Quantum Monte Carlo in Fermionic Models

Fakher F. Assaad (Field Theoretic Computer Simulations for Particle Physics and Condensed Matter, BU 8.5.2015)

<u>Outline</u>

Correlated electrons in bosonic and fermionic baths (CT-INT)

Electron-phonon interaction
Correlated helical liquids

Determinantal quantum Monte Carlo for lattice models

Entanglement spectra for correlated topological insulators
Mott transition

Conclusions

Weak coupling CT-QMC for the SIAM.

A. N. Rubtsov et al. Phys. Rev. B72, 035122, (2005)

E. Gull, A. J. Millis, A. I. Lichtenstein, A. N. Rubtsov, M. Troyer, and P. Werner Rev. Mod. Phys., 83, 349, (2011)

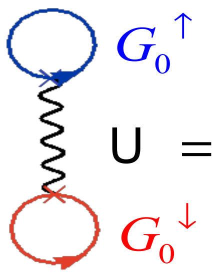
$$S = \underbrace{-\int d\tau\, d\tau'\, d_\sigma^+(\tau)\,\mathcal{G}_0^{-1}(\tau-\tau')\,d_\sigma(\tau')}_{S_0} + U\int_0^\beta d\tau\ \underbrace{d_\uparrow^+(\tau)\,d_\uparrow(\tau)}_{n_\uparrow(\tau)}\,d_\downarrow^+(\tau)\,d_\downarrow(\tau)$$

Dyson. Expansion around U=0.

$$\frac{\mathrm{Tr}\left[e^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H_0}\right]} = \sum_n \int_0^\beta d\tau_1 \cdots \int_0^{\tau_{n-1}} d\tau_n\,(-U)^n \left\langle n_\uparrow(\tau_1)n_\downarrow(\tau_1)\cdots n_\uparrow(\tau_n)n_\downarrow(\tau_n)\right\rangle_0$$

Wick

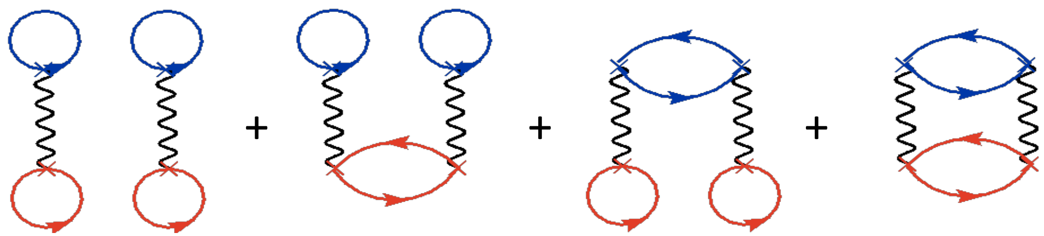
n=1



$$= -U\det\begin{pmatrix} G_0^{\uparrow}(\tau_1,\tau_1) & 0 \\ 0 & G_0^{\downarrow}(\tau_1,\tau_1)\end{pmatrix} \equiv -U\det\left[M_1(\tau_1)\right]$$

$$G_0^{\sigma}(\tau_2,\tau_1) = \left\langle T\,\hat{d}_\sigma^+(\tau_2)\hat{d}_\sigma(\tau_1)\right\rangle_0$$

n=2



$$= U^2\det\left[M_2(\tau_1,\tau_2)\right]$$

$$\frac{\mathrm{Tr}\left[e^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H_0}\right]} = \underbrace{\sum_n \int_0^{\beta} d\tau_1 \cdots \int_0^{\tau_{n-1}} d\tau_n}_{\textit{Sum}\text{ with Monte Carlo}} \underbrace{(-U)^n \det\left[M_n\left(\tau_1,\cdots,\tau_n\right)\right]}_{\text{Weight}}$$

Weight / Sign.

- $H_U \rightarrow \frac{U}{2}\sum_{s=\pm 1}\left(n_\uparrow^{\ d} - \left[1/2 - s\delta\right]\right)\left(n_\downarrow^{\ d} - \left[1/2 + s\delta\right]\right) = -\frac{K}{2\beta}\sum_s e^{s\alpha\left(n_\uparrow^{\ d} - n_\downarrow^{\ d}\right)}$

$$K = U\beta(\delta^2 - 1/4),\ \cosh(\alpha) - 1 = \frac{1}{2(\delta^2 - 1/4)}\ , \qquad \delta > 1/2$$

→ New dynamical variable s. Exact mapping onto CT-Hirsch-Fye, (i.e. CT-AUX)

K. Mikelsons et al Phys. Rev. E 79, 057701 (2009)
S. Rombouts et al. PRL 99, E. Gull et. al EPL (2008)

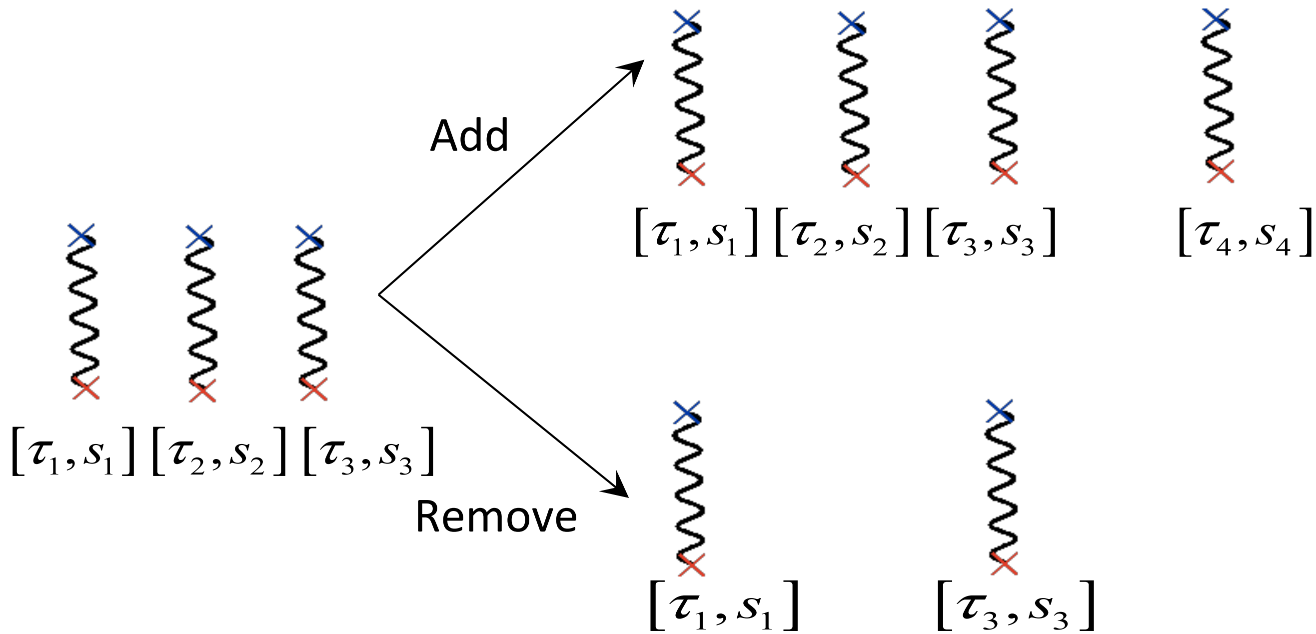
→ Sign problem behaves as in Hirsch-Fye.
Absent for one-dimensional chains, particle-hole symmetry, impurity models

→ Spin polarized particle-hole symmetric problems (Fermion bag)
E. Huffman, S. Chandrasekharan Phys. Rev. B 89, 111101 (2014)

$$\frac{\mathrm{Tr}\left[e^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H_0}\right]} = \underbrace{\sum_n \int_0^\beta d\tau_1 \sum_{s_1} \cdots \int_0^{\tau_{n-1}} d\tau_n \sum_{s_n}}_{\textit{Sum}\text{ with Monte Carlo}} \underbrace{\left(-\frac{U}{2}\right)^n \det\left[M_n\left(\tau_1, s_1 \cdots, \tau_n, s_n\right)\right]}_{\text{Weight}}$$

Sampling.

Configuration C: set of n-vertices at imaginary times $[\tau_1,s_1][\tau_2,s_2]\cdots,[\tau_n,s_n]$



$$\frac{\mathrm{Tr}\left[e^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H_0}\right]} = \underbrace{\sum_n \int_0^\beta d\tau_1 \sum_{s_1} \cdots \int_0^{\tau_{n-1}} d\tau_n \sum_{s_n}}_{\textit{Sum} \text{ with Monte Carlo}} \underbrace{\left(-\frac{U}{2}\right)^n \det\left[M_n\left(\tau_1, s_1 \cdots, \tau_n, s_n\right)\right]}_{\text{Weight}}$$

Measurements.

$$G^\sigma{}_C(\tau,\tau') \equiv \frac{\left\langle T\ H_U\left[\tau_1,s_1\right]\cdots H_U\left[\tau_n,s_n\right]\ \hat{d}^+_\sigma(\tau)\hat{d}_\sigma(\tau')\right\rangle_0}{\left\langle T\ H_U\left[\tau_1,s_1\right]\cdots H_U\left[\tau_n,s_n\right]\right\rangle_0} = G^\sigma{}_0(\tau,\tau') - \sum_{\alpha,\beta=1}^{n} G^\sigma{}_0(\tau,\tau_\alpha)\left(M^\sigma{}_n{}^{-1}\right)_{\alpha\beta} G^\sigma{}_0(\tau_\beta,\tau')$$

Wick theorem applies for each configuration C of vertices.

Note: Direct calculation of Matsubara Green functions.

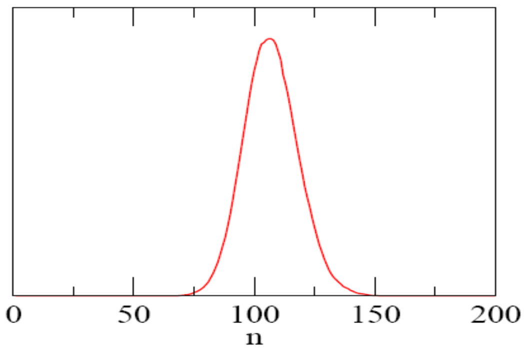
$$G^\sigma{}_C(i\omega_m) = G^\sigma{}_0(i\omega_m) - G^\sigma{}_0(i\omega_m)\sum_{\alpha,\beta=1}^{n} e^{-i\omega_m\tau_\alpha}\left(M^\sigma{}_n{}^{-1}\right)_{\alpha\beta} G^\sigma{}_0(\tau_\beta,0)$$

$$\frac{\mathrm{Tr}\left[e^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H_0}\right]} = \underbrace{\sum_n \int_0^\beta d\tau_1 \sum_{s_1} \cdots \int_0^{\tau_{n-1}} d\tau_n \sum_{s_n}}_{\textit{Sum}\text{ with Monte Carlo}} \underbrace{\left(-\frac{U}{2}\right)^n \det\left[M_n\left(\tau_1, s_1 \cdots, \tau_n, s_n\right)\right]}_{\text{Weight}}$$

Average Expansion parameter.

$$\langle n \rangle = -\beta U \left\langle \left(n_\uparrow^d - 1/2\right)\left(n_\downarrow^d - 1/2\right) - \delta^2 \right\rangle$$

- CPU time scales as <n>^{3} → (βV)^3



Histogram of expansion parameter.

Bosonic Baths → Electron-phonon problems

F. F. Assaad and T. C. Lang Phys. Rev. B76, 035116 (2007).

Hubbard-Holstein:

$$\hat{H} = \sum_{i,j,\sigma} t_{i,j}\, \hat{c}^{+}_{i,\sigma}\hat{c}_{j,\sigma} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + g\sum_i \hat{Q}_i(\hat{n}_i - 1) + \sum_i \frac{\hat{P}_i^2}{2M} + \frac{k}{2}\hat{Q}_i^2$$

Integrate out the phonons

$$Z = \int \left[dc^+ dc \right] \exp\left[-S_0 - U\int_0^\beta d\tau\; n_{i\uparrow}(\tau) n_{i\downarrow}(\tau) + \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{i,j} [n_i(\tau)-1] D^0(i-j,\tau-\tau')[n_j(\tau')-1] \right]$$

$$D^0(i-j,\tau-\tau') = \delta_{i,j}\frac{g^2}{2k}P(\tau-\tau'), \quad \lambda = \frac{g^2}{W\,k}$$

$$P(\tau) = \frac{\omega_0}{2(1-e^{-\beta\omega_0})}\left[e^{-|\tau|\omega_0} + e^{-(\beta-|\tau|)\omega_0} \right], \quad \omega_0 = \sqrt{k/M}$$

Attractive, retarded interaction (time scale $1/w_0$).

Antiadiabatic limit: $\lim_{\omega_0\to\infty} P(\tau) = \delta(\tau)$ → Attractive Hubbard.

Bosonic Baths → Electron-phonon problems

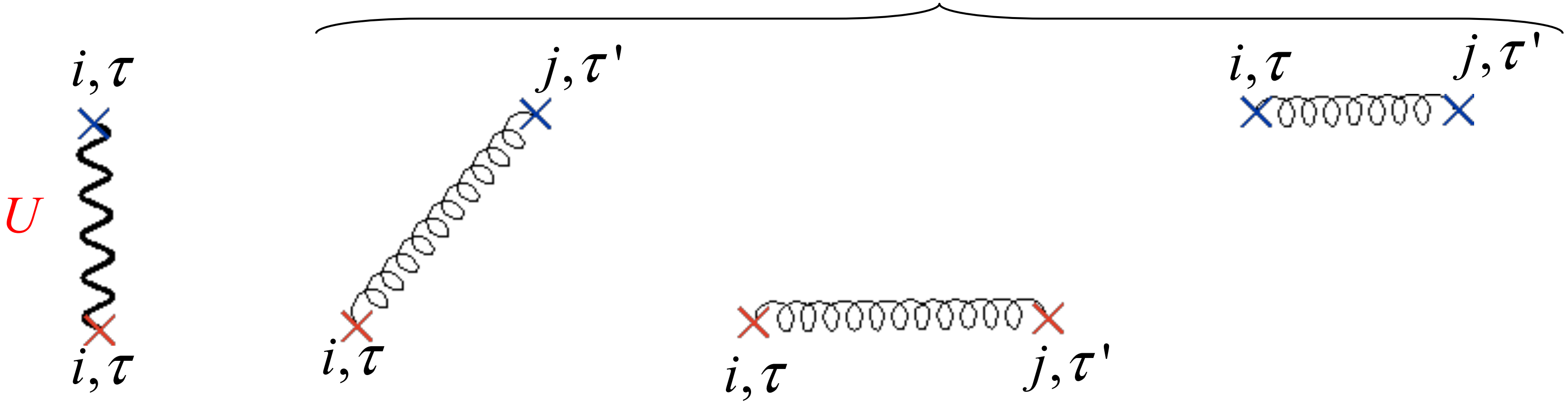
F. F. Assaad and T. C. Lang Phys. Rev. B76, 035116 (2007).

$$Z = \int \left[dc^+ dc \right] \exp\left[-S_0 - U \int_0^\beta d\tau \; n_{i\uparrow}(\tau) n_{i\downarrow}(\tau) + \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{i,j} [n_i(\tau) - 1] D^0(i-j, \tau-\tau') [n_j(\tau') - 1] \right]$$

QMC: Expand both in Hubbard and retarded phonon interaction.

Vertices:

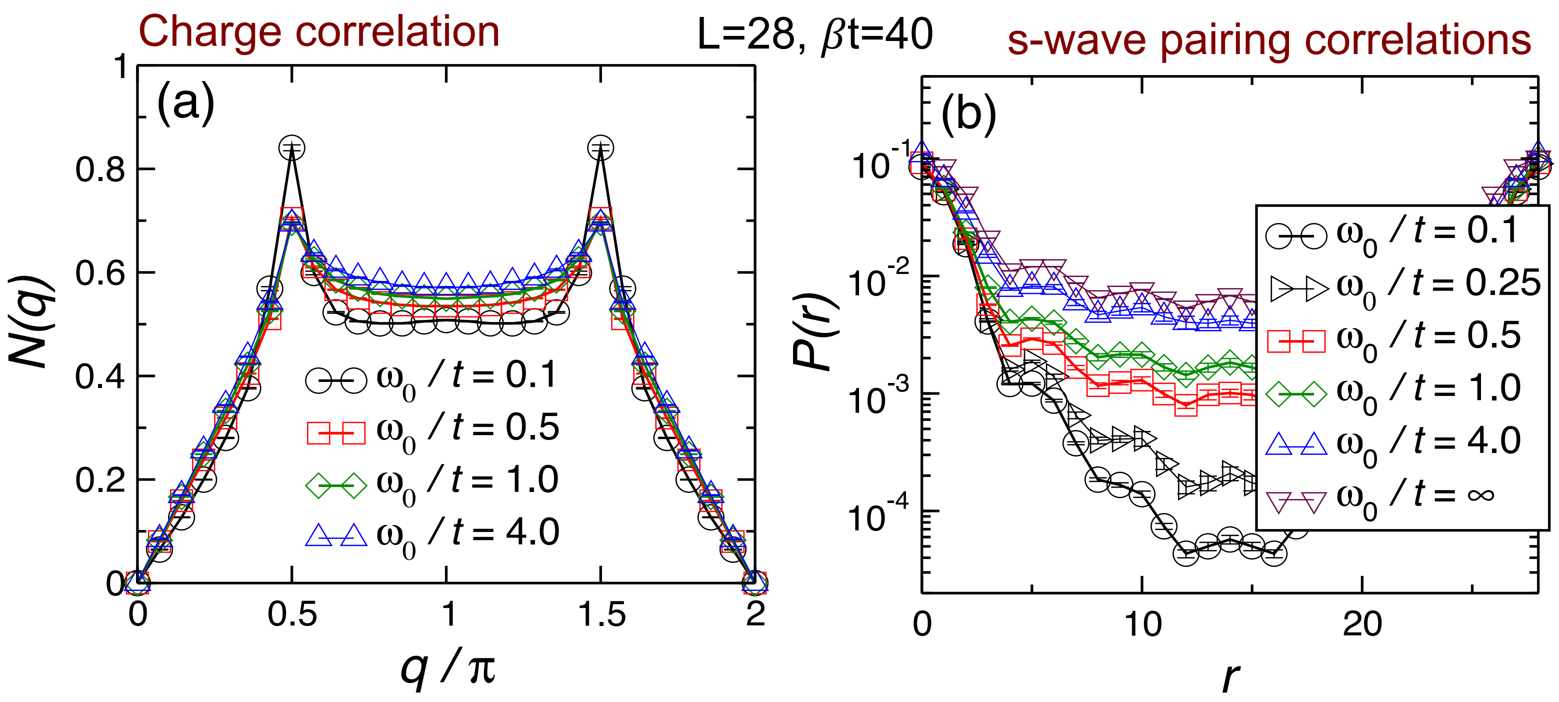
Hubbard. Phonon. $D^0(i-j,\tau-\tau')$



Bosonic Baths → Electron-phonon problems

M. Hohenadler, FFA. J. Phys.: Condens. Matter 25, 014005 (2013)

Peierls to superfluid crossover in the one-dimensional quarter filled Holstein model @ λ=0.35



$\omega_0 << t$ Pairs of electrons form a commensurate CDW (diagonal LRO).

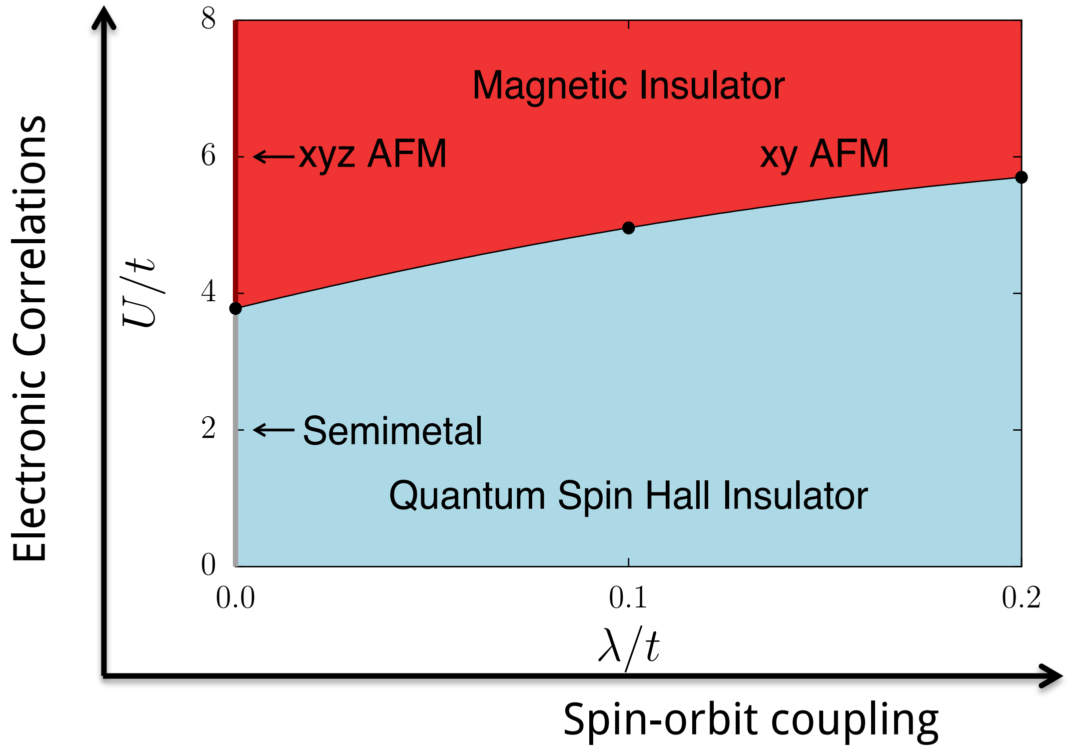
$\omega_0 >> t$ Pairs condense to form an s-wave superconductor (off diagonal LRO).

Fermionic Baths → Correlation effects on edge state of quantum Spin Hall states

Phases of the Kane Mele Hubbard model

S. Rachel & K. Le Hur, PRB 82, 075106, (2010)

M. Hohenadler et al. PRB 85 (2012)



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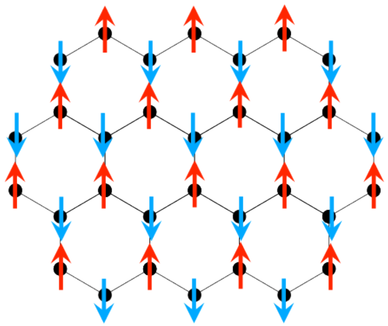


Antiferromagnetic
Mott insulator

S. Sorella and E. Tosatti.
EPL, 19, 699, (1992)

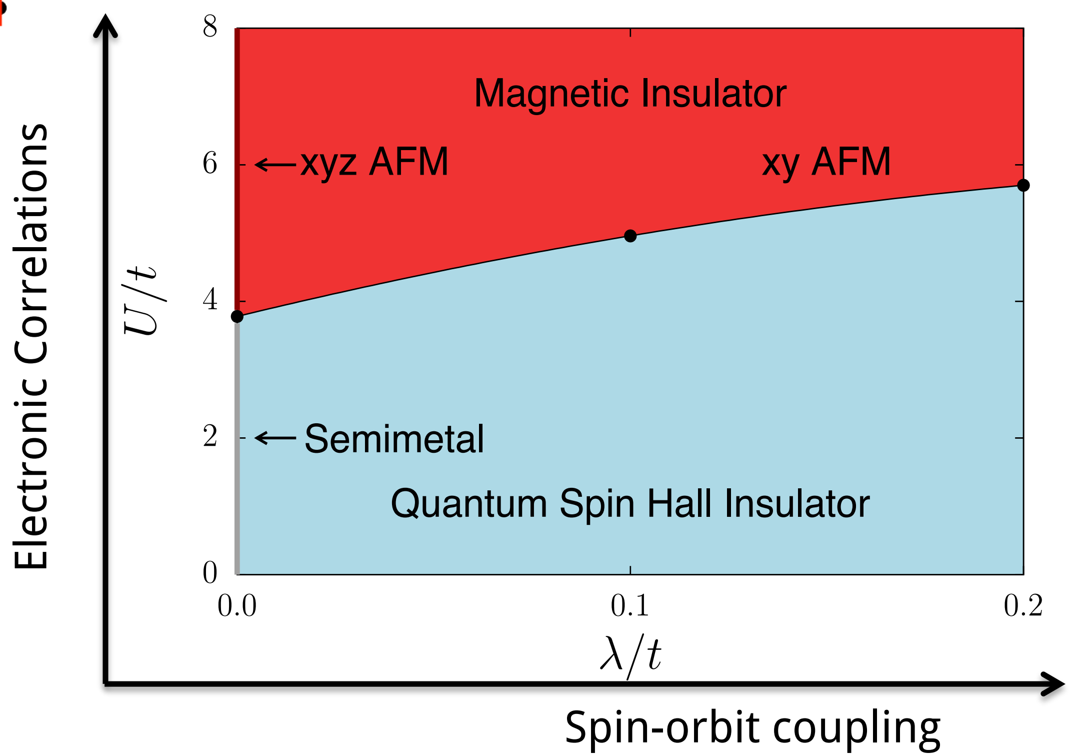
T. Paiva et al.
Phys. Rev. B, 72, 085123 (2005)

Z. Y. Meng et al.
Nature 464, 847 (2010)

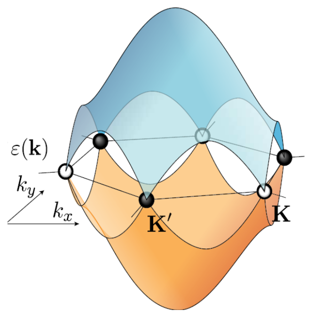
S. Sorella et al.
Scientific Reports 2, 992 (2012)

F. Assaad & I. Herbut
PRX 3, 031010 (2013)

B. Clark arXiv:1305.0278



Dirac fermions



Tight binding model on
Honeycomb lattice at
half-filling

Symmetries
SU(2) spin ✓
Sublattice ✓
Time reversal ✓

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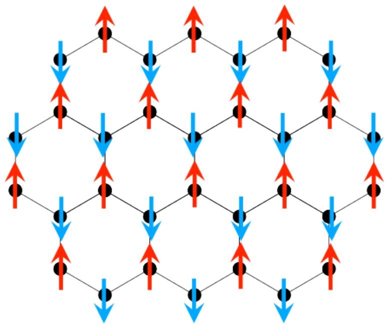

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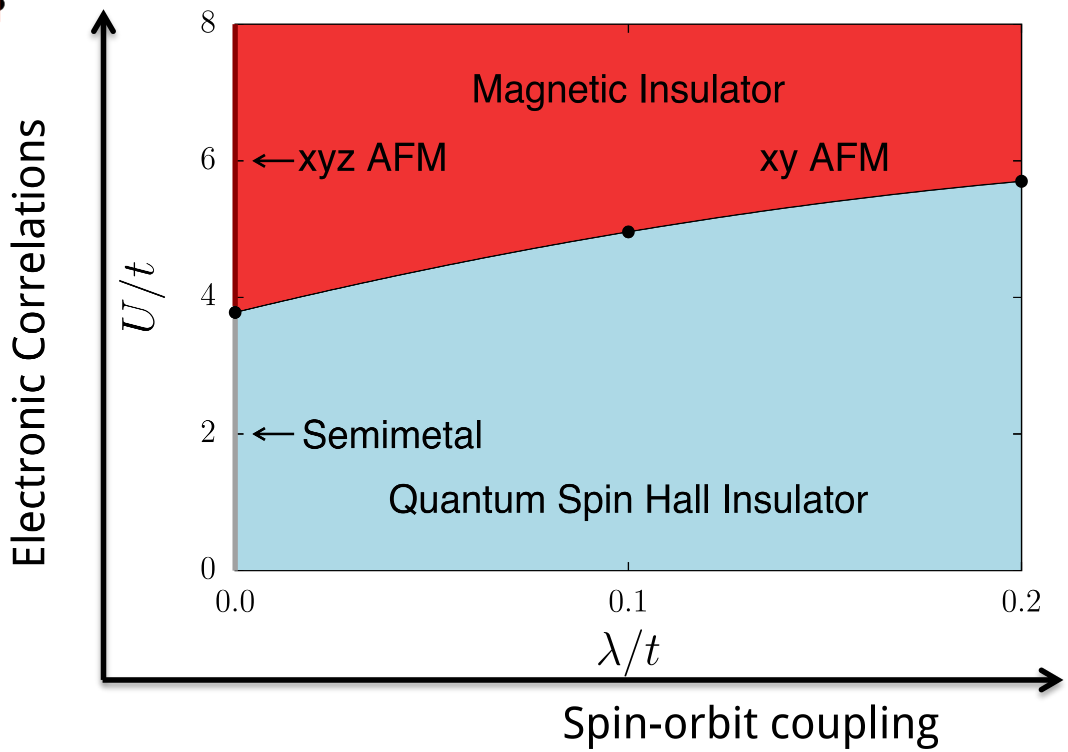
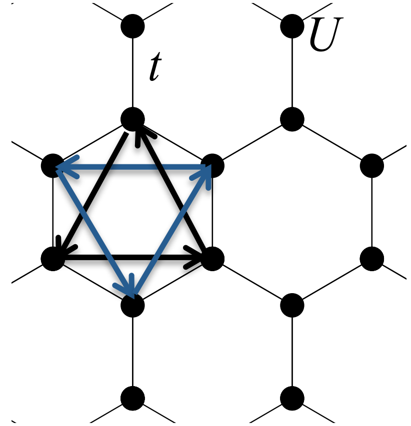
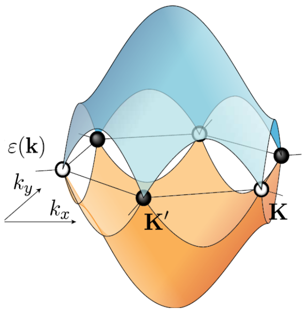


$$\mathbf{i} \longrightarrow \mathbf{j} = i\lambda\left(\hat{c}_{\mathbf{i}}^{\dagger}\sigma_z\hat{c}_{\mathbf{j}} - \hat{c}_{\mathbf{j}}^{\dagger}\sigma_z\hat{c}_{\mathbf{i}}\right)$$



Haldane model in each spin sector

Dirac fermions

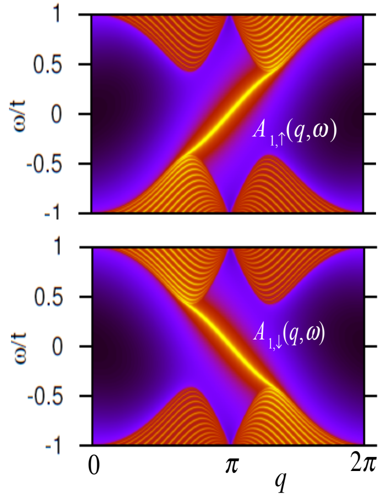


Tight binding model on Honeycomb lattice at half-filling

Symmetries
SU(2) spin ✓
Sublattice ✓
Time reversal ✓

SU(2) spin → U(1) spin
Sublattice ✗
Time reversal ✓

Opens gap → Quantum spin Hall with robust helical edge states.



Fermionic Baths → Correlation effects on edge state of quantum Spin Hall states

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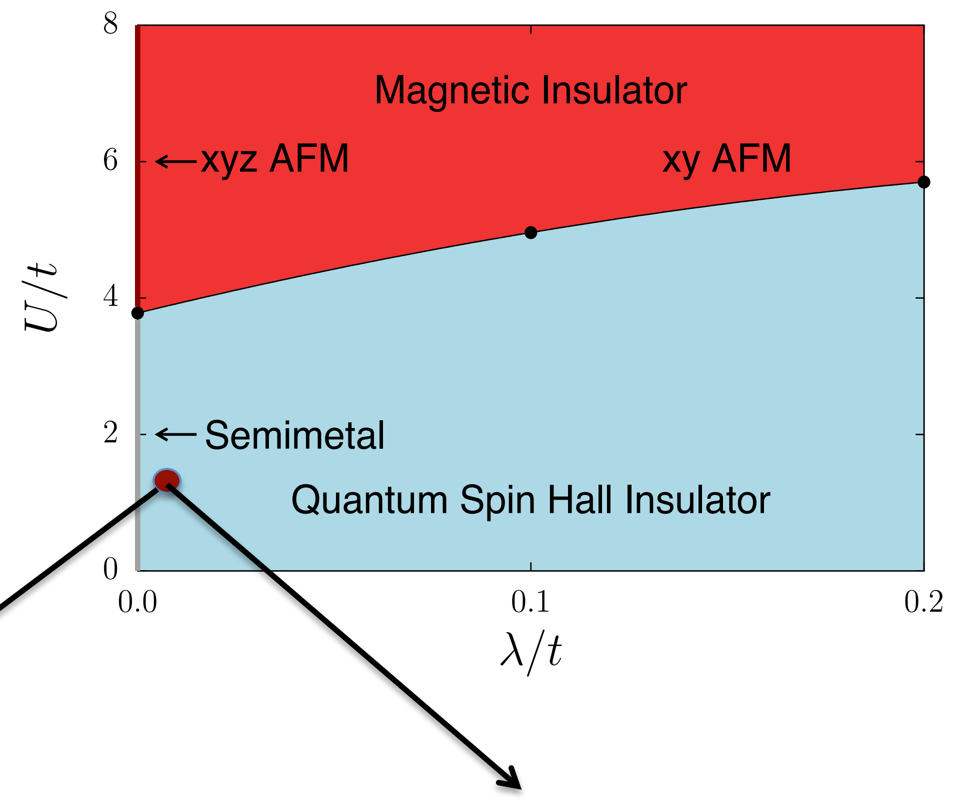
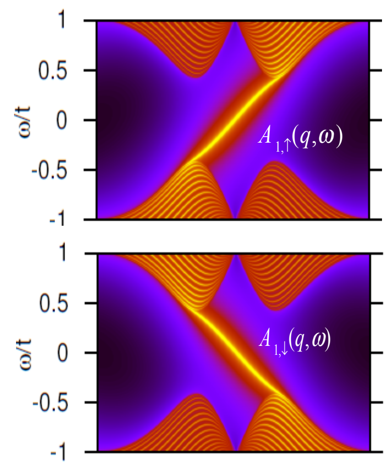


Bulk: $U/W \ll 1$

Ground state of bulk is well described by mean field

Edge: $U/v_F \gg 1$ ($v_F \sim \lambda$)

Edge states are exponentially localized
Strongly correlated problem

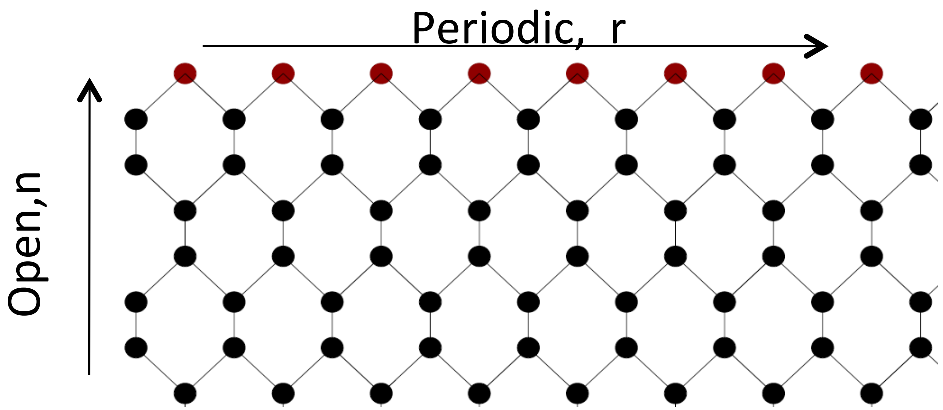


<u>Fermionic Baths</u> → Correlation effects on edge state of quantum Spin Hall states

M. Hohenadler, T. C. Lang and F. F. Assaad
Phys. Rev. Lett. 106, 100403 (2011)

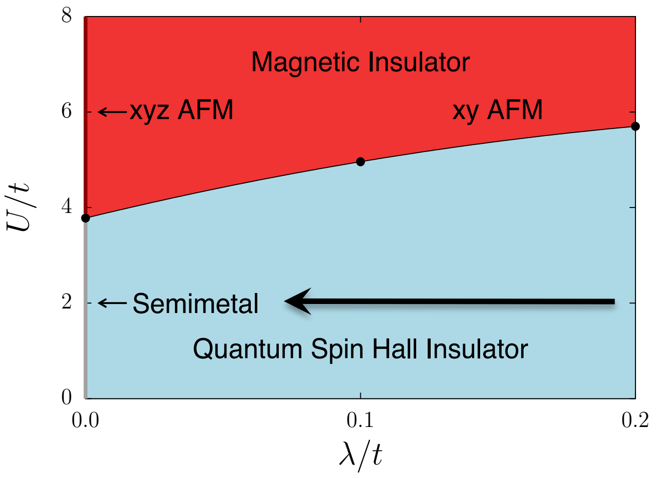
M. Hohenadler and F. F. Assaad
Phys. Rev. B 85, 081106(R) (2012)

→ Paramagnetic mean field for bulk. Retain all the fluctuations on the edge.



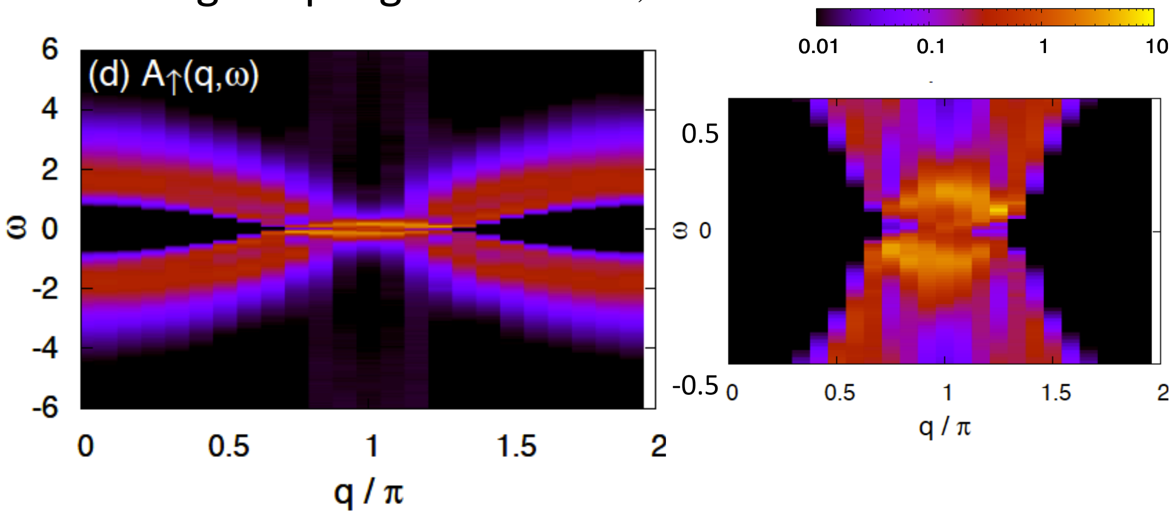
$$S = \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{r,r',\sigma} c^\dagger_{r,\sigma}(\tau) \underbrace{G^{-1}_{0,\sigma}(r-r',\tau-\tau')}_{\text{Green function of the model on the ribbon (e.g. paramagnetic mean-field)}} c_{r',\sigma}(\tau') + U \int_0^\beta d\tau \sum_r \left(n_{r,\uparrow}(\tau) - \frac{1}{2} \right) \left(n_{r,\downarrow}(\tau) - \frac{1}{2} \right)$$

Fermionic Baths → Correlation effects on edge state of quantum Spin Hall states



Single particle spectral function

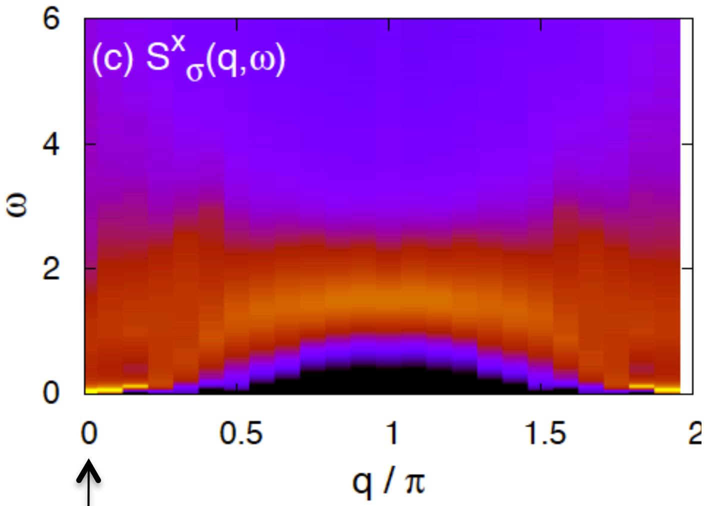
Strong coupling: $\lambda / t = 0.05, U / t = 2$



→ Inelastic scattering between left (spin do) and right (spin up) movers reduces substantially the spectral weight of the helical edge state.

Dynamic spin-structure factor

at $\lambda / t = 0.1, U / t = 2$



$2k_f$

$$S^x_\sigma(q,\omega) = \frac{1}{Z}\sum_{n,m} e^{-\beta E_n} \left|\langle m | S^x(q) | n \rangle\right|^2 \delta(E_m - E_n - \omega)$$

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Correlated helical liquids

Determinantal quantum Monte Carlo for lattice models

Entanglement spectra for correlated topological insulators
Mott transition

For $H = H_0 + H_1$

$$\lim_{L_\tau \to \infty,\, L_\tau \Delta\tau = \beta} \mathrm{Tr}\left[\left(e^{-\Delta\tau \hat{H}_0} e^{-\Delta\tau \hat{H}_1}\right)^{L_\tau}\right] = \mathrm{Tr}\left[e^{-\beta \hat{H}_0} T\, e^{-\int_0^\beta d\tau \hat{H}_1(\tau)}\right]$$

Auxiliary field QMC

$$\mathrm{Tr}\, e^{-\beta H} \propto \int D\{\Phi(\mathbf{i},\tau)\}\, e^{-S(\{\Phi(\mathbf{i},\tau)\})}$$

Trotter, Hubbard-Stratonovich

MC importance sampling

One body problem in external field

Example

For $$\hat{H} = \hat{H}_{KM} + \frac{1}{4}\sum_{\mathbf{i},\mathbf{j}} V_{\mathbf{i},\mathbf{j}} (\hat{c}^{\dagger}_{\mathbf{i}}\hat{c}_{\mathbf{i}} - 1)(\hat{c}^{\dagger}_{\mathbf{j}}\hat{c}_{\mathbf{j}} - 1), \quad \hat{c}^{\dagger}_{\mathbf{i}} = (\hat{c}^{\dagger}_{\mathbf{i},\uparrow}, \hat{c}^{\dagger}_{\mathbf{i},\downarrow})$$

$$S(\{\Phi(i,\tau)\}) = \sum_{\mathbf{i},\mathbf{j},\tau} \Delta\tau\, \Phi(\mathbf{i},\tau) V^{-1}_{\mathbf{i},\mathbf{j}} \Phi(\mathbf{j},\tau) - \ln \mathrm{Tr}\left[\prod_{\tau=1}^{L_\tau} e^{-\Delta\tau \hat{H}_{KM}} e^{-\Delta\tau \sum_{\mathbf{i}} i\Phi(\mathbf{i},\tau)\left[\hat{c}^{\dagger}_{\mathbf{i}}\hat{c}_{\mathbf{i}} - 1\right]}\right]$$

- The action is real! → positive weights
  (U(1) spin symmetry, particle-hole symmetry, V positive definite)

M. V. Ulybyshev, P. V. Buividovich, M. I. Katsnelson, and M. I. Polikarpov. Phys. Rev. Lett., 111, 056801, (2013).

M. Hohenadler, T.C. Lang and FFA PRL 106, 2011
D. Zheng, C. Wu and G.-M. Zhang PRB 84, 2011

Implementation.

S. R. White, D. J. Scalapino, R. L. Sugar, E. Y. Loh, J. E. Gubernatis, and R. T. Scalettar. Phys. Rev. B40, 506 (1989)
S. Sorella, S. Baroni, R. Car, and M. Parrinello. Europhys. Lett., 8, 663, (1989)
G. Sugiyama and S. Koonin. Anals of Phys., 168, (1986)
M. Imada and Y. Hatsugai. J. Phys. Soc. Jpn., 58, 3752 (1989) ....

$$\mathrm{Tr}\left[\prod_{\tau=1}^{L_\tau} e^{-\Delta\tau \hat{H}_{KM}} e^{-\Delta\tau \sum_{\mathbf{i}} i\Phi(\mathbf{i},\tau)\left[\hat{c}^\dagger_{\mathbf{i}}\hat{c}_{\mathbf{i}} - 1\right]}\right] = e^{i\Delta\tau\sum_{\mathbf{i},\tau}\Phi(\mathbf{i},\tau)} \det\left[1 + B_{L_\tau}\cdots B_1\right]$$

With $\hat{H}_{KM} = \hat{c}^\dagger T \hat{c}$ and $\sum_{\mathbf{i}} i\Phi(\mathbf{i},\tau)\hat{c}^\dagger_{\mathbf{i}}\hat{c}_{\mathbf{i}} = \hat{c}^\dagger V(\tau)\hat{c}$ one obtains $B_\tau = e^{-\Delta\tau T} e^{-\Delta\tau V(\tau)}$

Sampling. Single “spin-flip” sequential updating

Measurements.

$$\frac{\mathrm{Tr}\left[e^{-\beta\hat{H}}\hat{c}^\dagger_x\hat{c}_y\right]}{\mathrm{Tr}\, e^{-\beta\hat{H}}} = \int D\Phi\; P(\Phi)\; G_{x,y}(\Phi)$$

$$P(\Phi) = \frac{e^{-S(\Phi)}}{\int D\Phi\; e^{-S(\Phi)}}, \qquad G(\Phi) = (1 + B_{L_\tau}\cdots B_1)^{-1}$$

Wicks theorem holds for a given field configuration → Any equal time observable can be computed from G

Computational cost. $V^3\beta$

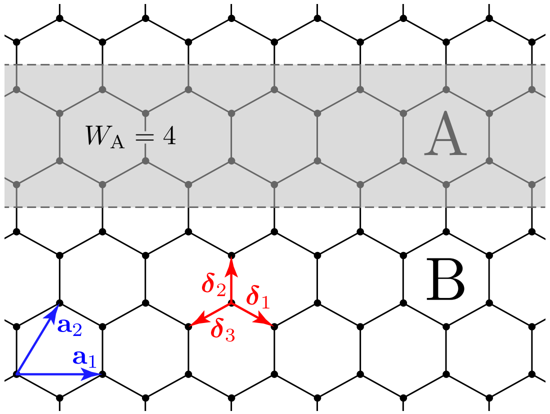
Is it possible to better? In principle yes → Hybrid molecular-dynamics hints to a $V\beta$ scaling

R. T. Scalettar, D. J. Scalapino, R. L. Sugar, and D. Toussaint. Phys. Rev. B36, 8632 (1987). In practice?

Recent developments: Renyi entanglement entropies.

A different way of writing Wick's theorem. T. Grover Phys. Rev. Lett., 111, 130402, (2013).

$$\hat{\rho} \equiv \frac{e^{-\beta \hat{H}}}{Z} = \int d\Phi\, P(\Phi) \hat{\rho}(\Phi), \qquad \hat{\rho}(\Phi) = \det\left[1 - G(\Phi)\right] e^{-\hat{c}^{\dagger} \ln\left[G^{-1}(\Phi) - 1\right] \hat{c}}$$



$$\hat{\rho}_A = \mathrm{Tr}_B \hat{\rho} \equiv \int d\Phi\, P(\Phi) \hat{\rho}_A(\Phi) \qquad \hat{\rho}_A(\Phi) : G \to G_A$$

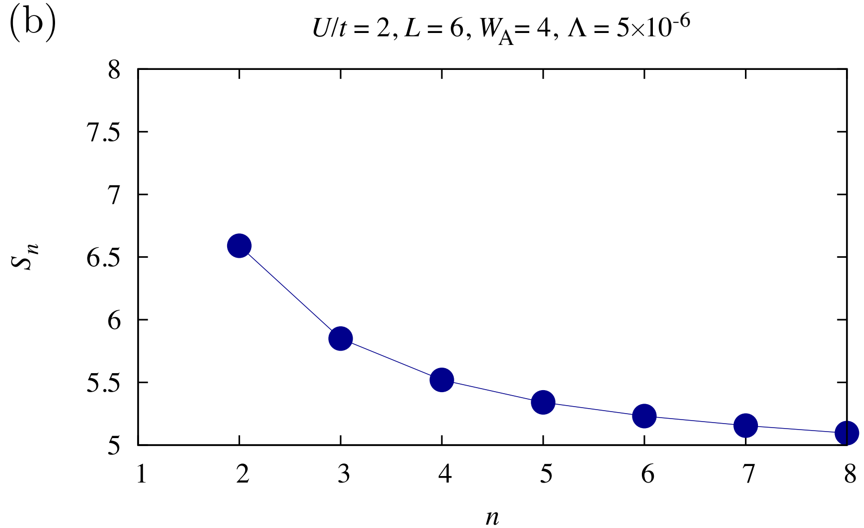
$$\mathrm{Tr} \hat{\rho}_A{}^n = \int \underbrace{d\Phi^1 \cdots d\Phi^n P(\Phi^1) \cdots P(\Phi^n)}_{\text{n-replicas}} \mathrm{Tr}\left[\hat{\rho}_A(\Phi^1) \cdots \hat{\rho}_A(\Phi^n)\right]$$

$$S_n = -\frac{1}{n-1} \ln \mathrm{Tr} \hat{\rho}_A{}^n$$

F. F. Assaad, T. C. Lang, and F. Parisen Toldin
Phys. Rev. B, 89, 125121, (2014)

Peter Bröcker and Simon Trebst arXiv:1404.3027

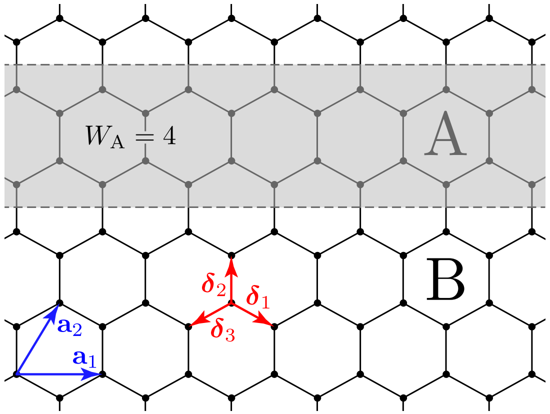
(b)



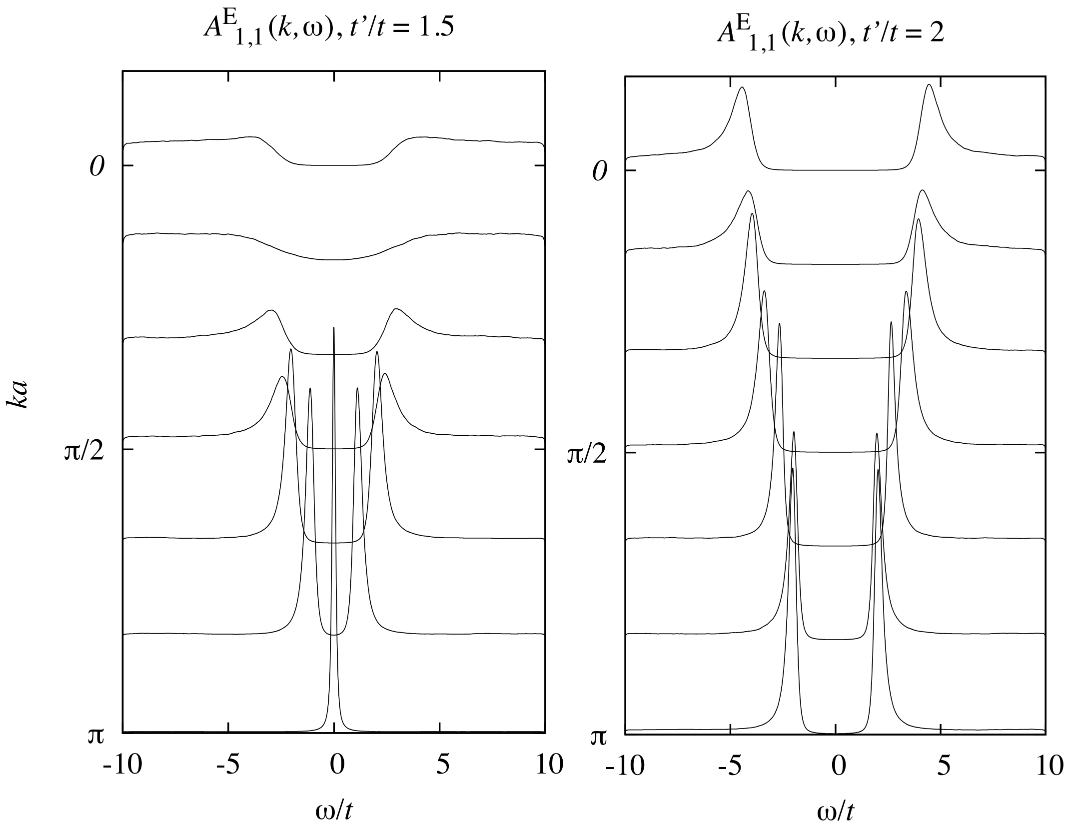
Recent developments: Entanglement spectrum $\hat{\rho}_A = e^{-\hat{H}_E}$

$$\left\langle a_x^\dagger(\tau_E) a_y \right\rangle_E = \frac{\mathrm{Tr}\left[\hat{\rho}_A^{\,n-\tau_E} a_x^\dagger \hat{\rho}_A^{\,\tau_E} a_y\right]}{\mathrm{Tr}\left[\hat{\rho}_A^{\,n}\right]} \quad \rightarrow \quad \left\langle a_x^\dagger(\tau_E) a_x \right\rangle_E = \int d\omega \, \frac{e^{-\tau_E \omega}}{1+e^{-n\omega}} A^E(x,\omega)$$

Example: Single particle entanglement spectral function for dimerized Kane-Mele Hubbard model.



U/t = 2, λ/t = 0.2



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F. F. Assaad, T. C. Lang, and F. Parisen Toldin Phys. Rev. B, 89, 125121, (2014)

The Mott transition

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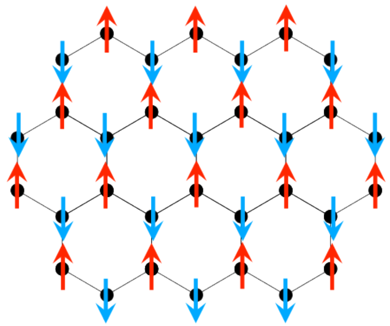
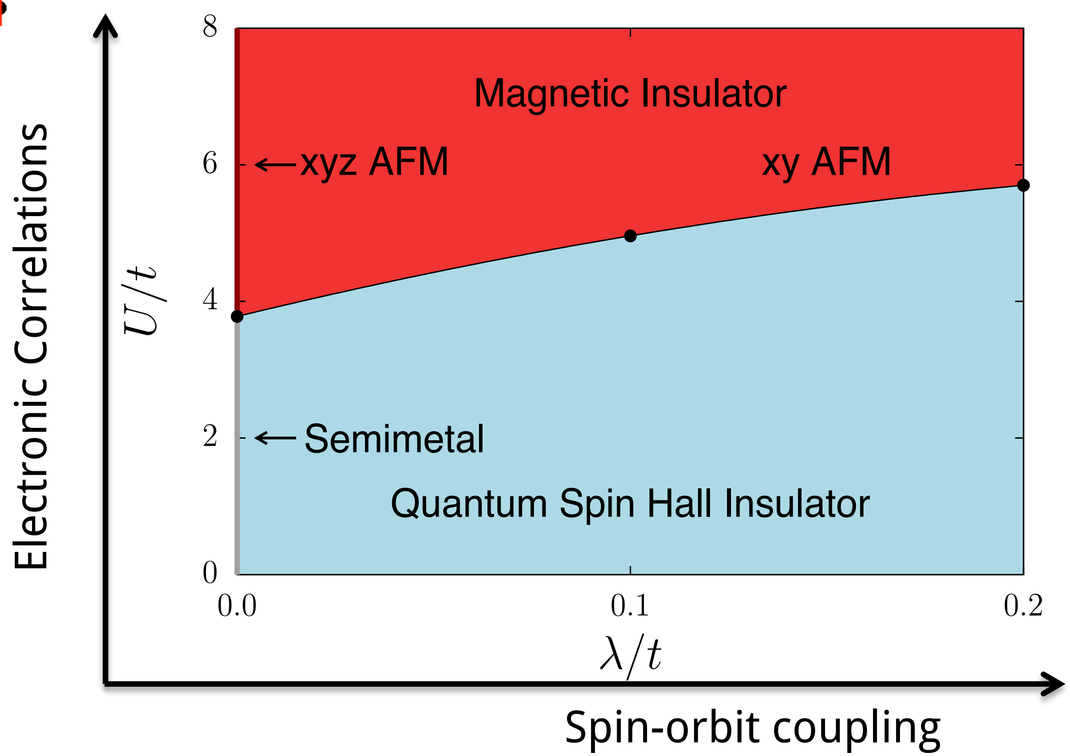
Phases of the Kane Mele Hubbard model



Antiferromagnetic
Mott insulator



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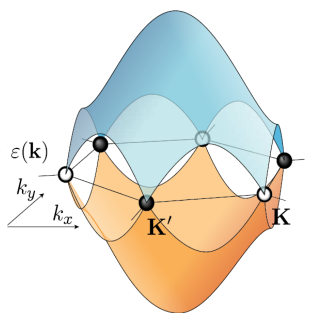
S. Sorella et al.
Scientific Reports 2, 992 (2012)

F. Assaad & I. Herbut
PRX 3, 031010 (2013)

B. Clark arXiv:1305.0278

Dirac fermions



Tight binding model on
Honeycomb lattice at
half-filling

Symmetries
SU(2) spin ✓
Sublattice ✓
Time reversal ✓

The Mott transition

Measuring the magnetic moment

FFA & I. Herbut PRX 3, 031010 (2013)

Steven R. White and A. L. Chernyshev Phys. Rev. Lett. 99, 127004

Introduce pinning fields $H = H_{tU} + h_0 (n_{0,\uparrow} - n_{0,\downarrow})$

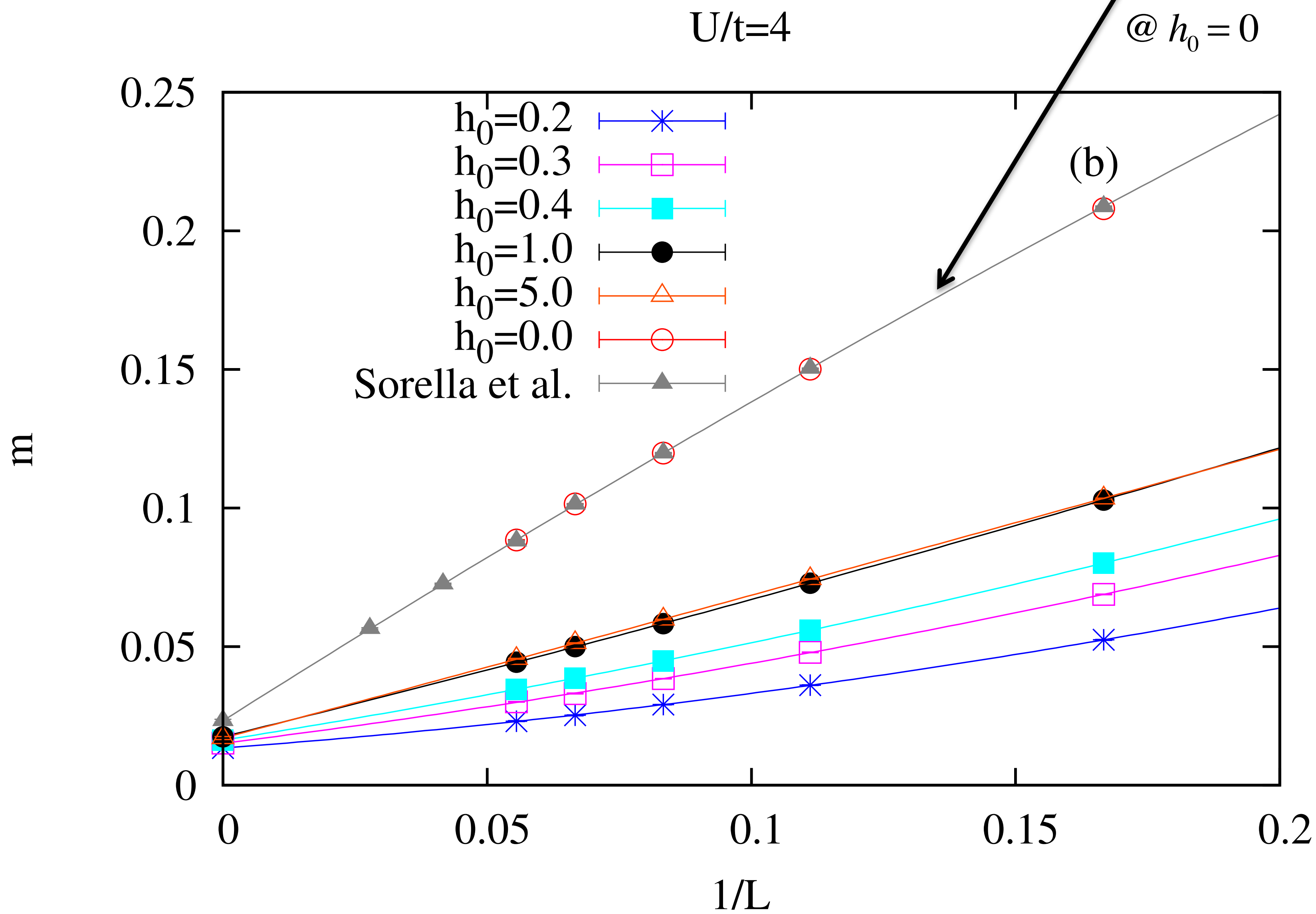
$$m = \lim_{R\to\infty} \lim_{L\to\infty} \left\langle S^z(R) \right\rangle e^{i\mathbf{Q}\cdot\mathbf{R}}$$

$$m = \lim_{L\to\infty} \frac{1}{L^2} \sum_i e^{i\mathbf{Q}\cdot\mathbf{i}} \left\langle S^z(i) \right\rangle$$

The Mott transition

$$\sqrt{\frac{1}{L^2}\sum_{\mathbf{i}=1}^{L^2}\left\langle \mathbf{S}_{A,\mathbf{0}}\bullet\mathbf{S}_{A,\mathbf{i}}\right\rangle}$$

$@\ h_0=0$



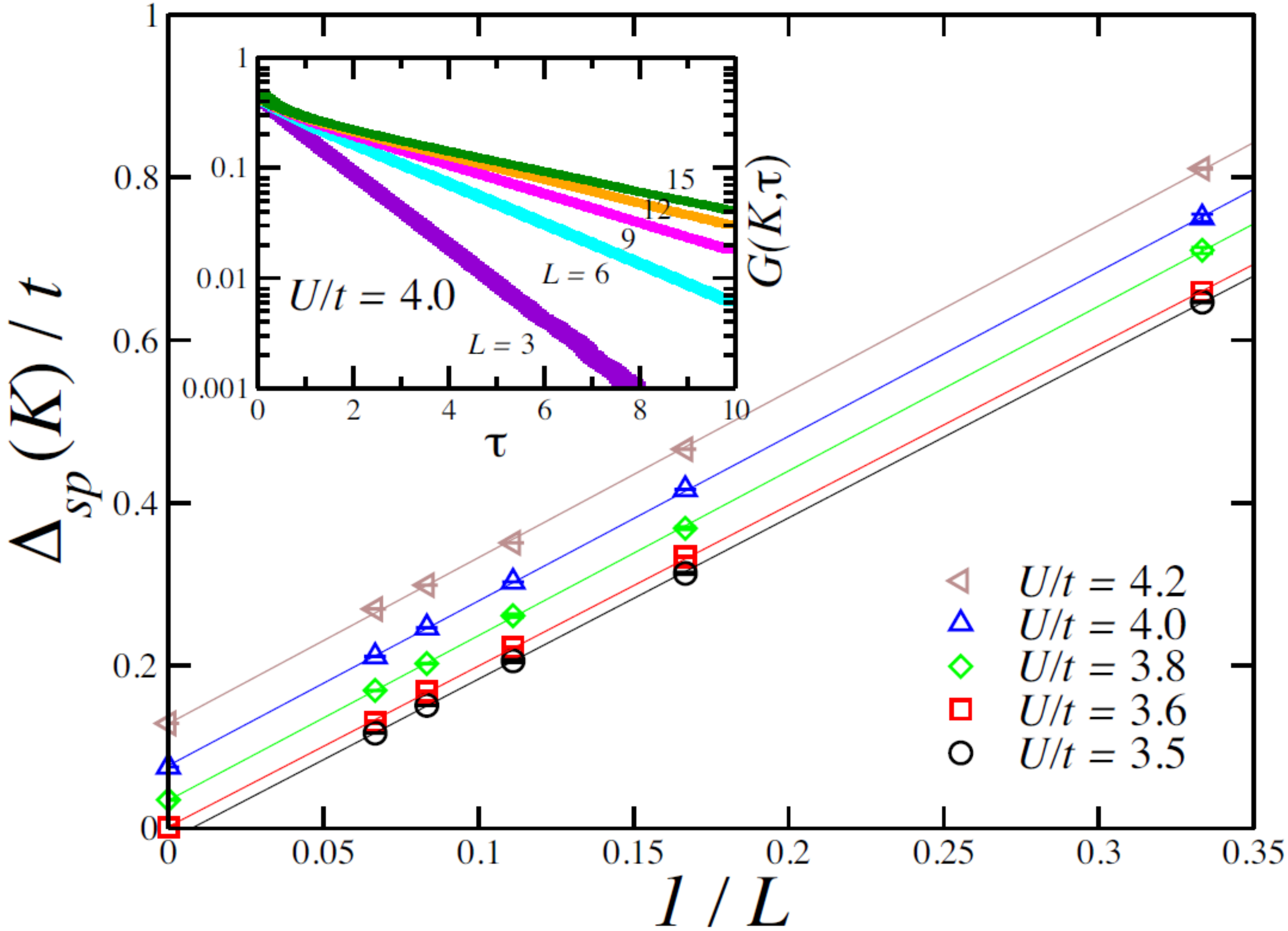
S. Sorella et al. Scientific Reports 2, 992 (2012)

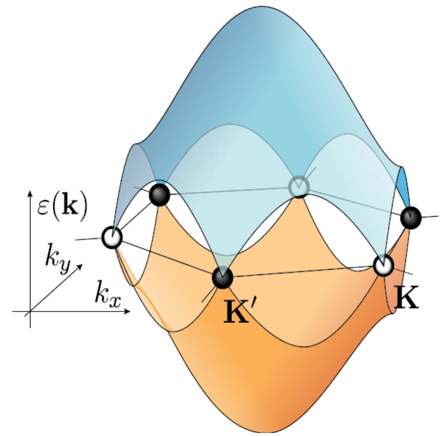
Measuring the single particle gap

Single particle Green's function

$$G(\vec{\mathbf{k}},\tau) = \frac{1}{4}\sum_{\alpha,\sigma}\left\langle c^{\dagger}_{k,\alpha,\sigma}(\tau)\, c_{k,\alpha,\sigma}\right\rangle$$

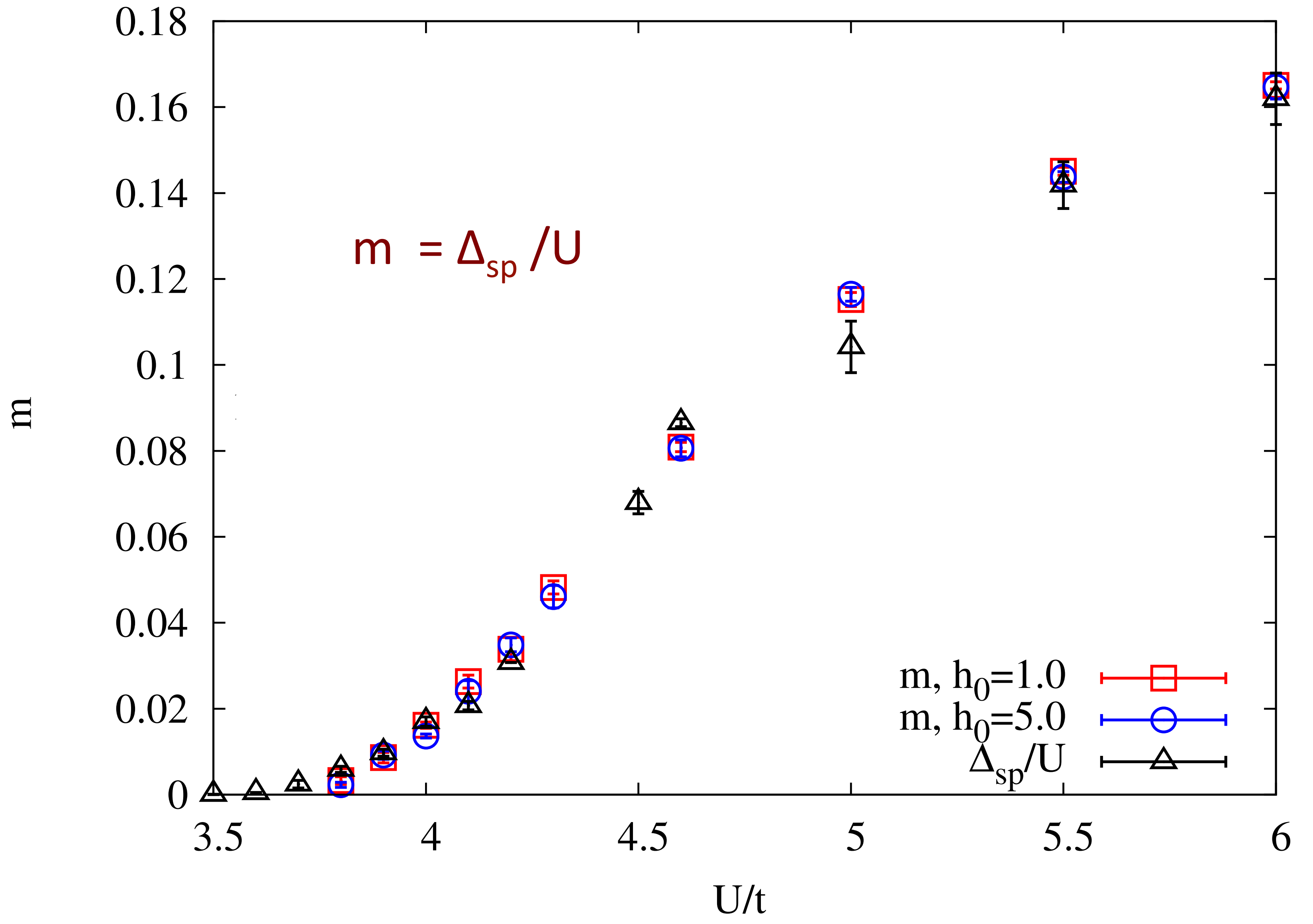
$$G(\vec{\mathbf{k}},\tau) \rightarrow Z_{\vec{\mathbf{k}}}\, e^{-\Delta_{sp}(\vec{\mathbf{k}})\tau}$$





The Mott transition

Polynomial extrapolation to L → ∞ of m and Δ_{sp}/U



Gross-Neveu Yukawa

I. Herbut, V. Juričić, O. Vafek   PRB 80, 075432, (2009)

$$L_0 = \sum_{\sigma} \bar{\psi}_\sigma(\mathbf{x},\tau)\partial_\mu \gamma_\mu \psi_\sigma(\mathbf{x},\tau)$$

Dirac fermions

$$L_b = \vec{\psi}_t(\mathbf{x},\tau)\cdot\left[-\partial_\tau^2 - v^2\vec{\nabla}^2 + t\right]\vec{\psi}_t(\mathbf{x},\tau) + \lambda\left(\vec{\psi}_t(\mathbf{x},\tau)\bullet\vec{\psi}_t(\mathbf{x},\tau)\right)^2$$

Order parameter

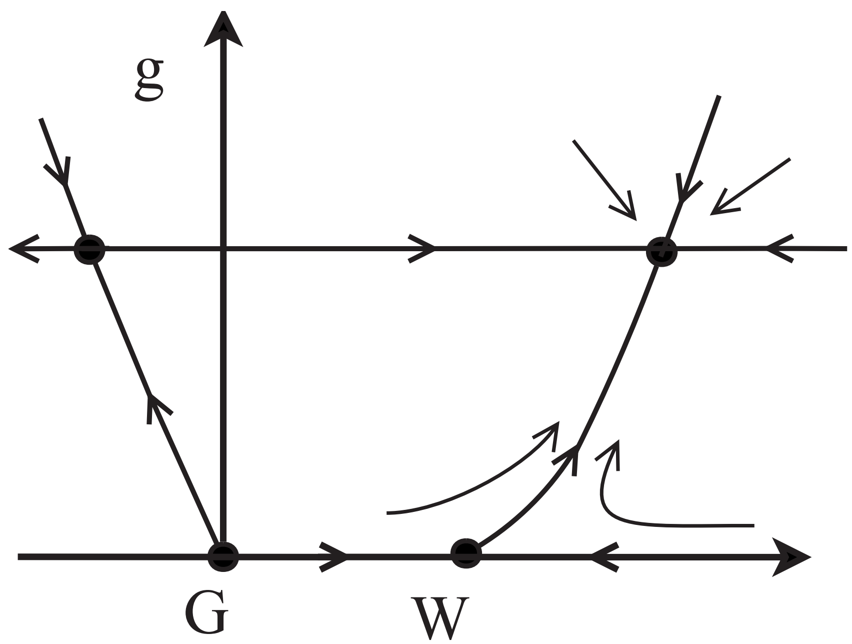
$$L_y = g\ \vec{\psi}_t(\mathbf{x},\tau)\cdot\sum_{\sigma,\sigma'}\bar{\psi}_\sigma(\mathbf{x},\tau)\vec{\sigma}_{\sigma,\sigma'}\psi_{\sigma'}(\mathbf{x},\tau)$$

Yukawa coupling

$$\boxed{\Delta_{sp} \propto g\left|\left\langle \vec{\psi}_t \right\rangle\right|}$$

Upper critical dimension d=3 → ε-expansion

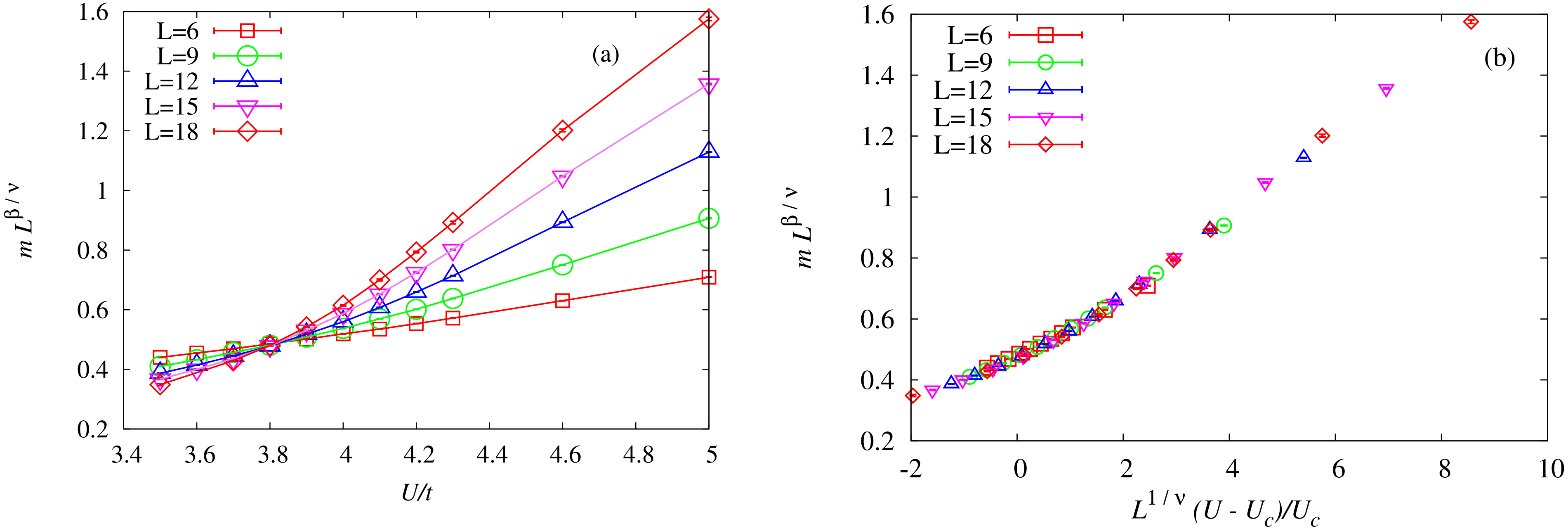
$$\boxed{\begin{aligned} \frac{\beta}{\nu} &= 1 - \frac{\varepsilon}{10} + O\left(\varepsilon^2\right) \\ \nu &= \frac{1}{2} + \frac{21}{55}\varepsilon + O\left(\varepsilon^2\right) \end{aligned}}$$



$$\frac{\beta}{\nu} = 1 - \frac{1}{10} + O\left(1^2\right)$$

$$\nu = \frac{1}{2} + \frac{21}{55}1 + O\left(1^2\right)$$

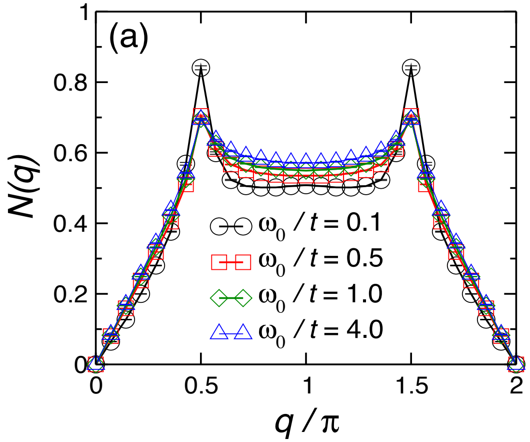
$$m(L,U) = L^{-\beta/\nu}\ F(\ L^{1/\nu}(U - U_c))$$



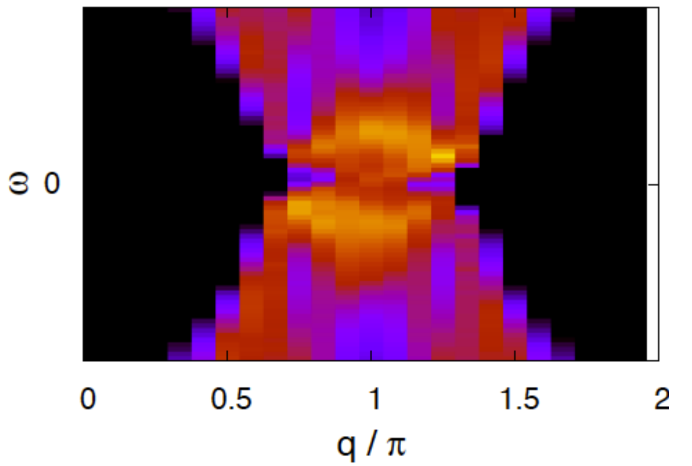
Conclusion

Correlated electrons in bosonic and fermionic baths (CT-INT)

Electron-phonon problem

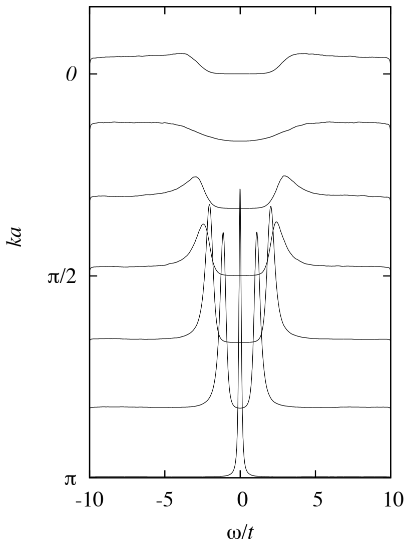


Correlation effects in helical liquids

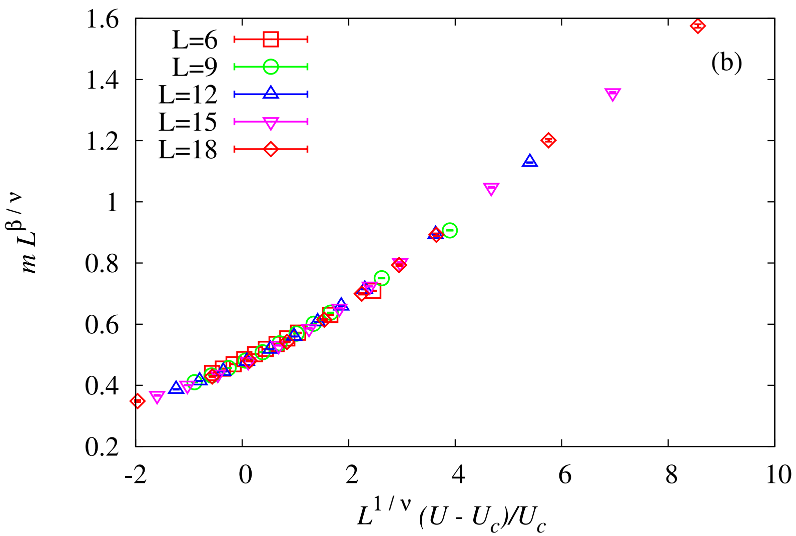


Determinantal quantum Monte Carlo for lattice models

Entanglement spectral functions



Mott transition. Fermionic criticality



Many thanks to

T. Lang

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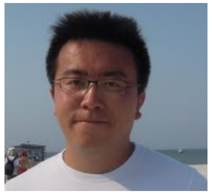

Z. Y. Meng

M. Hohenadler

M. Bercx

F. Goth

F. Parisen Toldin

I. Herbut