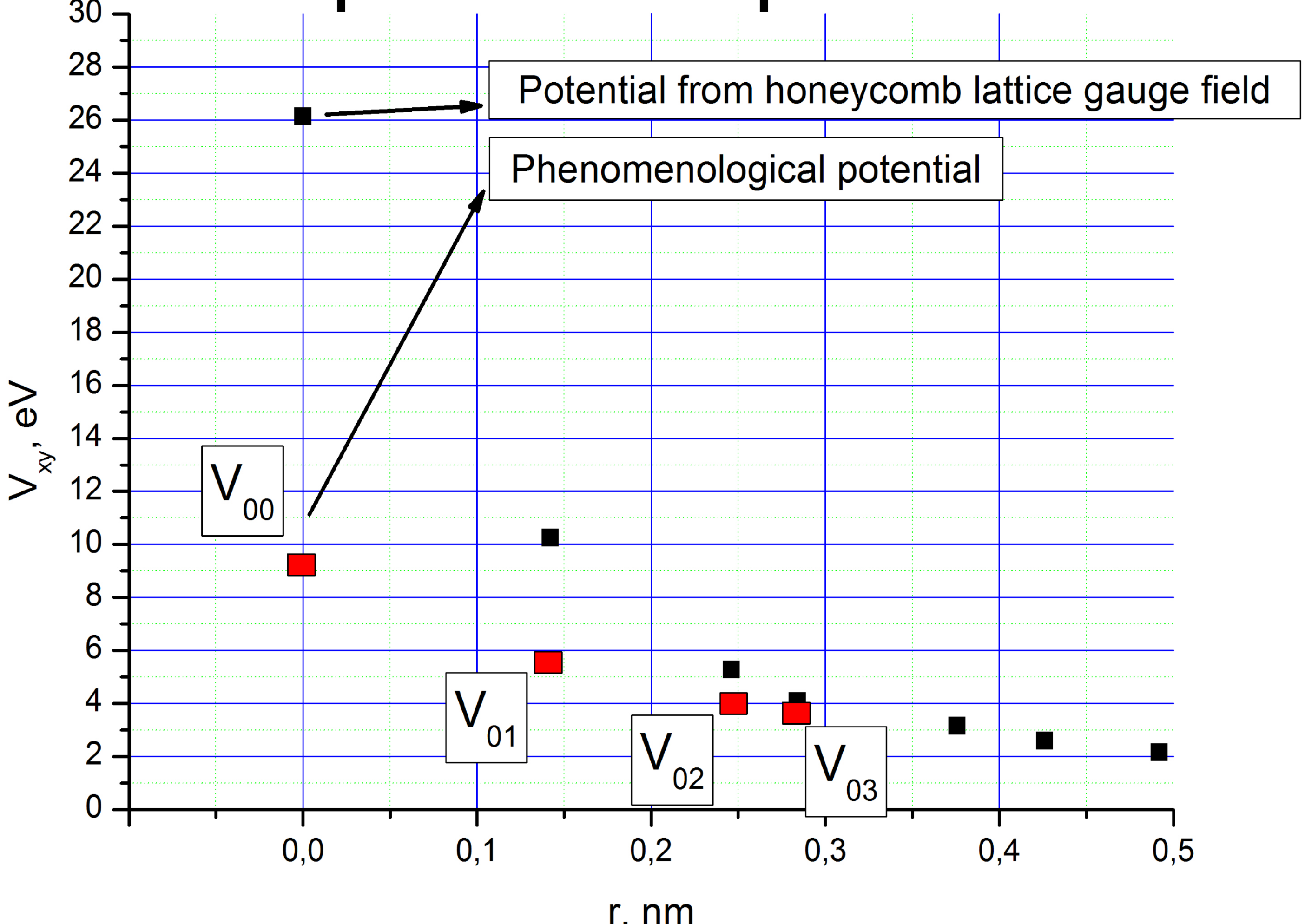
Forward Differences form $2\sin\delta E/2 = \pm i\delta e^{-i\delta E/2}\sqrt{\omega_k^2 + m^2}$ $O(\delta)$ Errors

A/B Time reversal form $2\sin E\delta/2 = \pm i\delta\sqrt{\dfrac{m^2 + \omega_k^2}{1-m}}$ $O(\delta^2)$ Errors

Staggered Time form $\sin\delta E = \pm\delta i\sqrt{\omega_k^2 + m^2}$ $O(\delta^2)$ Errors

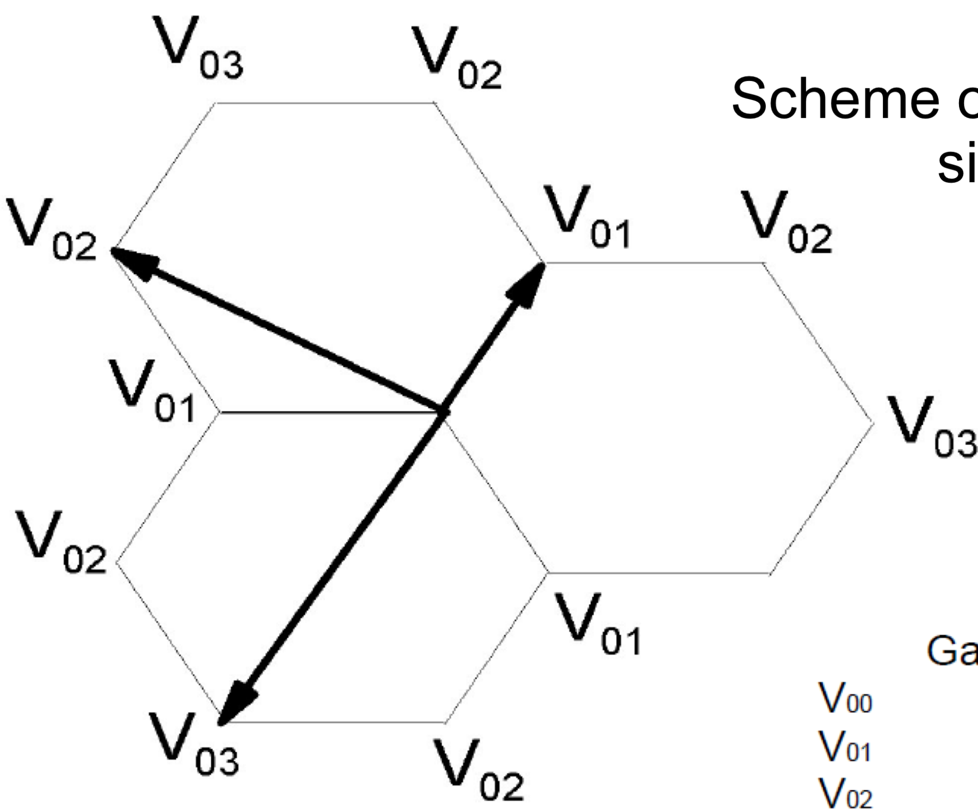
Comparison of 3 options need testing on realistic lattices with Coulomb term

Comparison of potentials



«Regularization» of Coulomb interaction at short distances

Comparison of potentials



Scheme of neighbour sites

	Gauge field	Phenomenology
V_{00}	26.2 eV	9.3 eV
V_{01}	10.2 eV	5.5 eV
V_{02}	5.3 eV	4.1 eV
V_{03}	4.1 eV	3.6 eV

see M. V. Ulybyshev, P. V. Buividovich, M. I. Katsnelson, ArXiv: 1304.3660, to appear in PRL

Conclusions

Regularization of Coulomb potential at short distances strongly affects the conductor-insulator phase transition in graphene.

Though the suspended graphene is a conductor, the phase transition still exists at unphysical dielectric permittivity $\varepsilon \sim 0.7$.

M. V. Ulybyshev, P. V. Buividovich, M. I. Katsnelson, ArXiv: 1304.3660, to appear in PRL

http://www.yale.edu/QCDNA/index14.html June 19-21, 2014