Higgsing lattice gauge theories with strongly interacting fermions

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Motivations

- Gauge symmetry breaking plays a crucial role in
  - Standard Model - electroweak symmetry breaking
  - Grand Unified theories
  - BCS theory of superconductivity
- Green typically put in ``by hand" thru vev of scalar
- Problematic in PP - A lot of fine tuning required to keep scalar sector insensitive to high energies (naturalness)

Long history of efforts to replace scalar by bound state of fermions or condensate - dynamical symmetry breaking

Dynamical symmetry breaking

- Problem: in PP context fermion condensates typically arise through non-perturbative effects in strongly coupled theories
- Difficult to study with analytical methods. But candidates for study using lattice gauge theory
- Lots of lattice work recently searching for models with near conformal dynamics - candidate walking technicolor theories as alternative to SM Higgs ... eg Meifeng's talk

Barriers to lattice approaches

- All lattice studies to date focus on the strong dynamics - and leave out the broken sector... (put in later using p theory ...)
- Why ? Several results seem to prohibit typical continuum symmetry breaking scenarios when moved over to the lattice
  - Can only gauge exact symmetries.
  - lattice constructions favor vector symmetries
  - Vafa-Witten theorem prohibits spontaneous symmetry breaking of vector symmetries
  - Forced to think of chiral/axial symmetries - but no lattice chiral fermion or exact lattice chiral symmetry

Nielson-Ninomyia

Punchline

- Possible to construct a model in which exact lattice symmetries are spontaneously broken due to strongly coupled fermion dynamics
- The broken symmetries which start out as vector-like transform into axial symmetries in continuum limit
- After gauging weak symmetries at non-zero lattice spacing yields dynamical Higgs mechanism.
- Higgs phase survive continuum limit ?

Staggered fermions

One popular lattice fermion used in QCD is staggered fermion

$$S = \sum_{x,\mu} \eta_\mu(x)\overline{\chi}(x)\left[\chi(x+\mu) - \chi(x-\mu)\right] + m\overline{\chi}(x)\chi(x)$$

$$\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu} x_i}$$

Describes 4 Dirac fermions in continuum limit.

Decompose $\overline{\chi} = (\overline{\psi}_+, \overline{\lambda}_-)$ $\quad \chi = (\psi_-, \lambda_+)$ $\quad \pm \equiv \text{parity of site}$

where eg. $\psi_-(x) = \frac{1}{2}\left(1 - \epsilon(x)\right)\chi(x)$ with $\epsilon(x) = (-1)^{\sum_{i=1}^{4} x_i}$

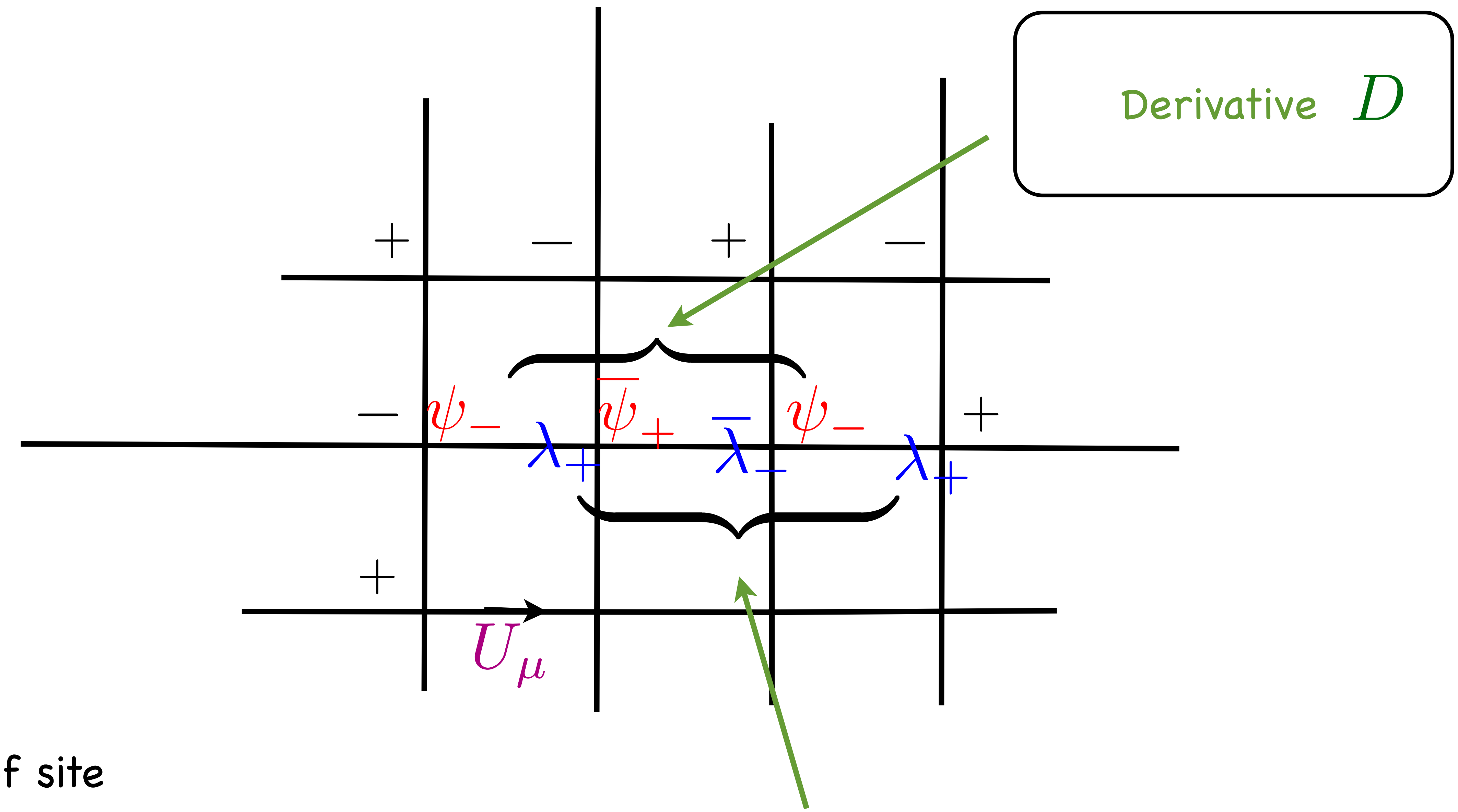
$$S = \sum_{x,\mu} \eta_\mu(x)\left[\overline{\psi}_+(x) D_\mu \psi_-(x) + \overline{\lambda}_-(x) D'_\mu \lambda_+(x)\right]$$

m=0

derivatives may be different

Symmetries:

$$\overline{\psi}_+(x) \to \overline{\psi}_+(x) G^\dagger(x) \qquad \overline{\lambda}_-(x) \to \overline{\lambda}_-(x) H^\dagger(x)$$
$$\psi_-(x) \to G(x)\psi_-(x) \qquad \lambda_+(x) \to H(x)\lambda_+(x)$$



Parity of site

$$P = (-1)^{\sum_{\mu=1}^{4} x_\mu}$$

$$D_\mu \psi(x) = \frac{1}{2}\left(U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)\right)$$

One example - technicolor-like model

Assume gauge group factors into strong and weak sectors:

$$SU(N)_{strong} \times [SU(M) \times SU(M)]_{weak}$$

$$\beta_S = \frac{2N}{g_S^2},\ \beta_W = \frac{2M}{g_W^2} \, and \ r = \frac{\beta_W}{\beta_S} >> 1$$

Fermions transforming as:

$$\psi : (\Box, \Box, 1) \qquad \lambda : (\Box, 1, \Box)$$

$$\Box \equiv \text{fundamental rep}$$

Condensates

First switch off weak gauge coupling.

For large enough strong coupling expect a condensate of form

$$< \overline{\psi}_+(x)\lambda_+(x) > + < \overline{\lambda}_-(x)\psi_-(x) > \neq 0$$

continuum

$\overline{\psi}_L\lambda_R + \ldots$

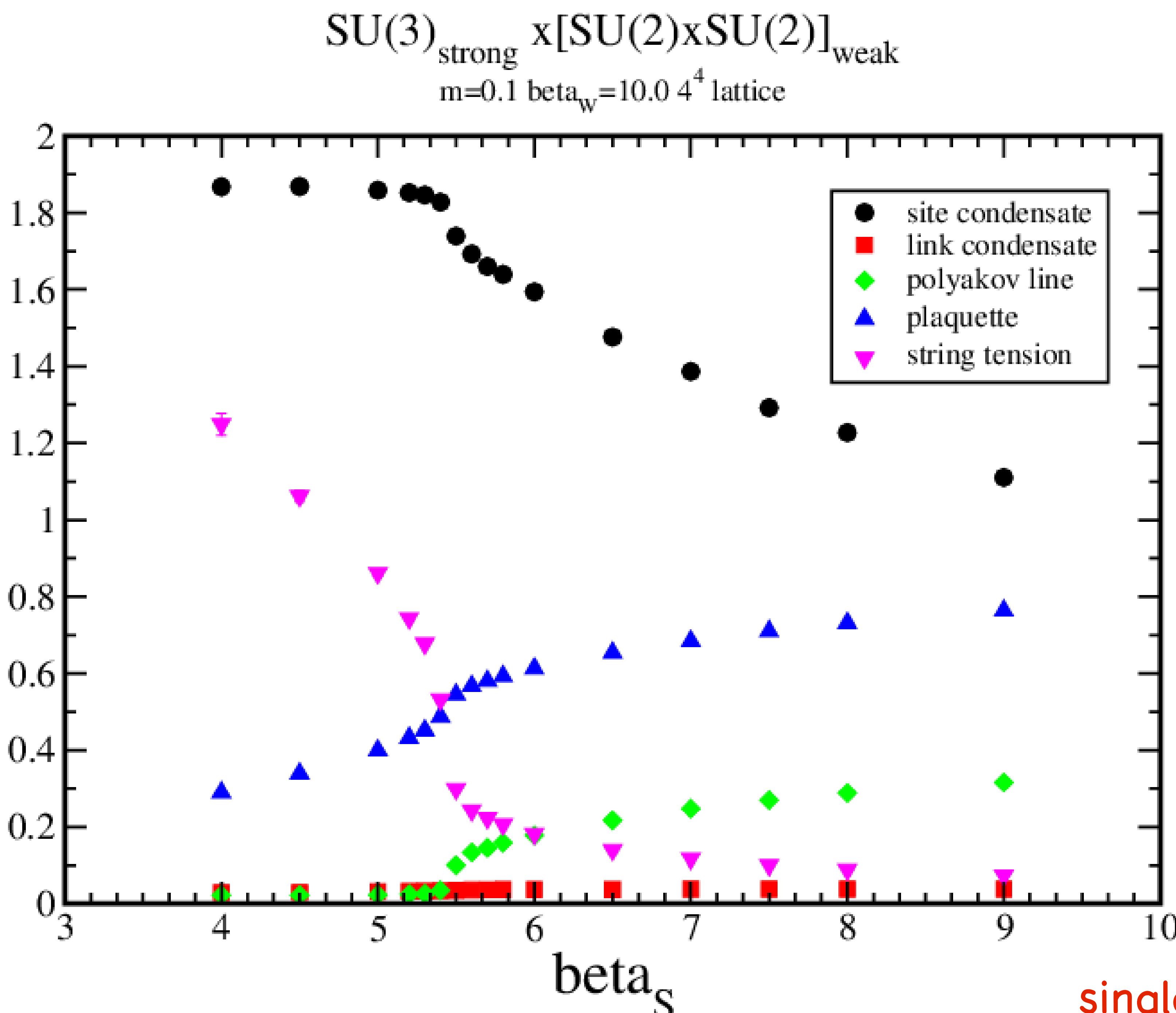
By construction singlet under strong force but will spontaneously break weak symmetries.

$$SU(M)\times SU(M) \to SU(M)_{diag}$$

Symmetry preserving condensate also possible

$$\epsilon(x)\xi_\mu(x) < \overline{\psi}_+(x)U_\mu(x)V_\mu(x)\psi_-(x+\mu) >$$

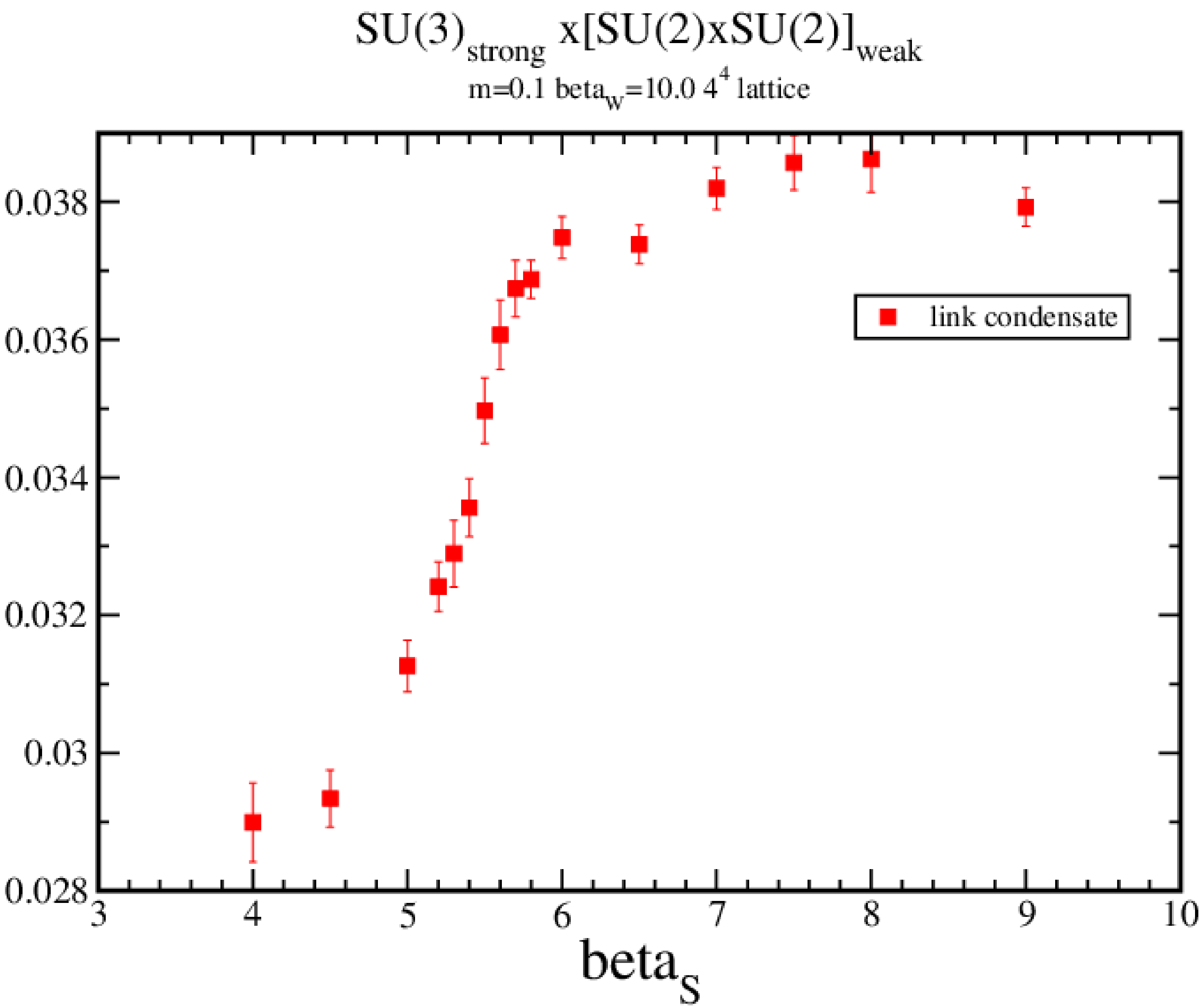
Phase structure vs β_S



SU(3) strong
SU(2) weak

single site condensate strongly preferred over symmetry preserving link condensate

Single link condensate



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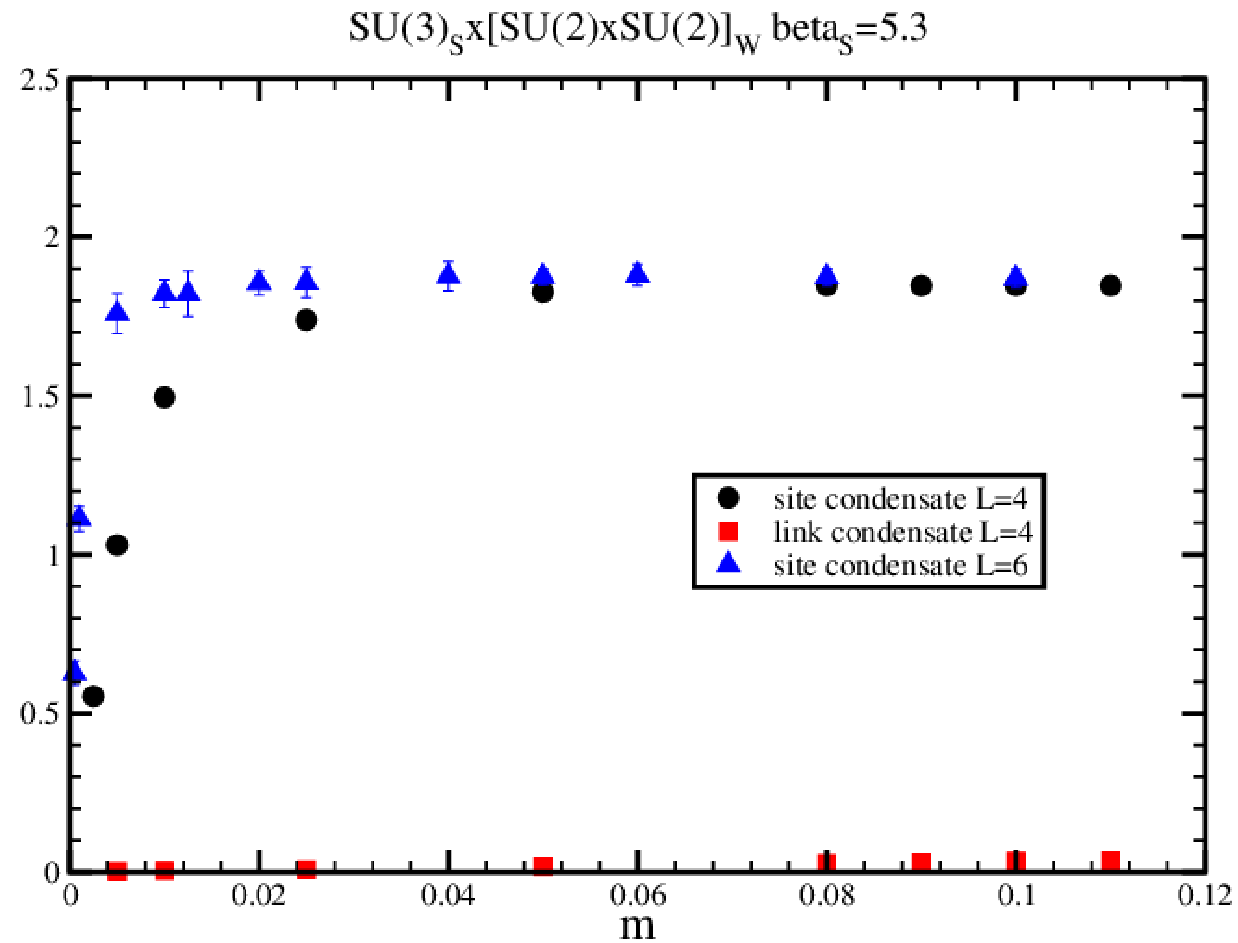
Spontaneous symmetry breaking

Condensates computed in presence of source m
Interested in m->0 after thermodynamic limit

Stable plateau
for small m
Turns over for
m<1/V



Vafa Witten theorem

- Theorem prohibits spontaneous breaking of vector symmetries - symmetries where

$$\psi \to G\psi, \qquad \overline{\psi} \to \overline{\psi}G^\dagger$$

- The symmetries here seem to be of this form .. can condensate survive continuum limit ?

- Yes ! In continuum replace staggered fields by Dirac $\Psi = (\psi, \lambda)$

$$\Psi^\alpha \Sigma^{\alpha\beta} \Psi^\beta = (\overline{\psi}\,\overline{\lambda}) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} \psi \\ \lambda \end{pmatrix}$$

Chiral transform $\Sigma \to I$
broken generators pick up factor of γ_5

Broken symmetries
axial in background of
this condensate!

No real surprise ...

Example: $SU(2)xSU(2)\text{->}SU(2)_{diag}$

Single site condensate is just the usual order parameter for chiral symmetry staggered fermions in QCD.

Continuum limit will enhance to $SU_V(8)xSU_A(8)$

To be compatible with VW broken SU(2) must reside in $SU_A(8)$

Gauging weak sector

When weak symmetry global expect M^2-1 massless GB after breaking

Once we gauge this symmetry these massless states should disappear from spectrum. Gauge bosons become massive - Higgs phase ...

What about Nielsen-Ninomiya, Karsten-Smit etc ? Continuum is vector-like - no anomalies.

Elitzur's theorem insists that no local order parameter in gauge theory. So fermion bilinear $\overline{\psi}\lambda$ is always zero

A Higgs phase must instead correspond to condensation of a (gauge invariant) four fermion operator

$$\Sigma \sim \overline{\psi}_+ \lambda_+ \overline{\lambda}_- \psi_-$$ ← non-zero vev always

Non-local order parameters allowed ...

Polyakov loops - non-local order parameter

Phases classified by behavior of Wilson line that wraps the lattice

$$P(x) = \frac{1}{N}\mathrm{Tr}\left(\prod_{t=1}^{T} U_t(x,t)\right)$$

Measures free energy of static source

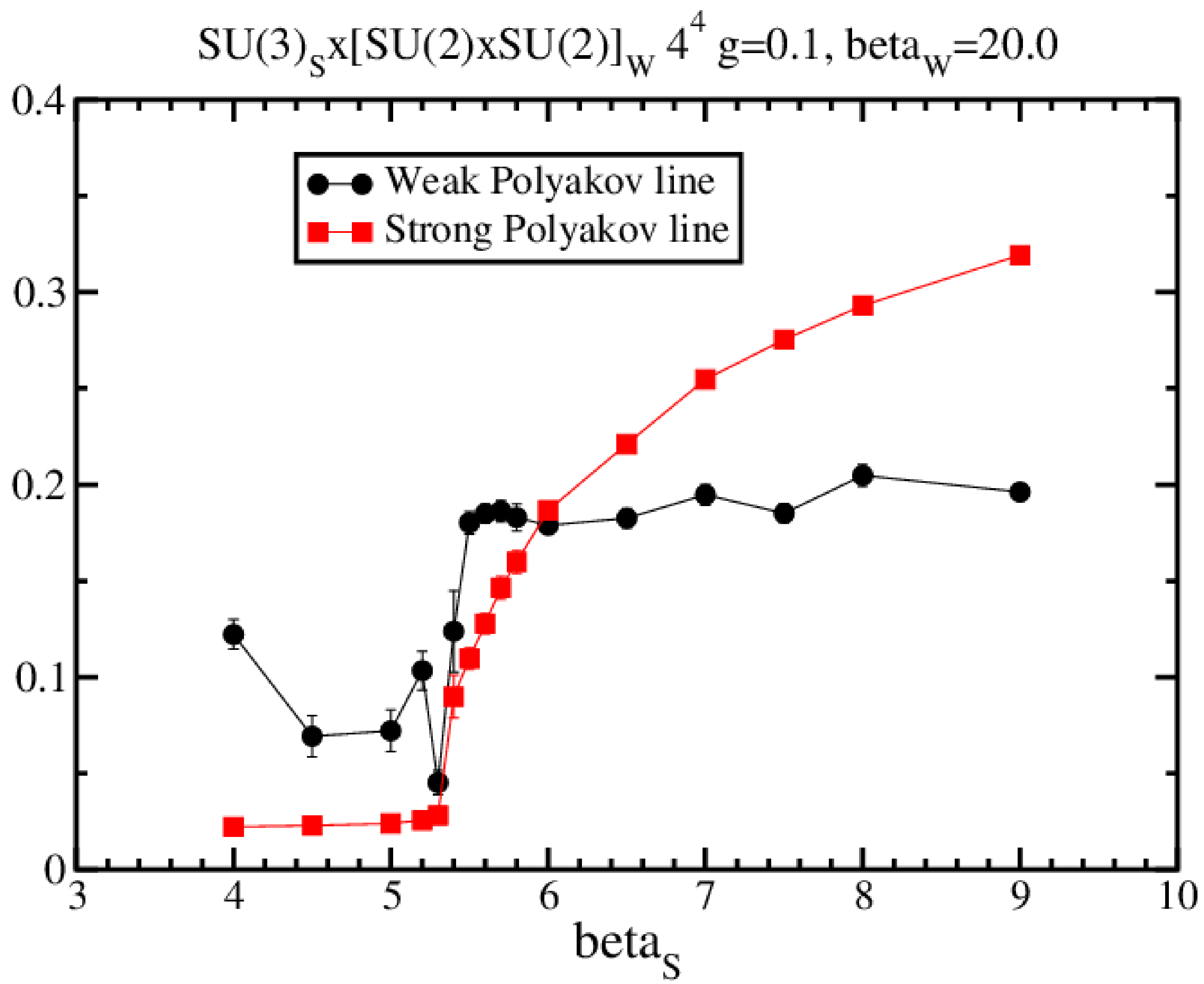
$$P \sim e^{-F_{q\overline{q}}T}$$

Weak sector source carries quantum numbers of $\overline{\psi}\lambda$

Hence corresponding observable:

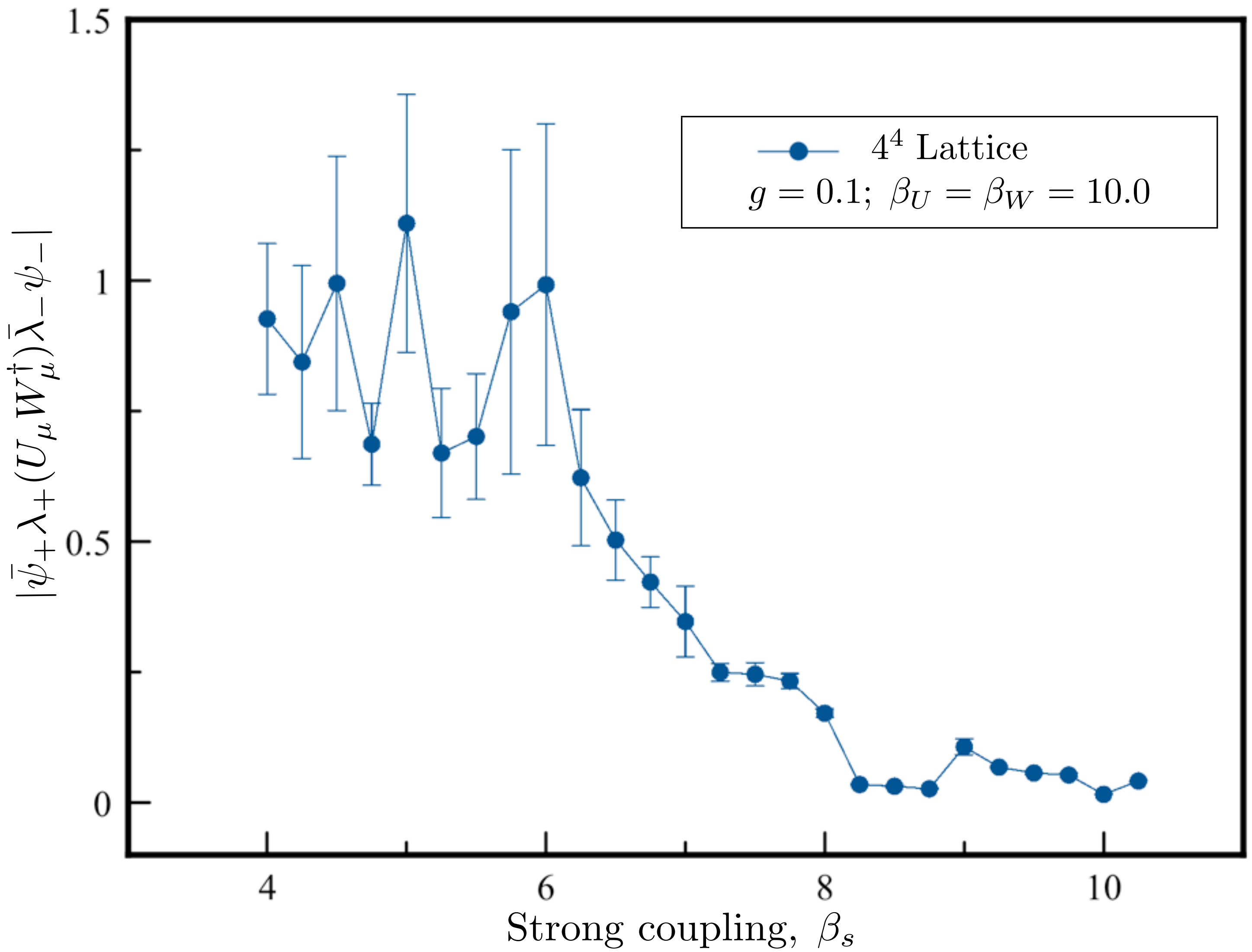
$$P_W = \left(\mathrm{Tr}\prod U_\mu\right)\left(\mathrm{Tr}\prod W^*\mu\right)$$

Polyakov lines



Once system confines discontinuous jump in free energy of weak source

Four fermion condensate



See enhanced 4 fermi condensate at strong coupling

Interpretation

- Evidence that system enters Higgs phase at strong coupling/coarse lattices.
- Does this phase survive continuum limit ?
- Naively it should not. At a=0 staggered fermions yield Dirac fermions - vector-like theory. Such a theory can always develop a fully gauge invariant condensate (eg single link).
- Vacuum alignment arguments in continuum would favor this symmetric condensate.
- Unravel by studying the theory in scaling region

$$\beta_S, \beta_W, L \to \infty \text{ with } \frac{\beta_S}{\beta_W} = r \text{ fixed } << 1$$

Vacuum alignment

- For zero weak coupling global symmetries are G=SU(8)xSU(8). Expect breaks to SU(8)
- Strong interaction vacuum highly degenerate corresponding to different embeddings of SU(2)xSU(2) into G
- Switch on weak coupling. Unique vacuum picked out. Usual folklore is that this vacuum does not break (weakly coupled) gauge symmetries in vector-like theory.
- In staggered lattice theory degeneracy of strong interaction vacuum is lifted and single site condensate breaking SU(2)xSU(2) dominates
- Strong interaction cut-off effects compete with weak interaction to determine true vacuum.

At r=0 SU(2) breaks .. does this survive to finite r ?

Summary

- Constructed a lattice model of (reduced) staggered fermions which exhibits a Higgs phase at non-zero lattice spacing
- Broken symmetry starts out as vector-like but reappears as an axial symmetry in continuum limit - consistent with VW theorem. No anomalies ..
- Correlated with formation of a gauge invariant four fermion condensate - consistent with Elitzur's theorem
- See abrupt transition in (weak) Polyakov line as system confines ...
- 60 million $ question: does this Higgs phase survive continuum limit ?

Need simulations on larger, finer lattices to see - in progress !

Kahler-Dirac fermions

Consider 4 degenerate Dirac fermions. Global symmetry
$SO_{Lorentz}(4) \times SU_{flavor}(4)$

$$\gamma_\mu^{\alpha\beta}\partial_\mu\psi_\beta^a = 0,\ a = 1\ldots 4$$

Consider twisted Lorentz symmetry corresponding to
$SO_{twist}(4)=diag(SO_{Lorentz}(4) \times SO_{flavor}(4))$ -- fermions become matrix!

$$S = \int \mathrm{Tr}\,\left(\overline{\psi}\gamma_\mu\partial_\mu\psi\right)$$ In massless case decomposes into 2 pieces

$$S = \int \mathrm{Tr}\left(\overline{\psi}_+\gamma_\mu\partial_\mu\psi_-\right) + \mathrm{Tr}\left(\overline{\psi}_-\gamma_\mu\partial_\mu\psi_+\right)$$

with $$\psi_\pm = \frac{1}{2}\left(\psi \pm \gamma_5\psi\gamma_5\right)$$

Like staggered story ...!

In fact can derive staggered action from this continuum KD action

Can gauge 2 pieces independently. Fermion bilinear invariant under twisted Lorentz symmetry is $\mathrm{Tr}\left(\overline{\psi}_+\psi_+\right) + \mathrm{Tr}\left(\overline{\psi_-}\psi_-\right)$