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Difficult (unsolvable) vs. easy (solvable)

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Solutions can give “physical insight”:
physics of the ground state
quantum entanglement(?)

A “simple” Yukawa lattice model

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Euclidean Action

$$S(\phi,\overline{\psi},\psi) \;=\; \sum_{x,y} \overline{\psi}_x D^0_{xy} \psi_y \;-\; g\; \rho_x\; e^{i(-1)^x \theta_x} \overline{\psi}_x \psi_x \;+\; S_b(\phi)$$

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free staggered fermions

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Theory of massless fermions interacting with a complex scalar field!

Traditional approach

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Severe sign problem!

But is it "difficult" or "easy" sign problem?

Fermion Bag solution

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S.C PRD(R)(2012)

Fermion Bag solution

S.C PRD(R)(2012)

Rewrite the partition function as

$$Z = \int [d\phi]\ \mathrm{e}^{-S_b([\phi])} \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi} D^0 \psi} \prod_x \left(\mathrm{e}^{g\rho_x \mathrm{e}^{i\varepsilon_x \theta_x}\ \overline{\psi}_x \psi_x} \right)$$

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Due to the Grassmann nature

$$\mathrm{e}^{g\rho_x \mathrm{e}^{i\varepsilon_x \theta_x}\ \overline{\psi}_x \psi_x} = 1 + g\rho_x \mathrm{e}^{i\varepsilon_x \theta_x} \overline{\psi}_x \psi_x = \sum_{n_x = 0,1} \left(g\rho_x \mathrm{e}^{i\varepsilon_x \theta_x} \overline{\psi}_x \psi_x \right)^{n_x}$$

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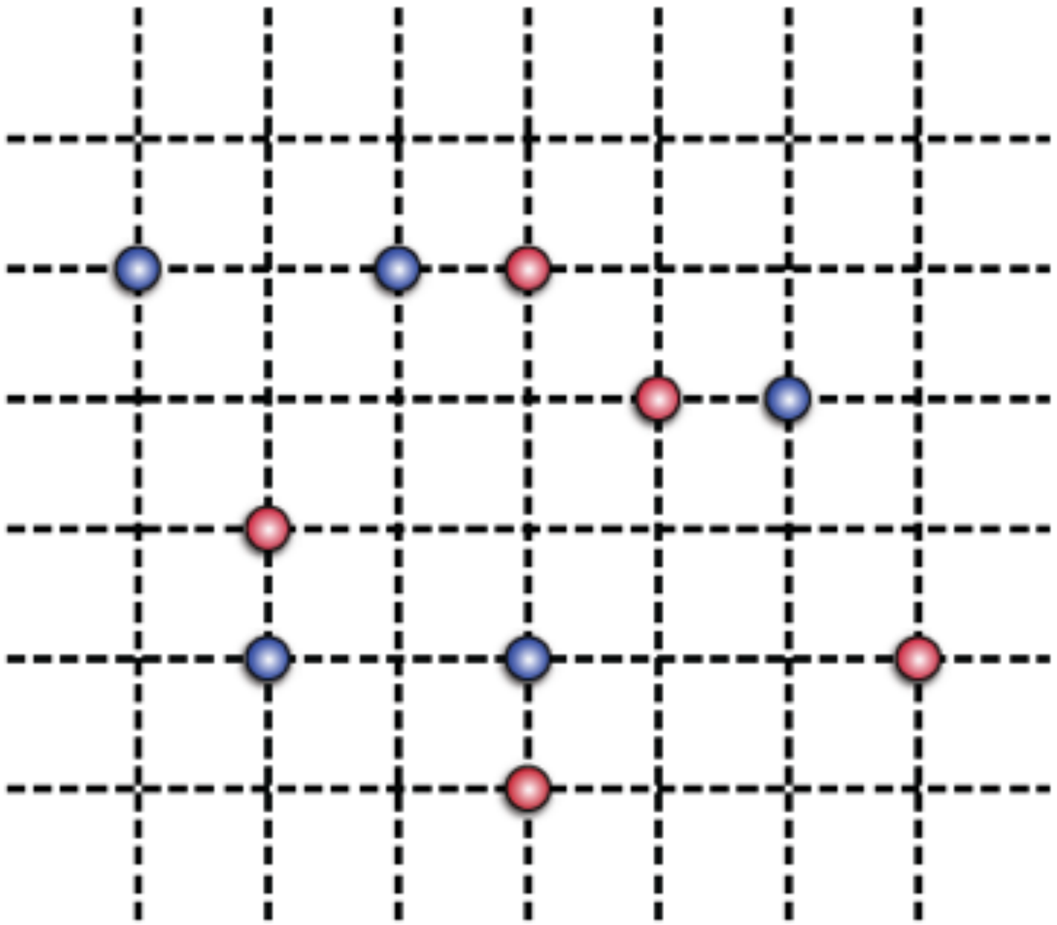
We can then rewrite

$$Z = \sum_{[n_x]} \int [d\phi] \mathrm{e}^{-S_b([\phi])} \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi} D^0 \psi} \prod_x \left(g\rho_x \mathrm{e}^{i\varepsilon_x \theta_x}\ \overline{\psi}_x \psi_x \right)^{n_x}$$

For a given configuration [n]
let z_1 z_2 ... z_k be the k sites
where $n_x = 1$
at all other sites $n_x = 0$

example of configuration $[n_x]$ with k = 10



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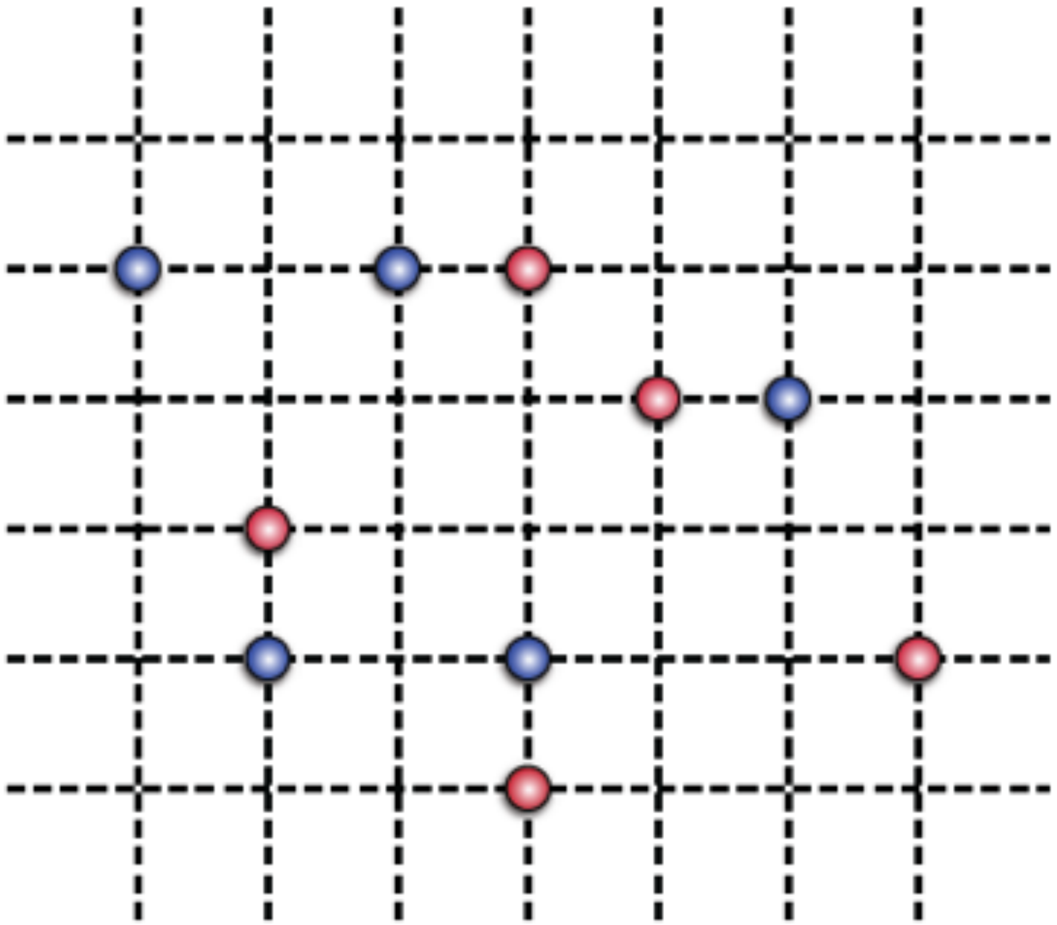

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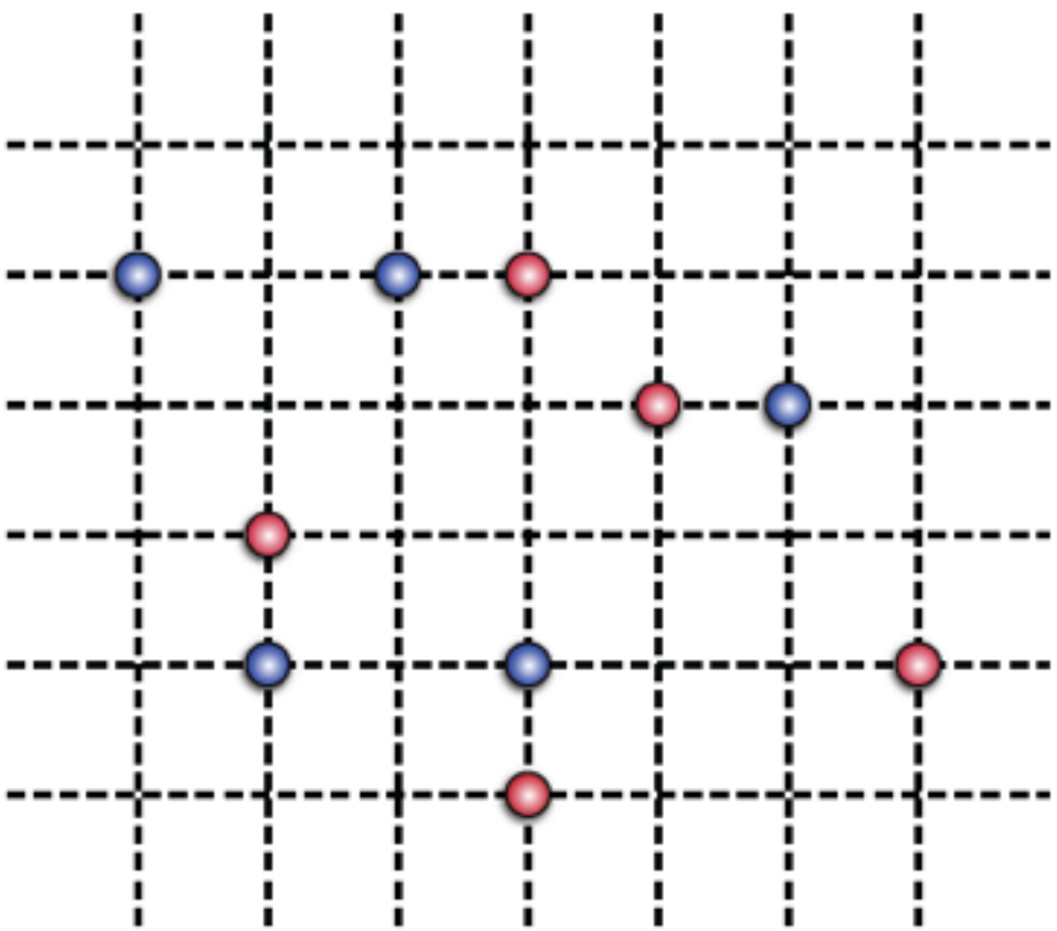

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Bosonic term
(k-point correlation function)

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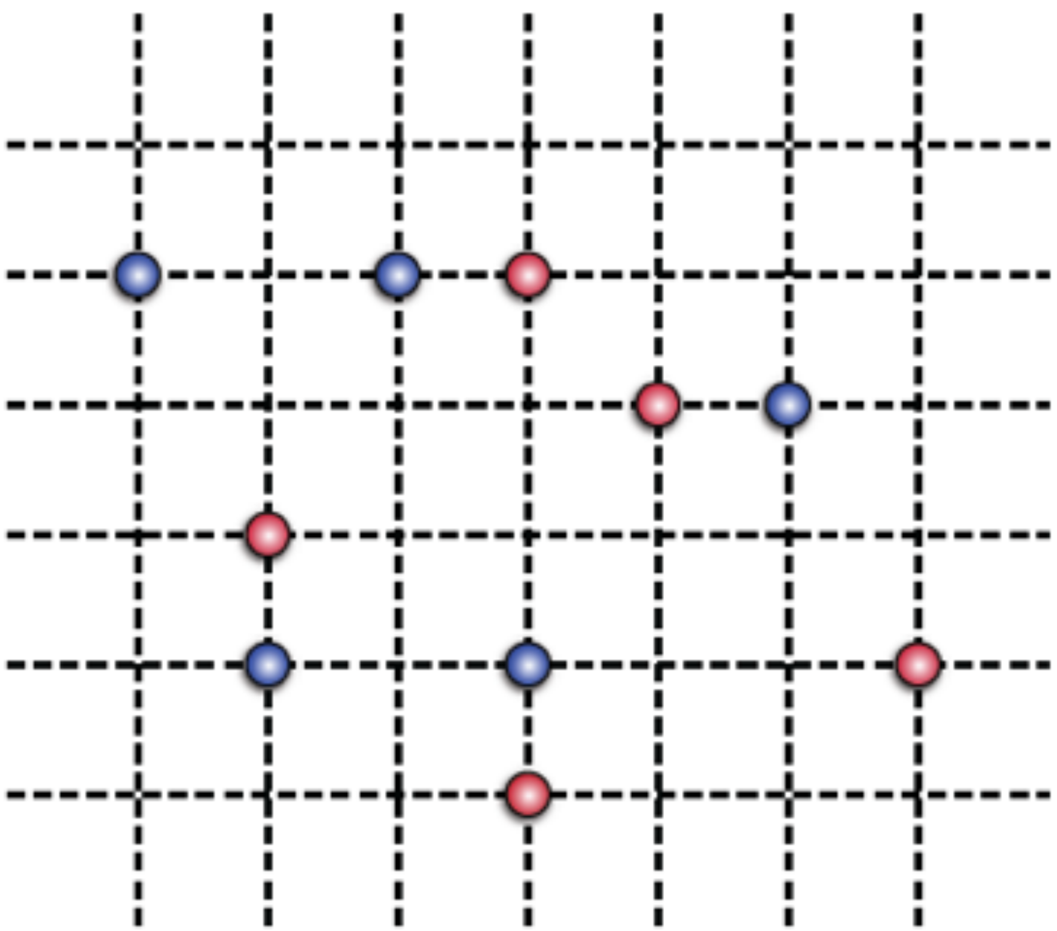

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Bosonic term
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Fermionic term
(k-point correlation function)

Fermion Bags

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S.C. Lattice 2008,2010
S.C, A.Li 2011,2012

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Fermion k-point correlation function

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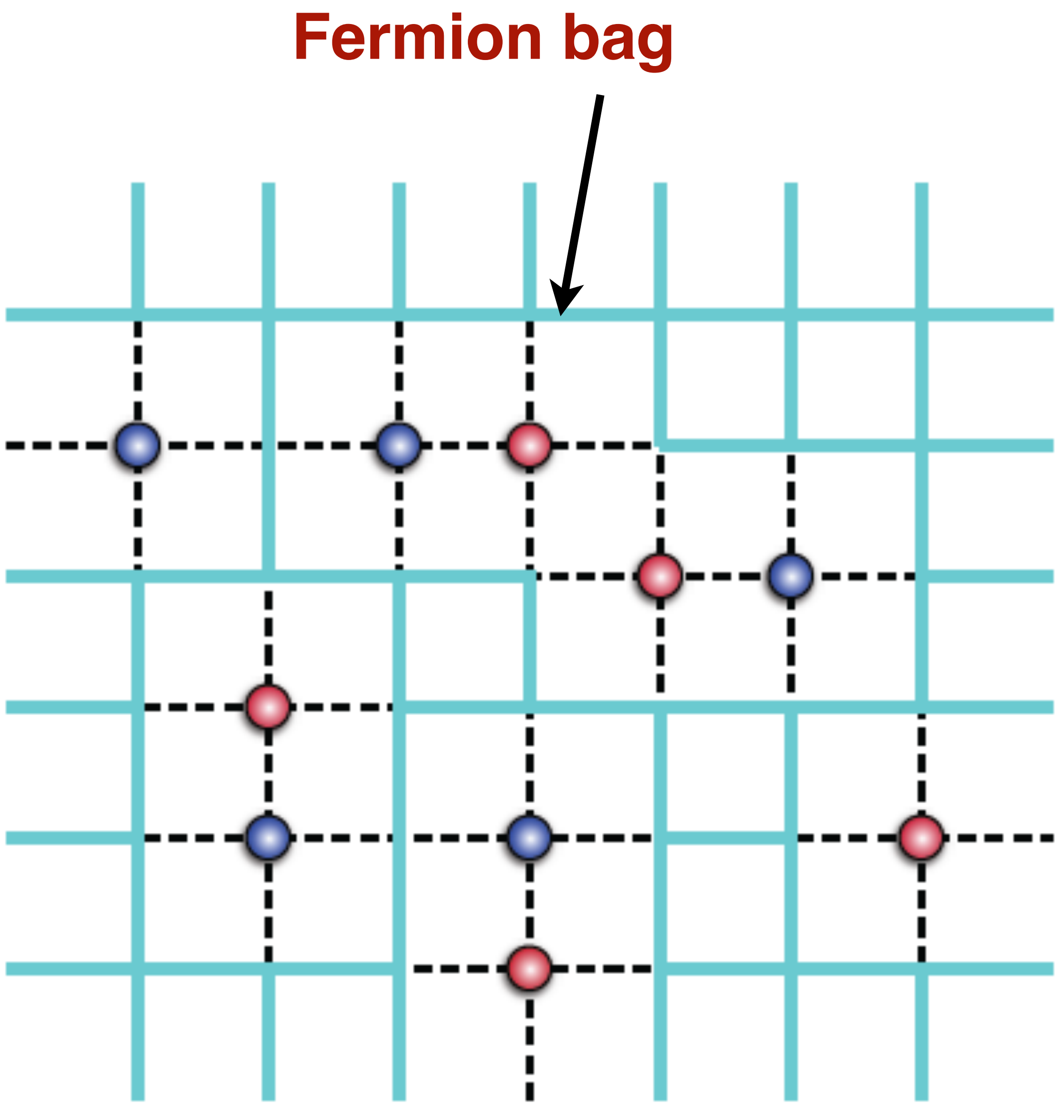

fermion bag configuration

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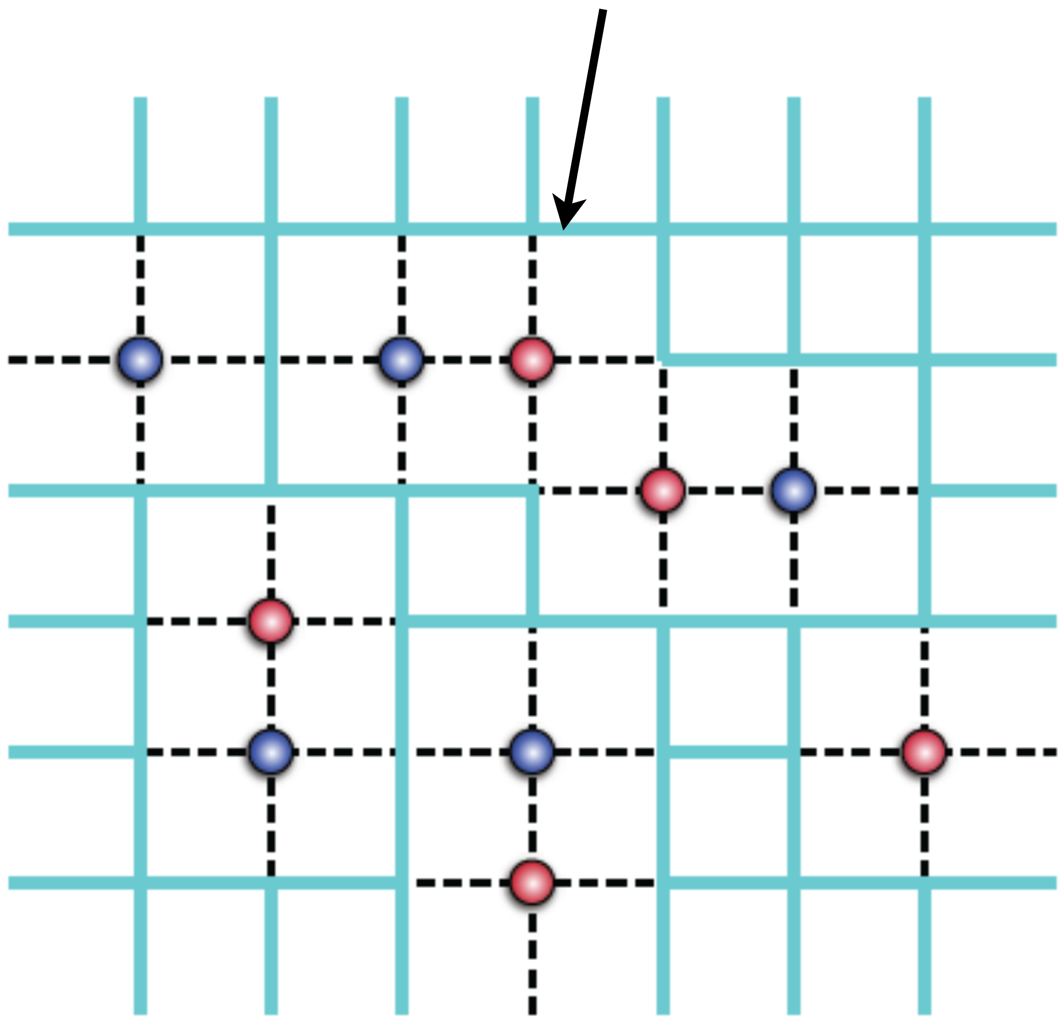

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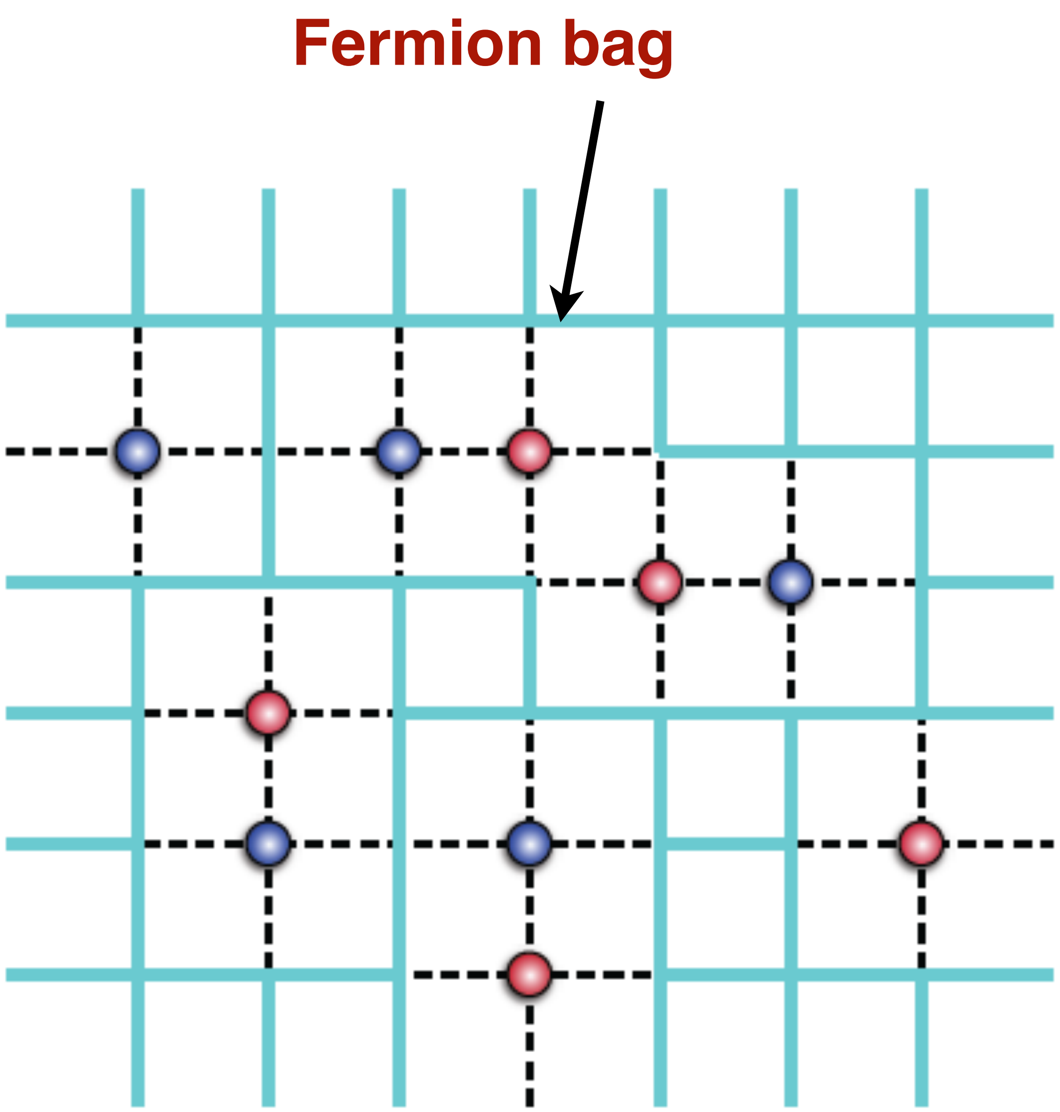
obtained by dropping sites z_1 ... z_k in D^0



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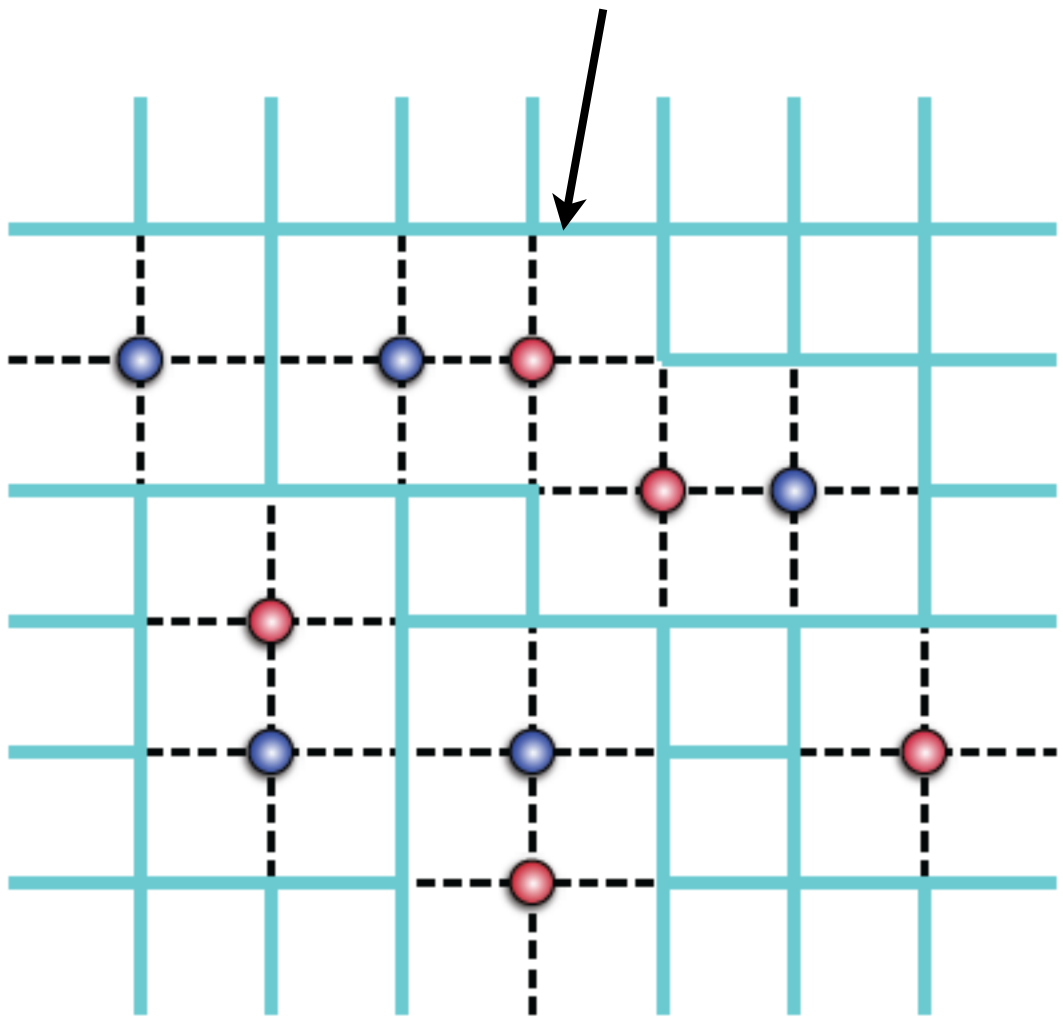
$$W^0_{[n]} = \begin{pmatrix} 0 & B([n]) \\ -B^T([n]) & 0 \end{pmatrix}$$

Fermion bag



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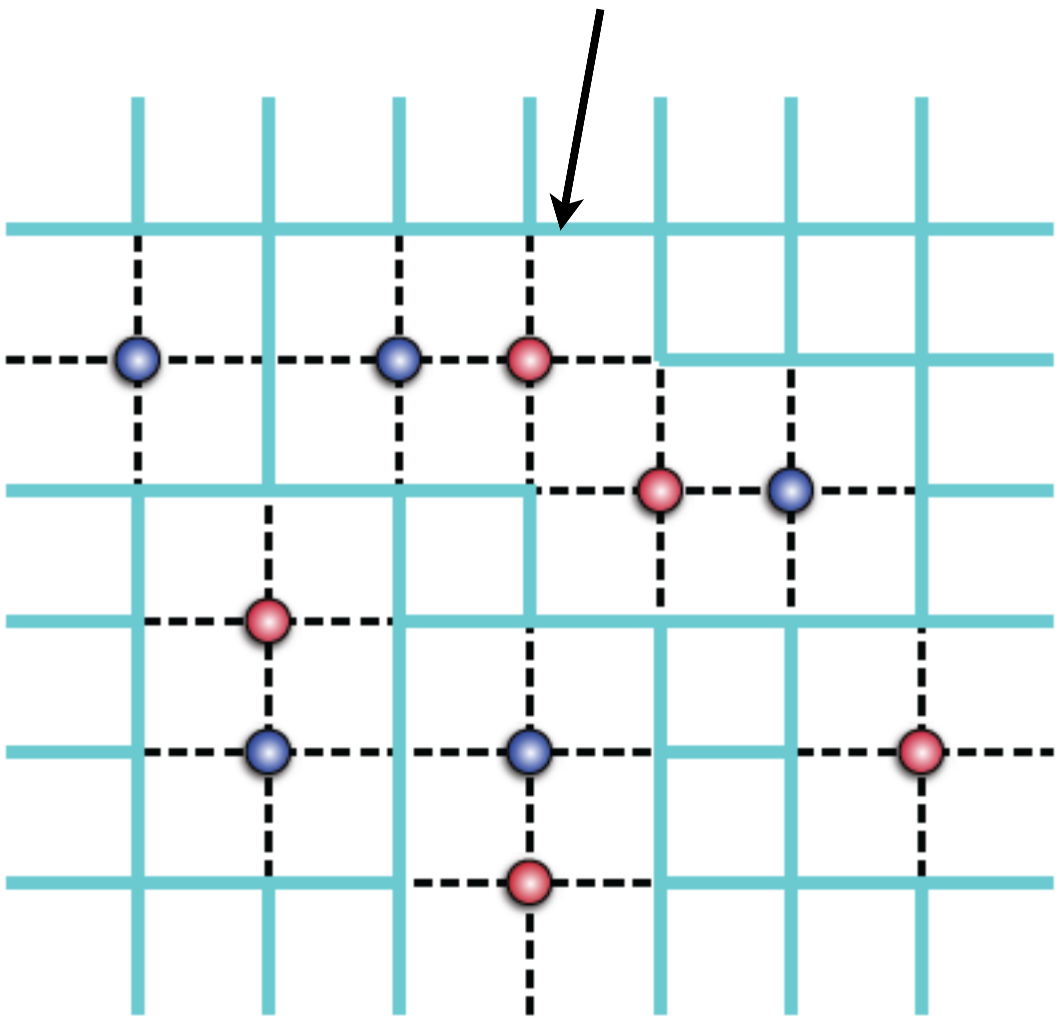
$$\mathrm{Det}(W^0_{[n]}) \geq 0$$



fermion bag configuration

Dual Fermion Bags

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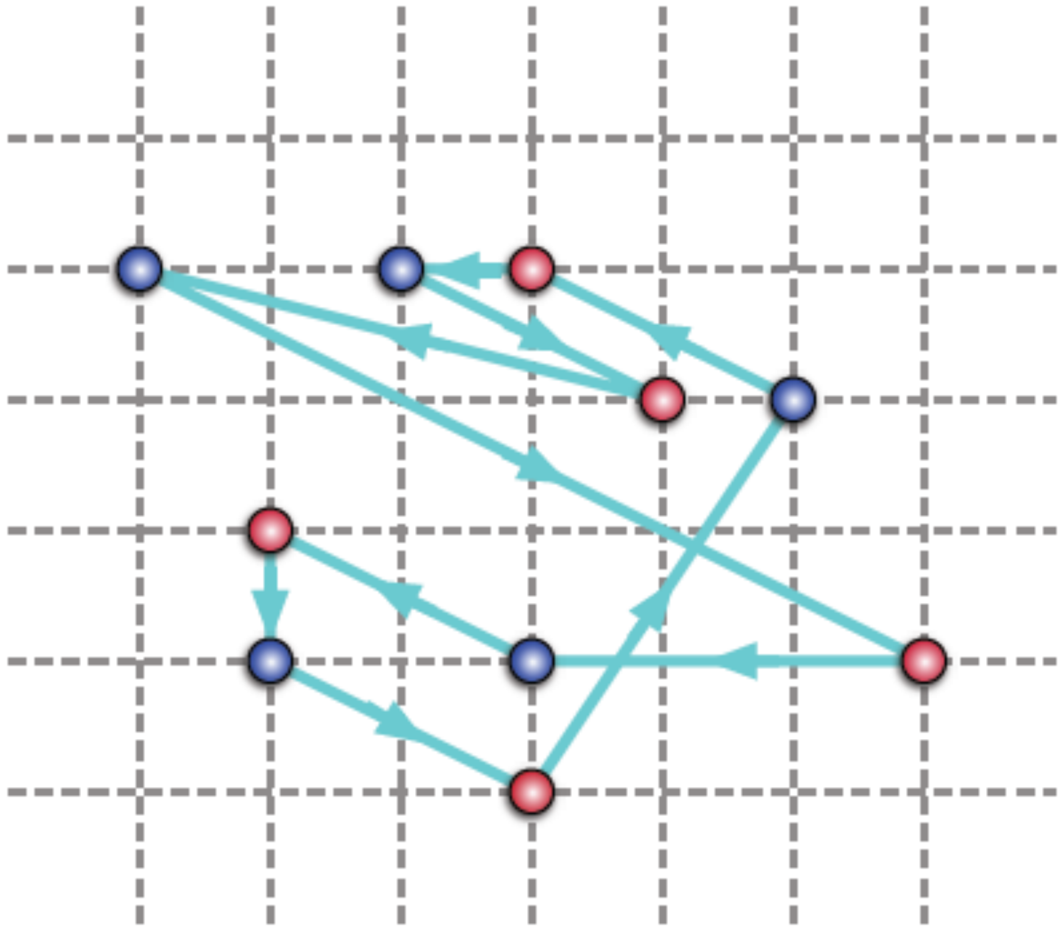

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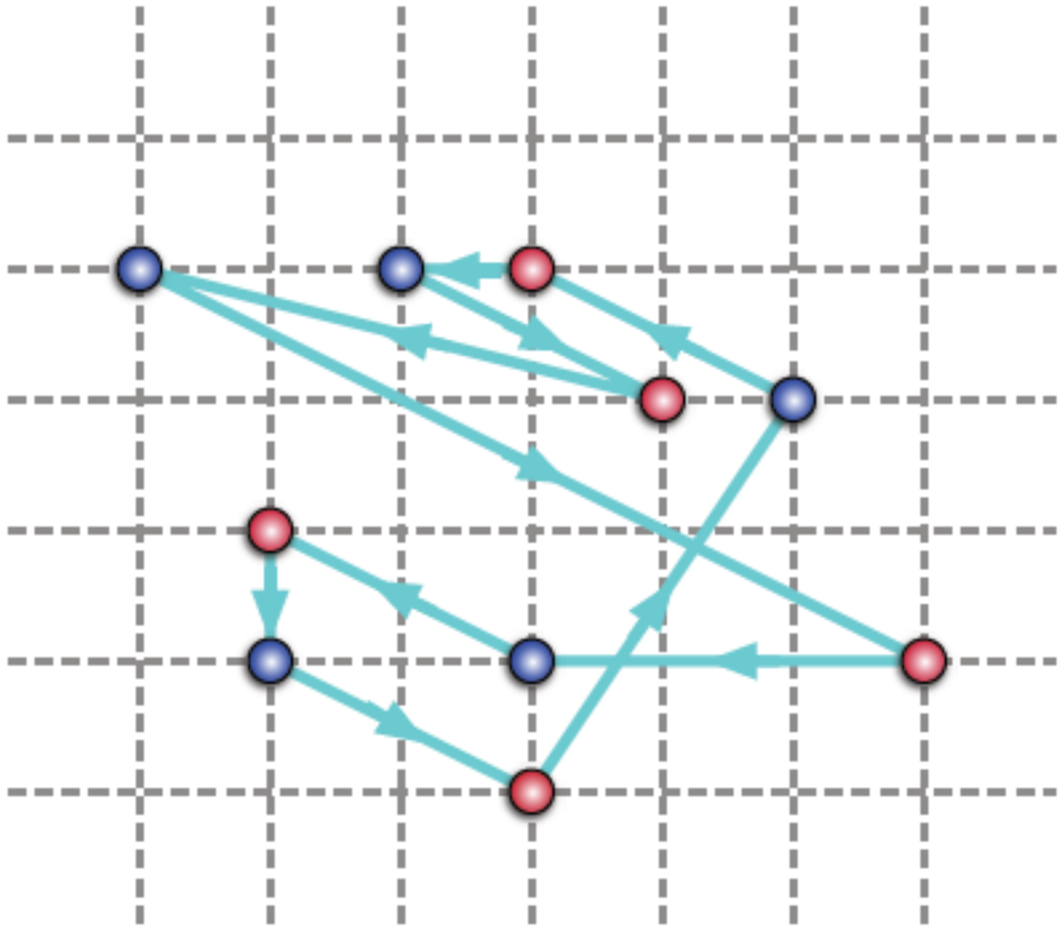

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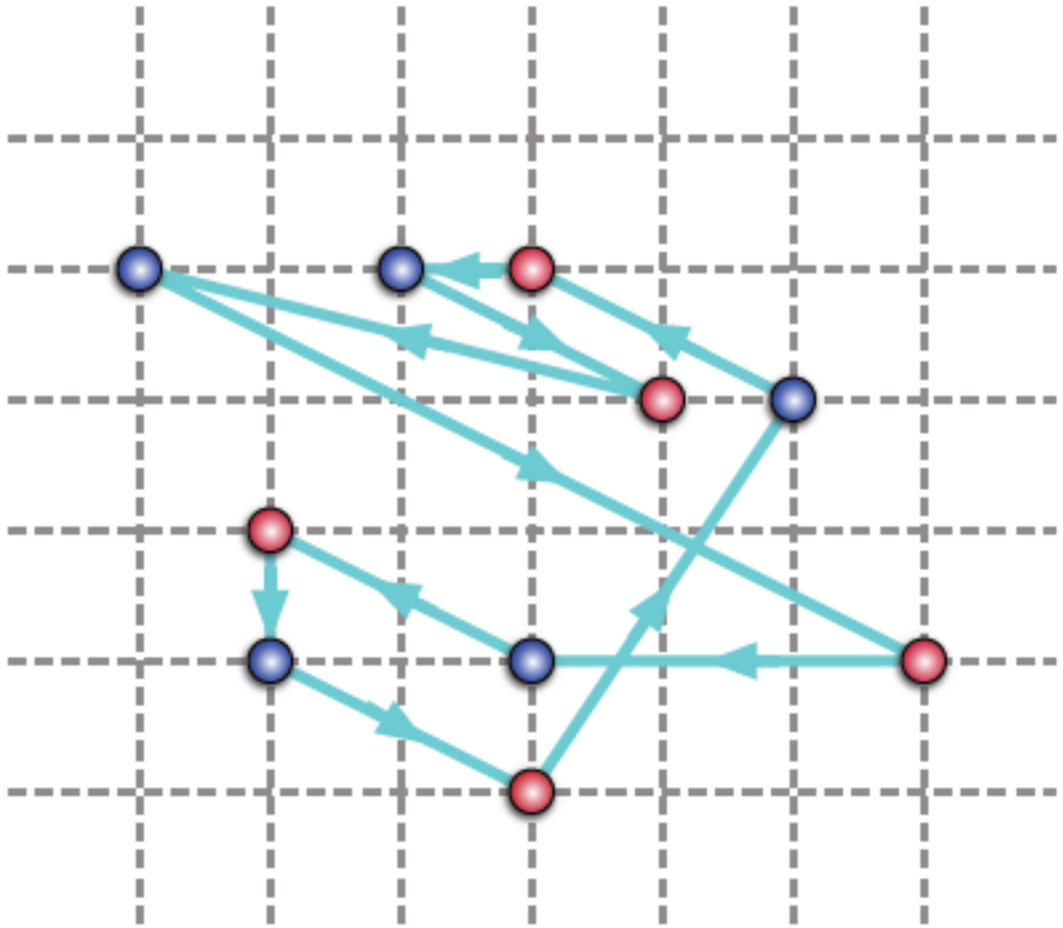

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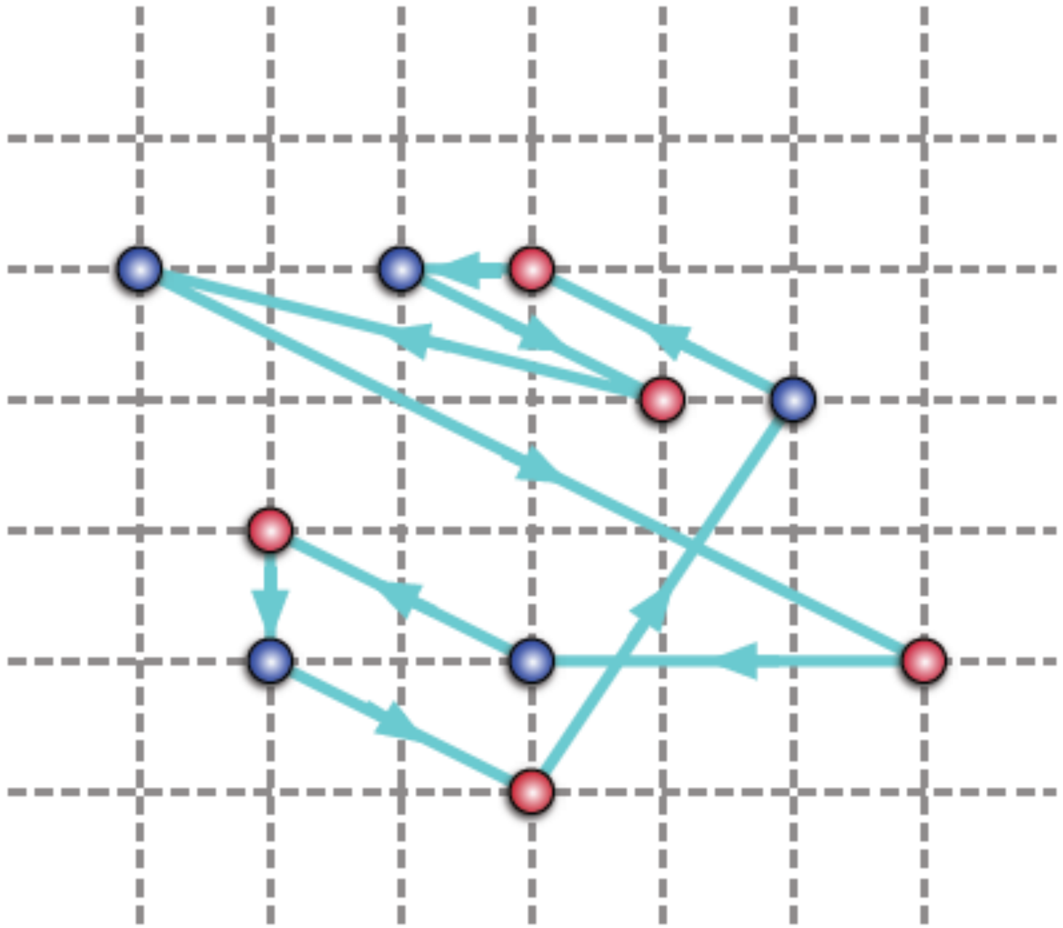

Dual Fermion Bag

Similar to the CT diagrammatic determinantal Monte Carlo

Rubtsov, Savkin, Lichtenstein, Prokofev, Svistunov, Troyer, ...

Talk by Assad

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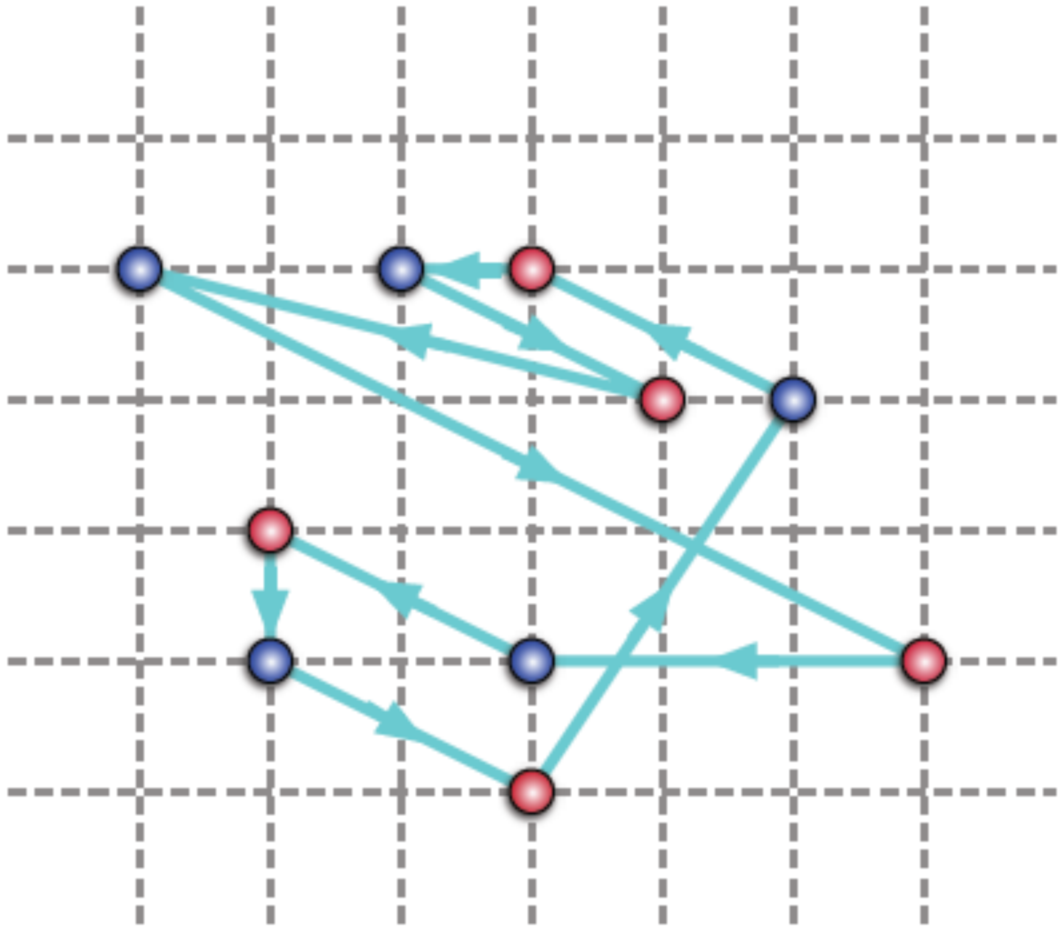

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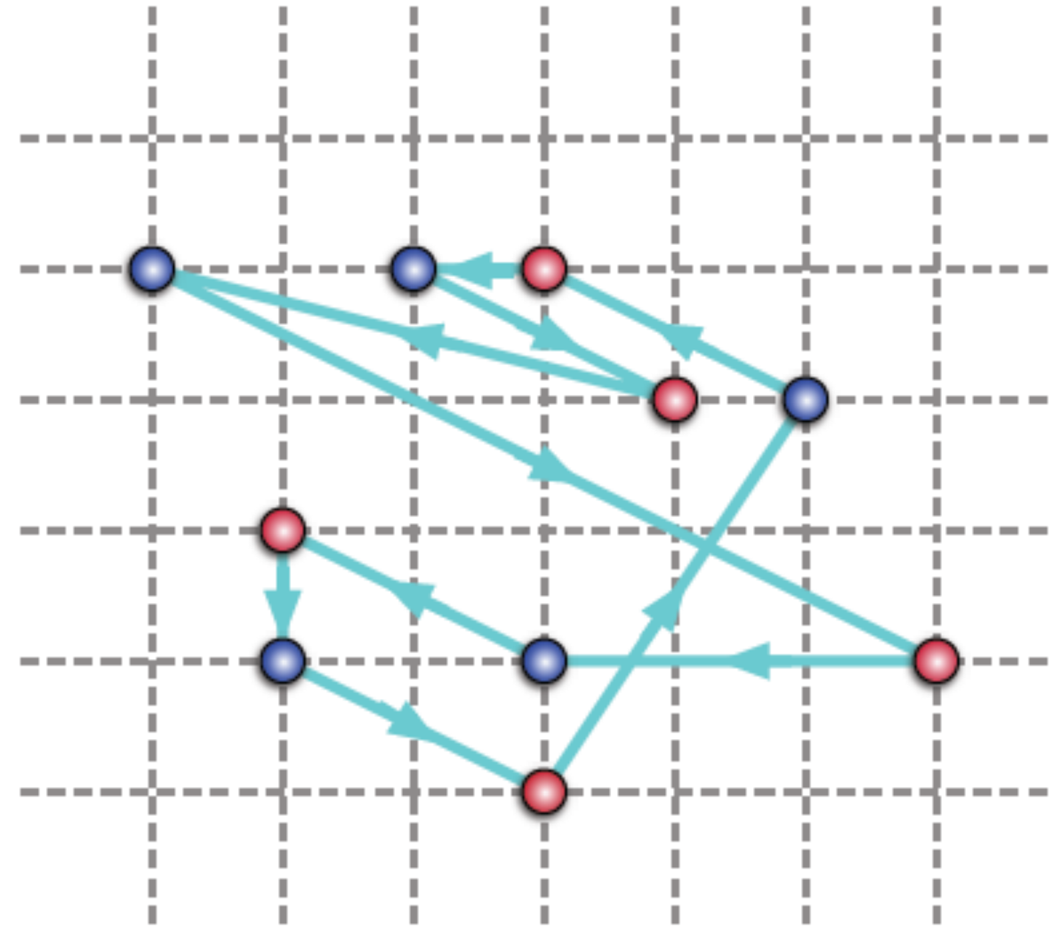
Det W^0 = Det D^0 Det $G_{[n]}$

strong coupling fermion bag



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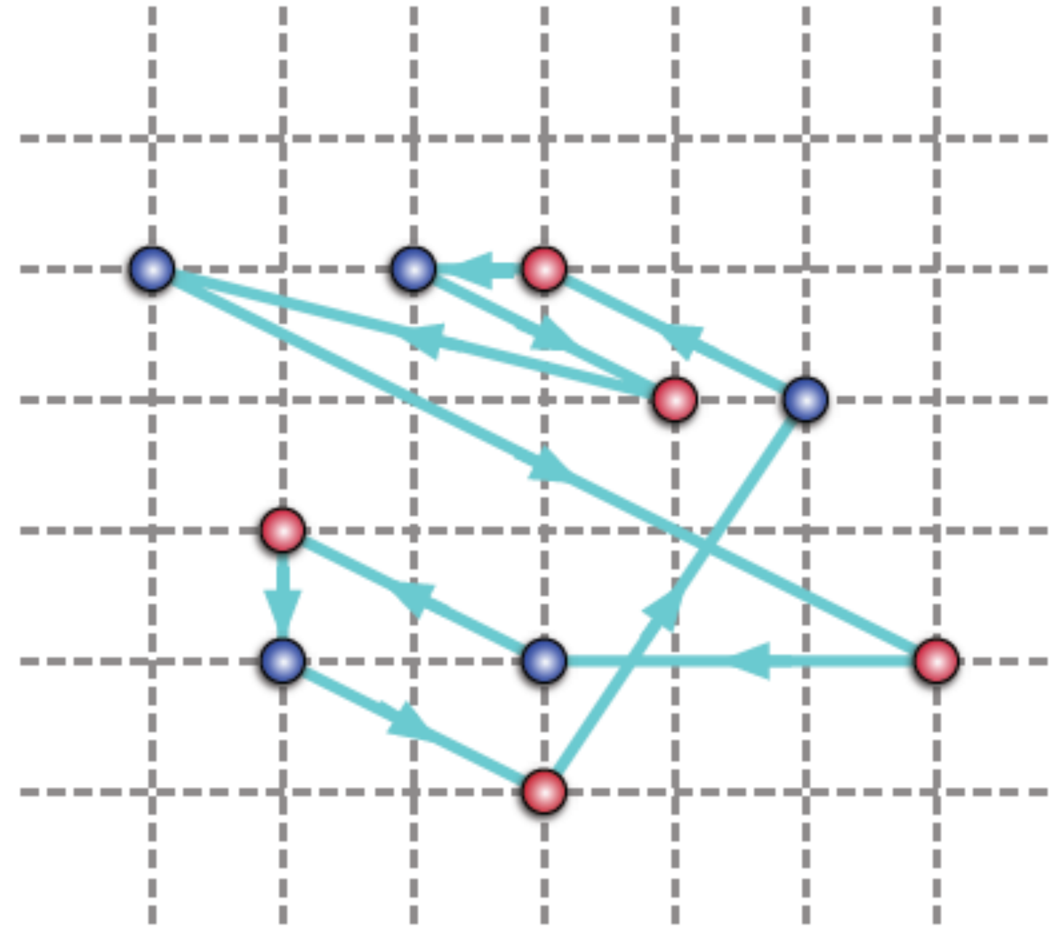
strong coupling fermion bag **weak coupling fermion Bag**



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Talk by Assad

Bosonic k-point correlation function

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We will assume that we can write

$$\int [d\phi] \mathrm{e}^{-S_b([\phi])} \; \rho_{z_1} \mathrm{e}^{i\varepsilon_{z_1}\theta_{z_1}} \ldots \rho_{z_k} \mathrm{e}^{i\varepsilon_{z_k}\theta_{z_k}}$$

$$= \sum_{[b]} \int [d\rho] \; \Omega([b, \rho, n])$$

where $\Omega([b, \rho, n]) \geq 0$

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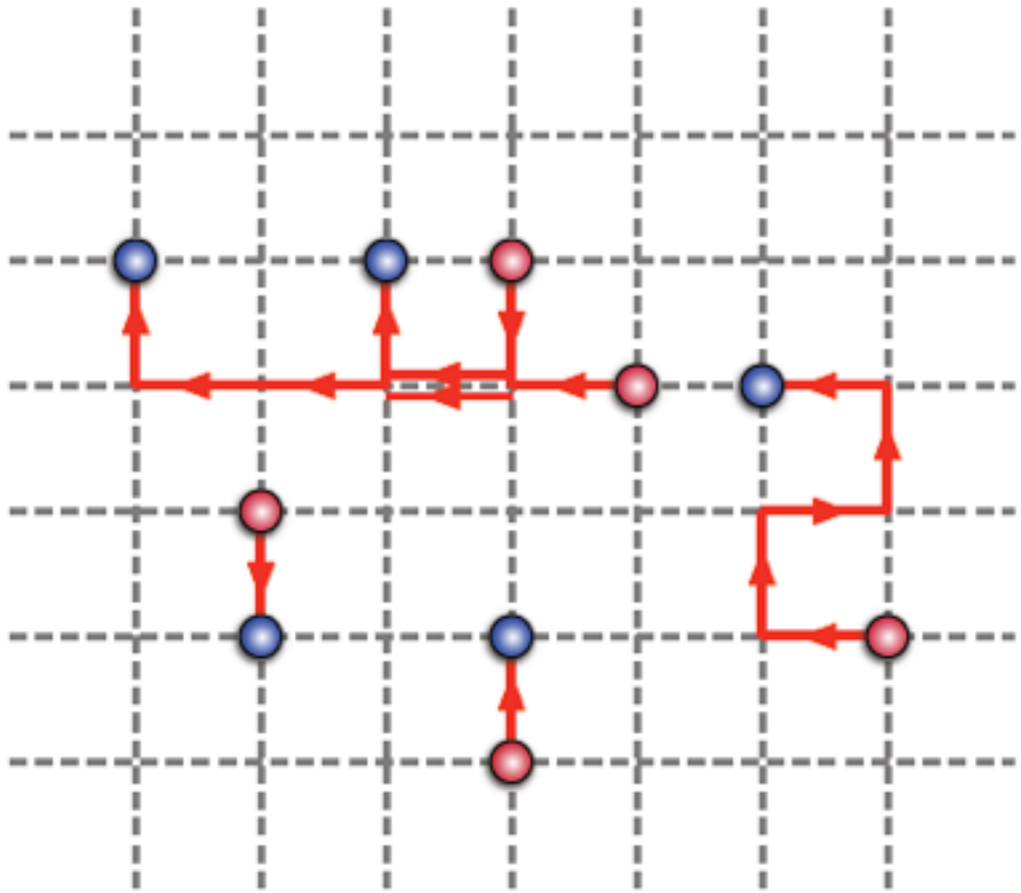

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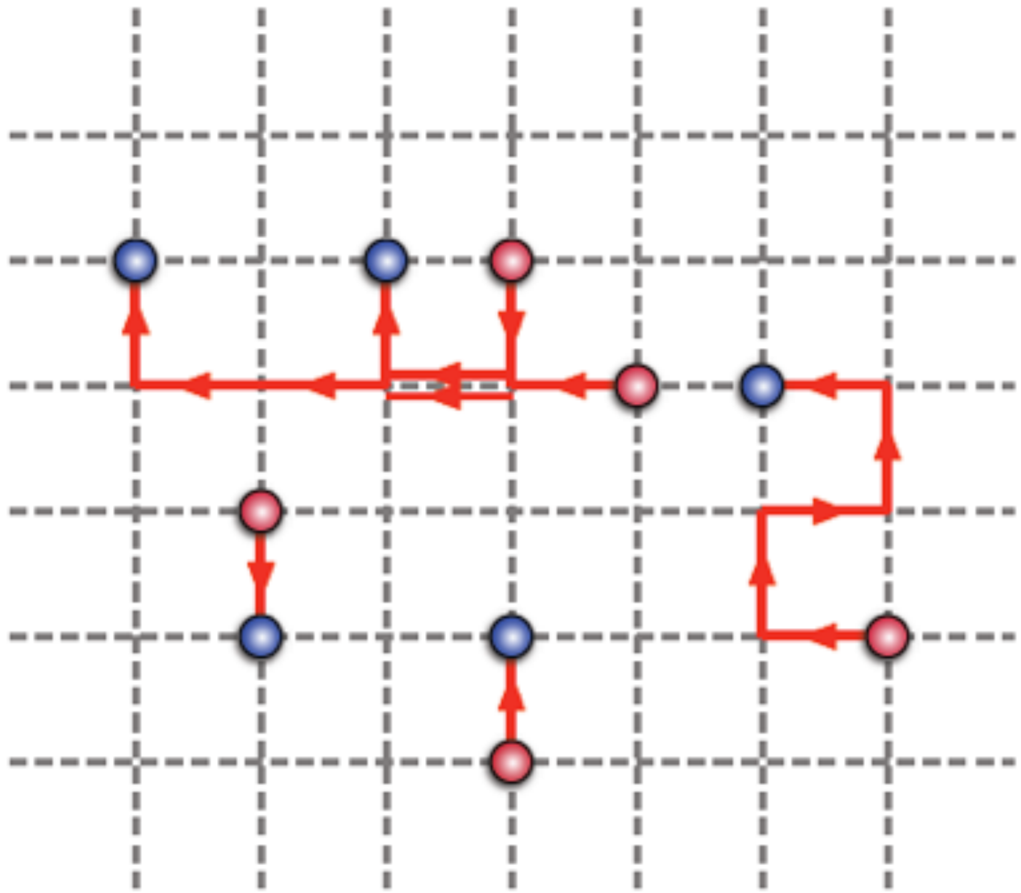

For the standard action this is nothing but the "worldline" approach to bosons

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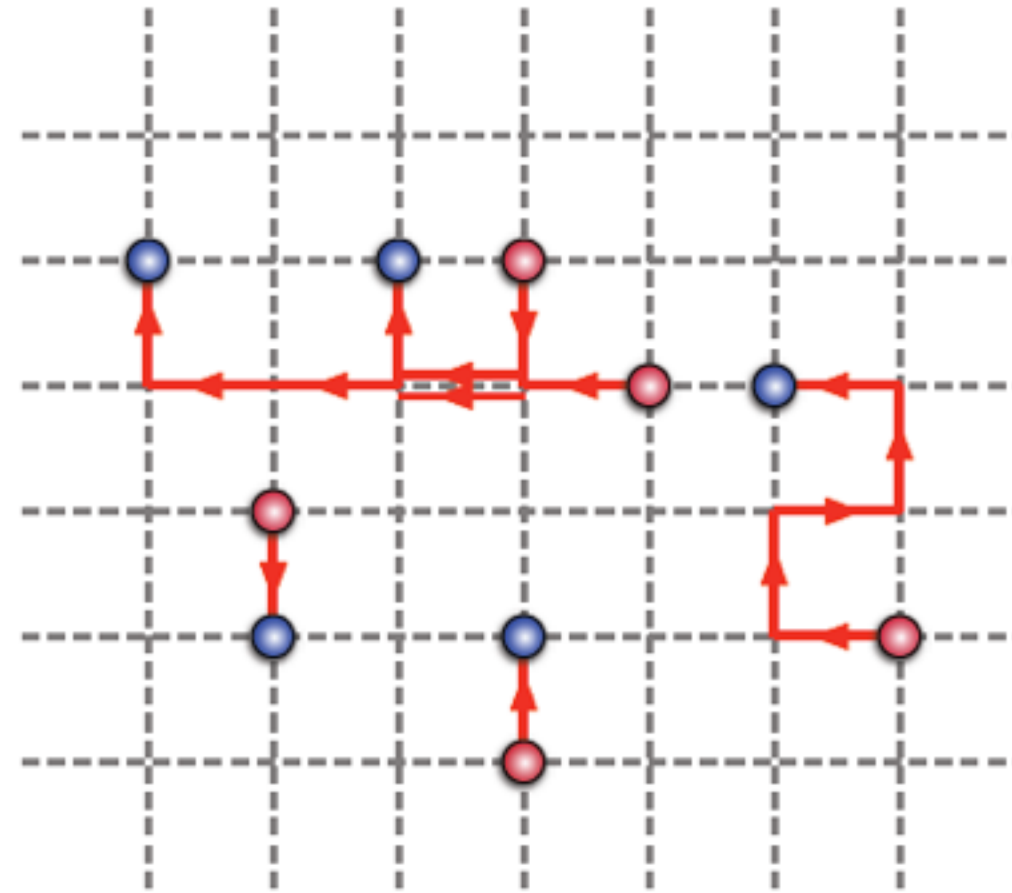

example of a [b,ρ,n] configuration

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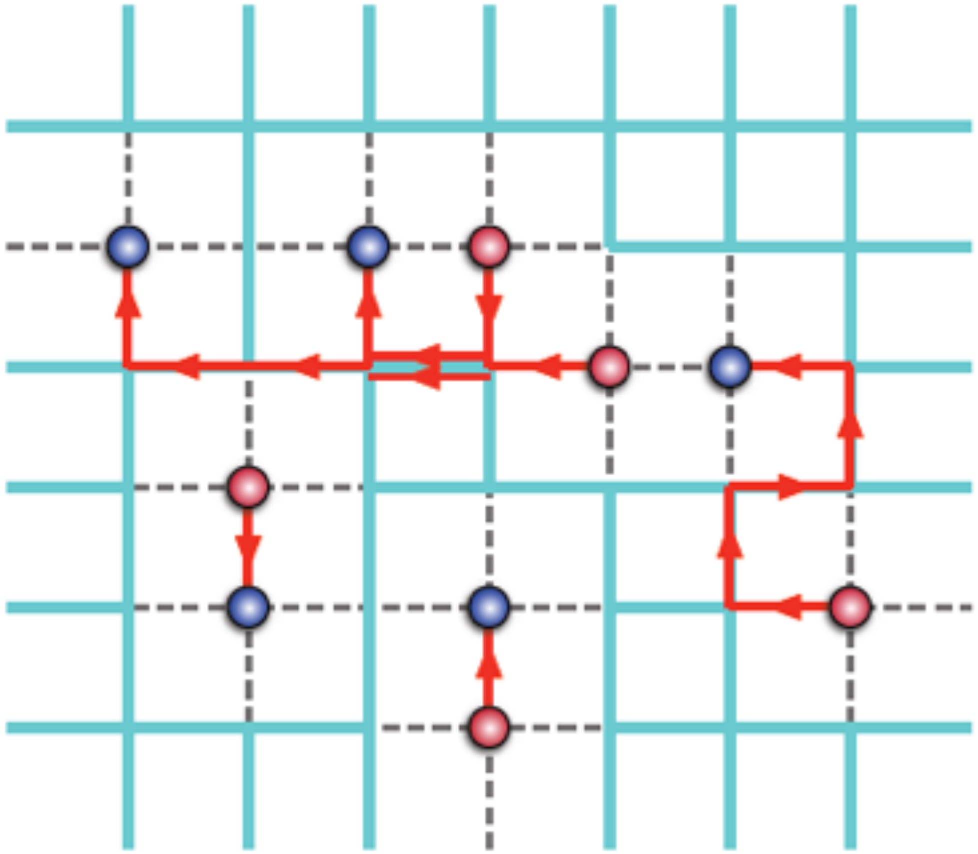

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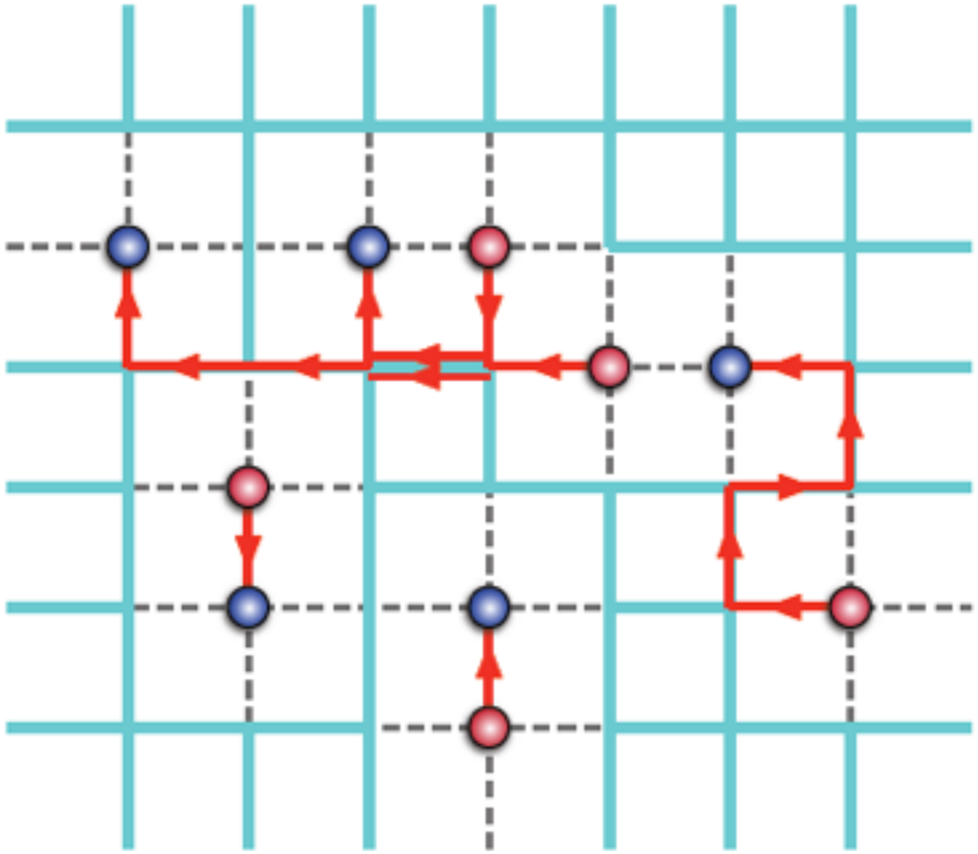

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A variety of Yukawa and Gross-Neveu Models

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some SU(3) symmetric fermion models

Some Results
with new solutions

SU(2) x U(1) Thirring model results

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Hands, Debbio, Jersak,....

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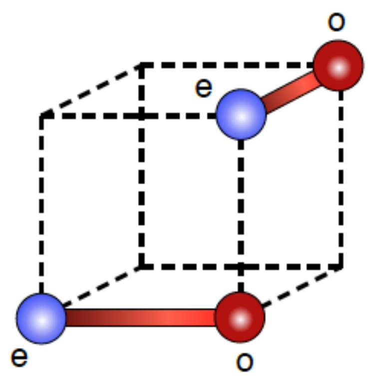

Thirring

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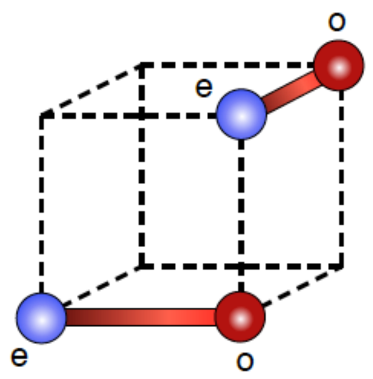

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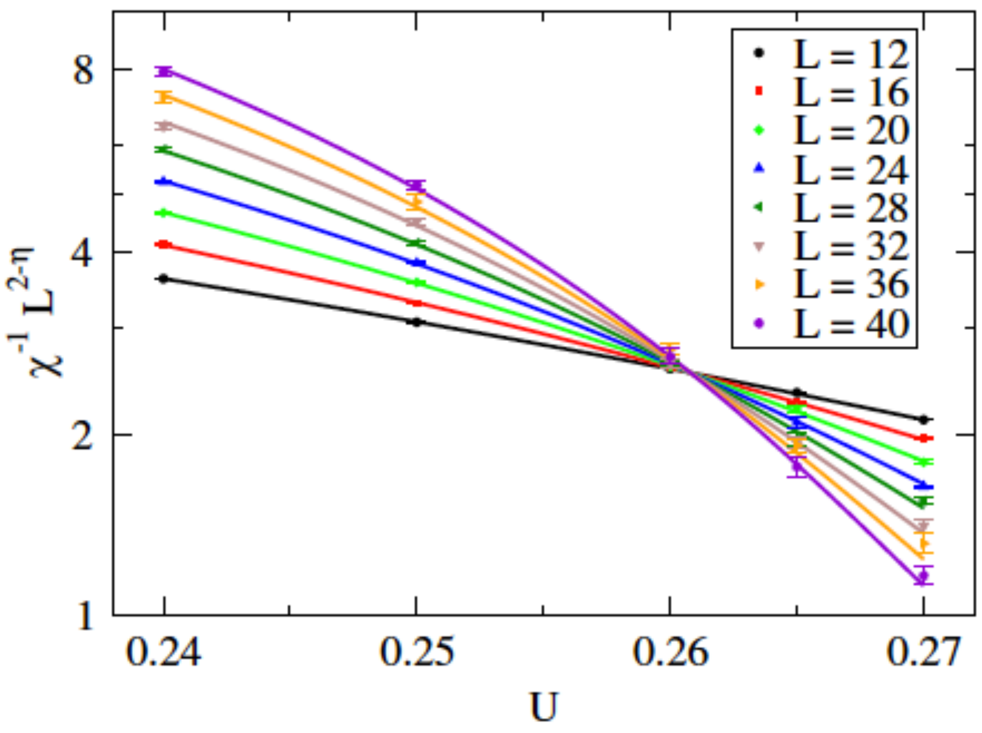

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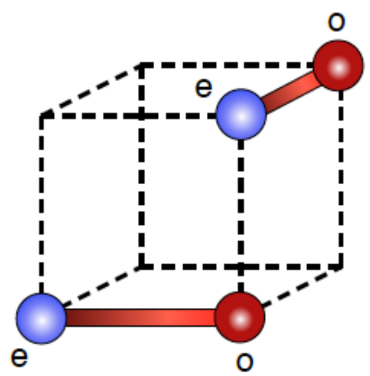

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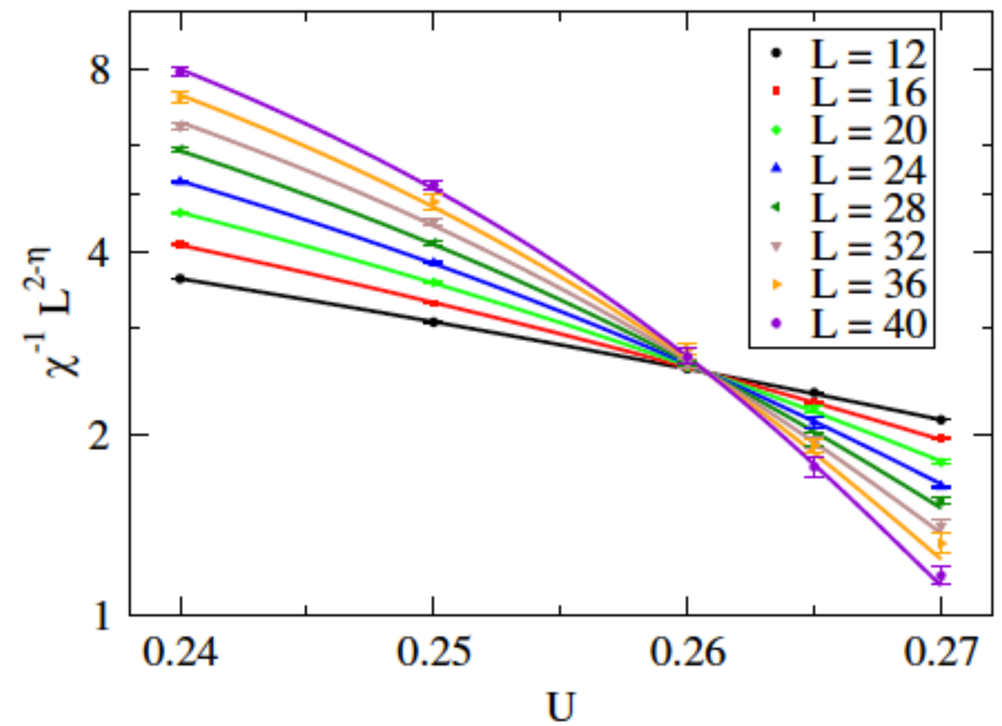

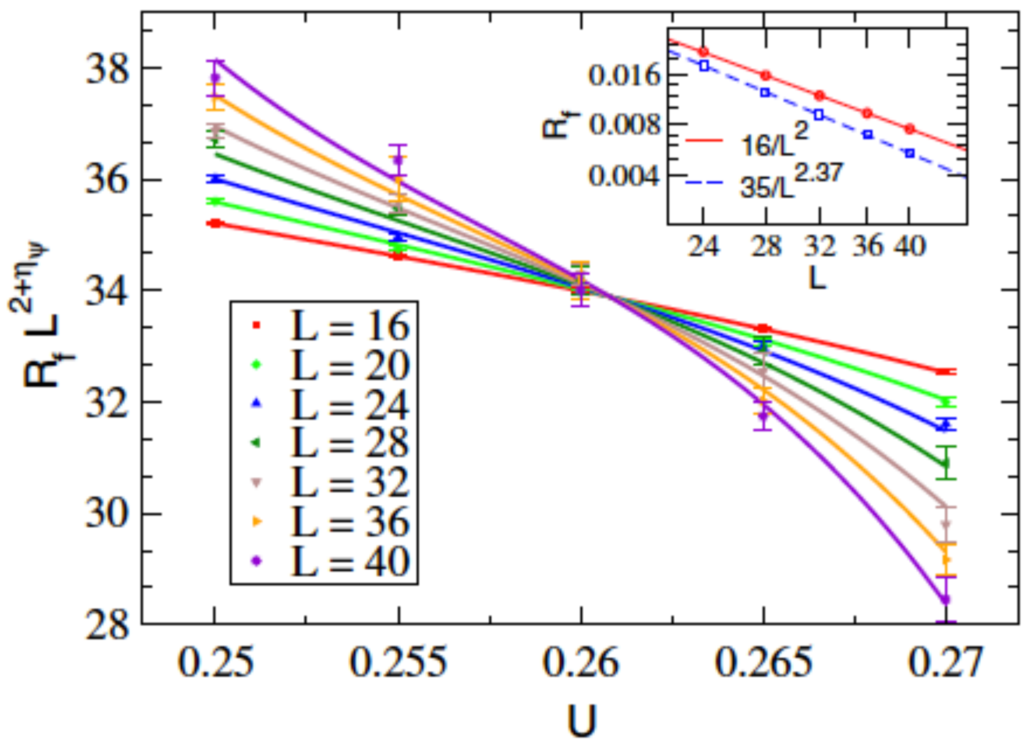

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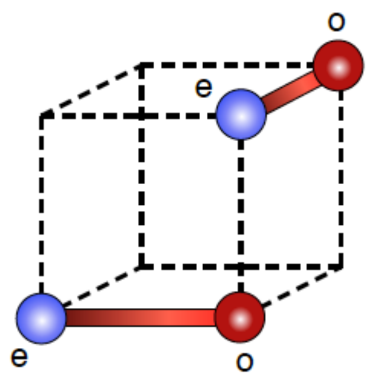
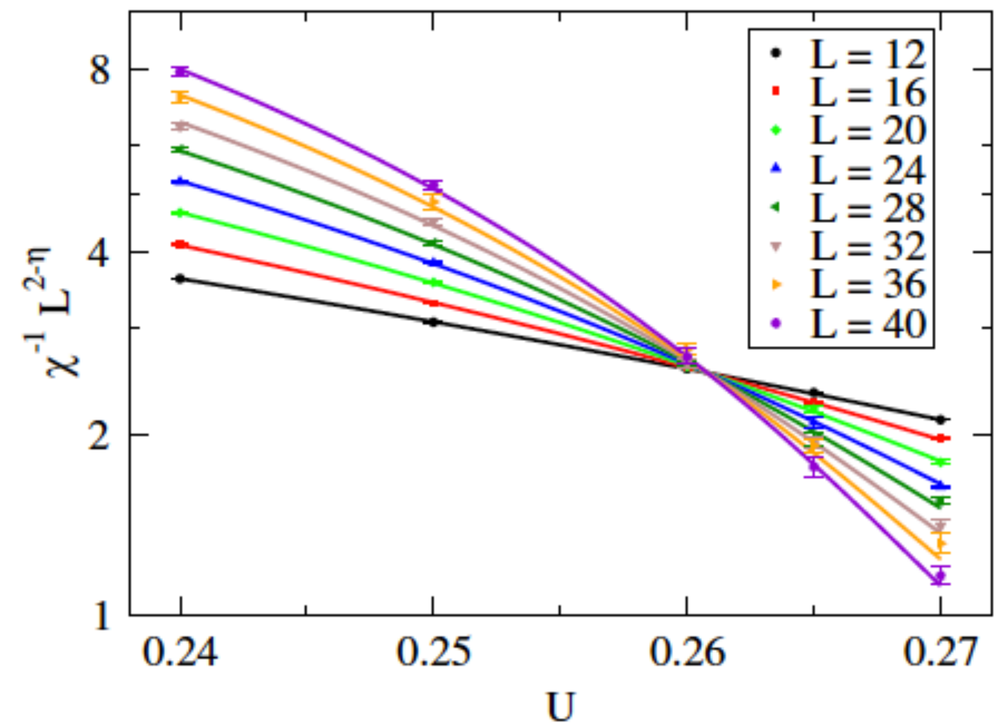




<u>**Combined fit results**</u>

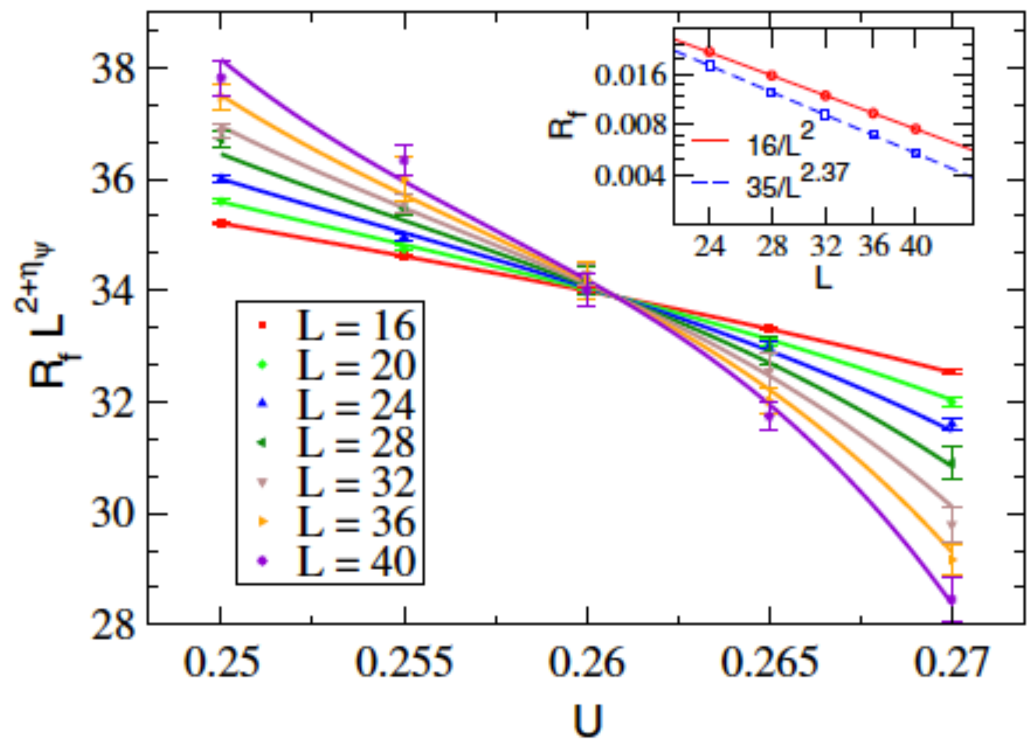
(PRL 108, 140404, 2012)

$U_c = 0.2608(2)$

$\nu = 0.85(1)$

$\eta = 0.65(1)$

$\eta_\psi = 0.37(1)$



SU(2) x U(1) Gross-Neveu model results

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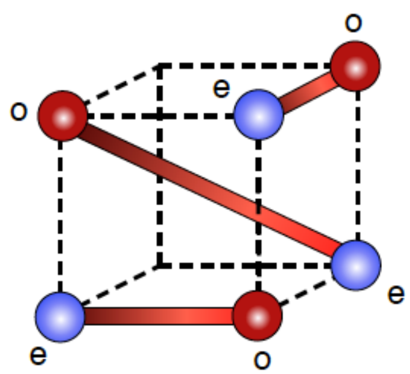

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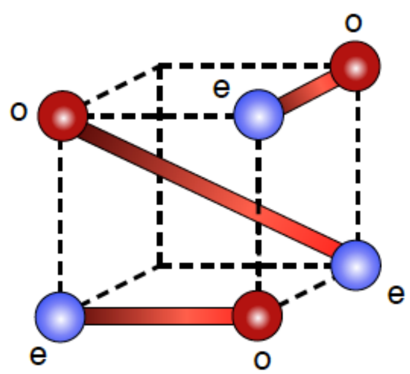

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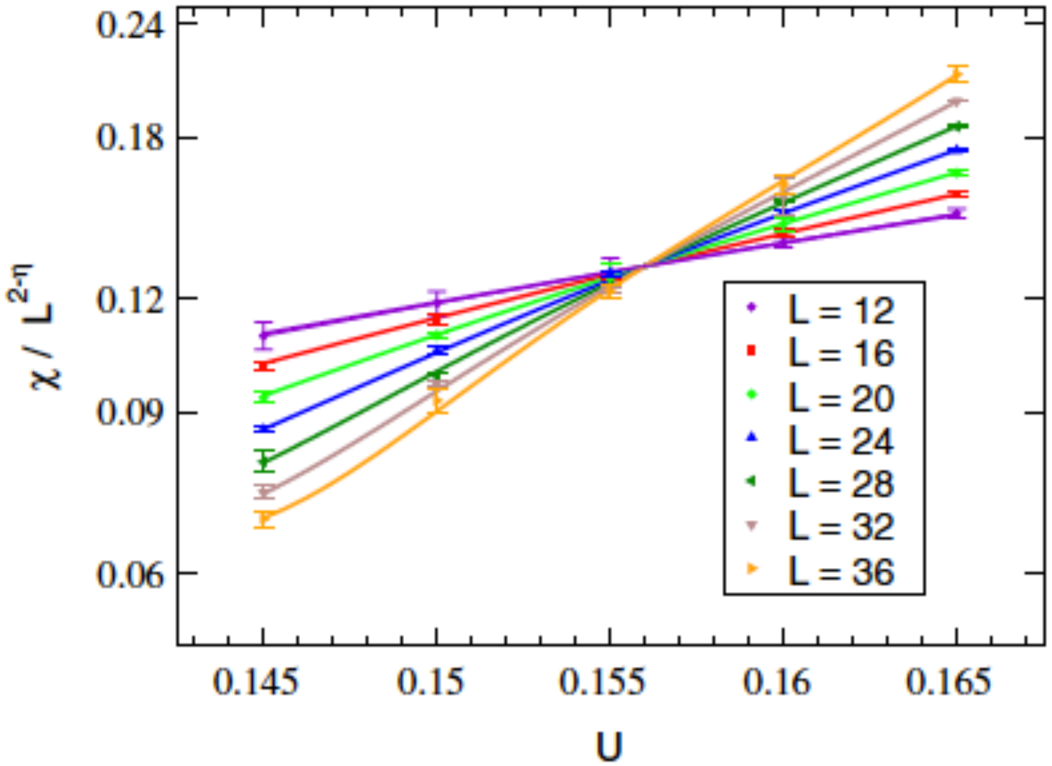

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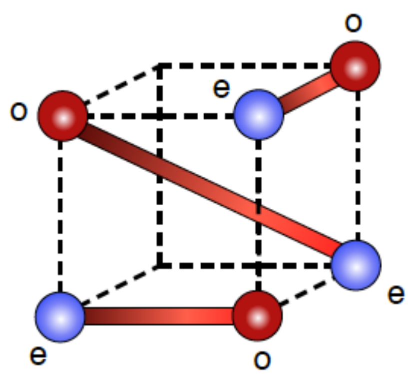

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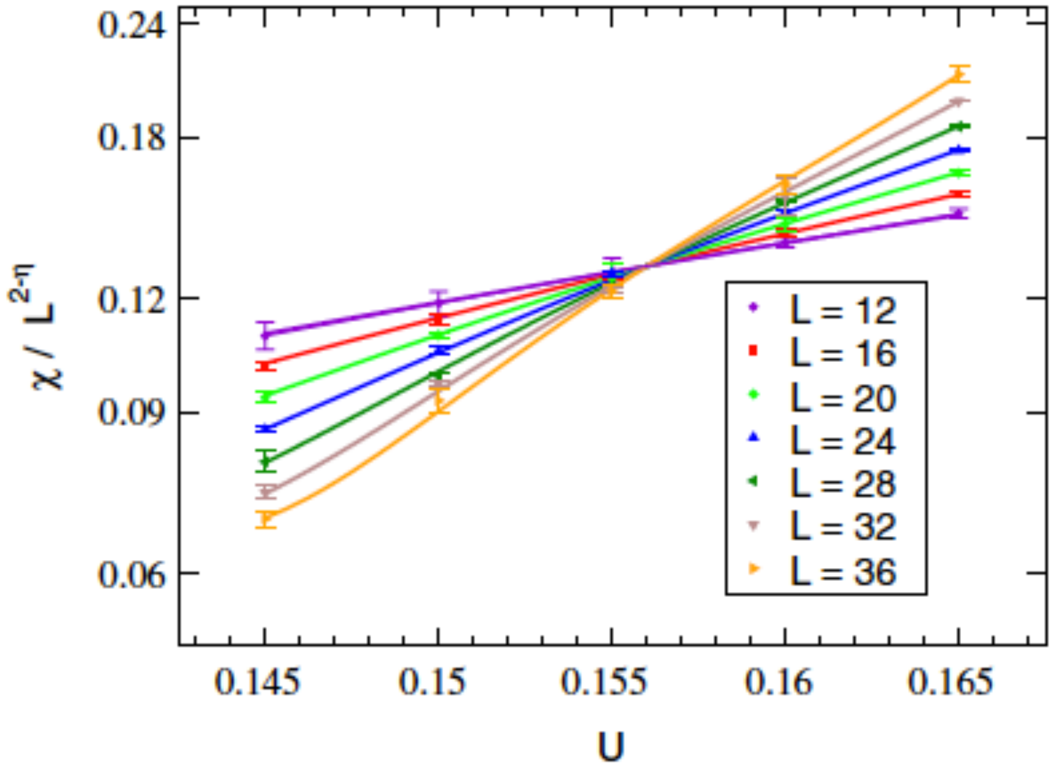

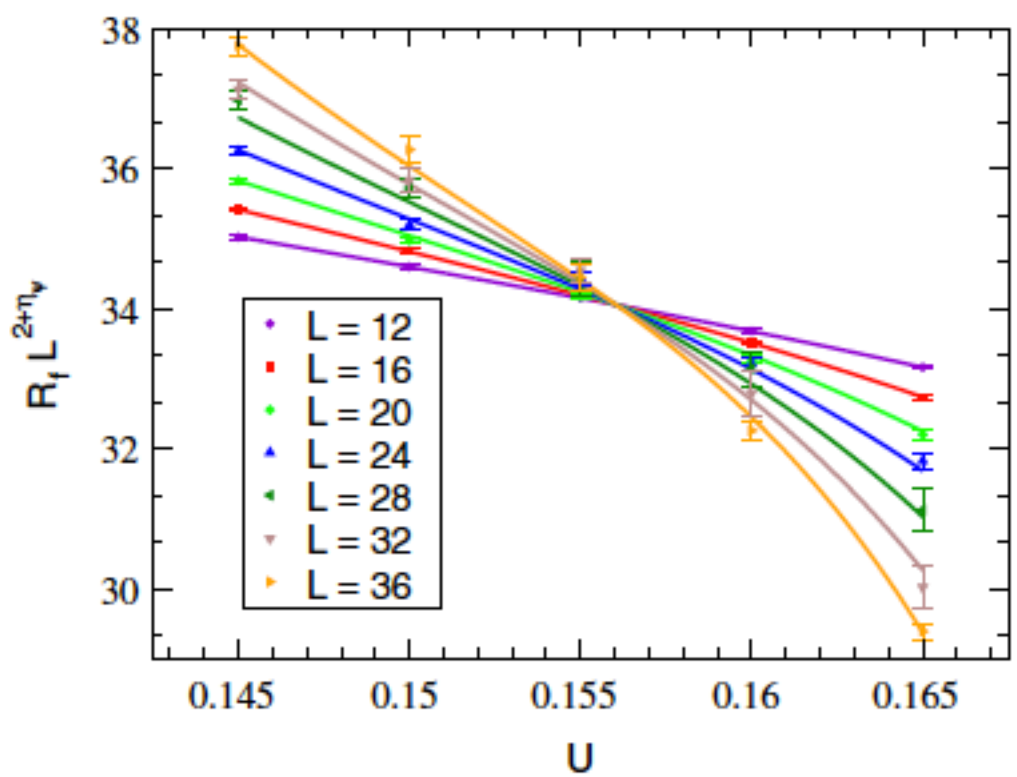

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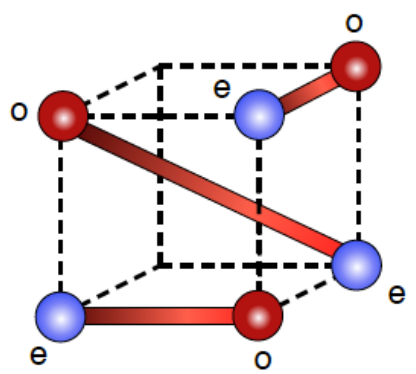

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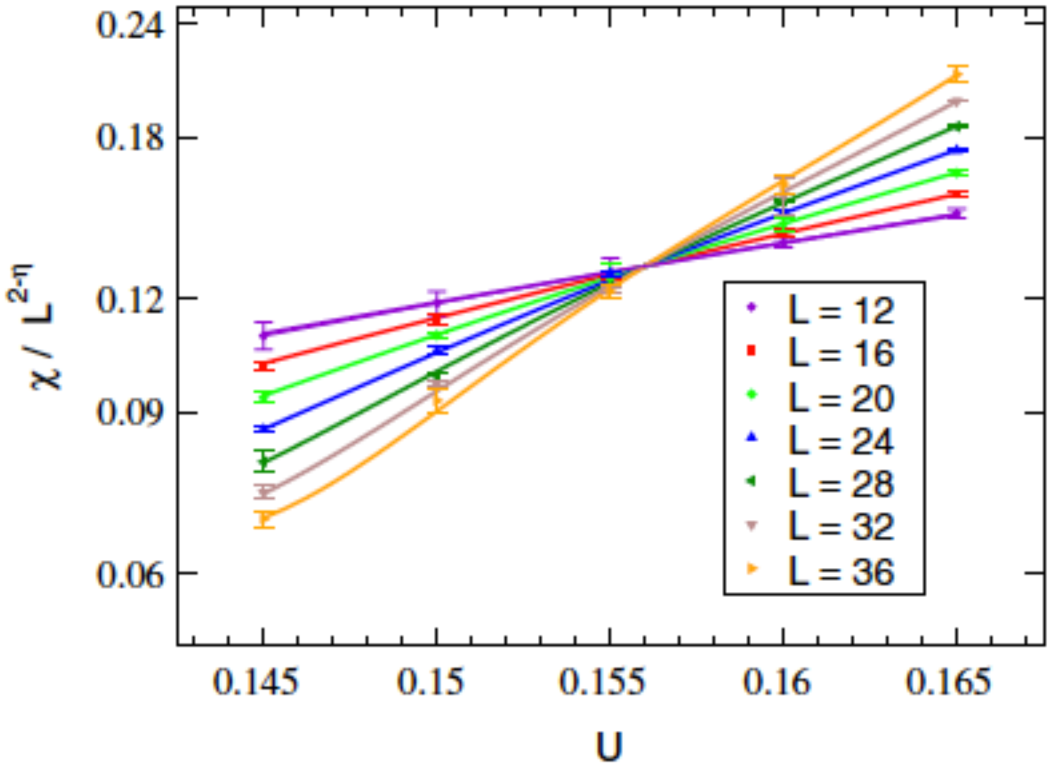

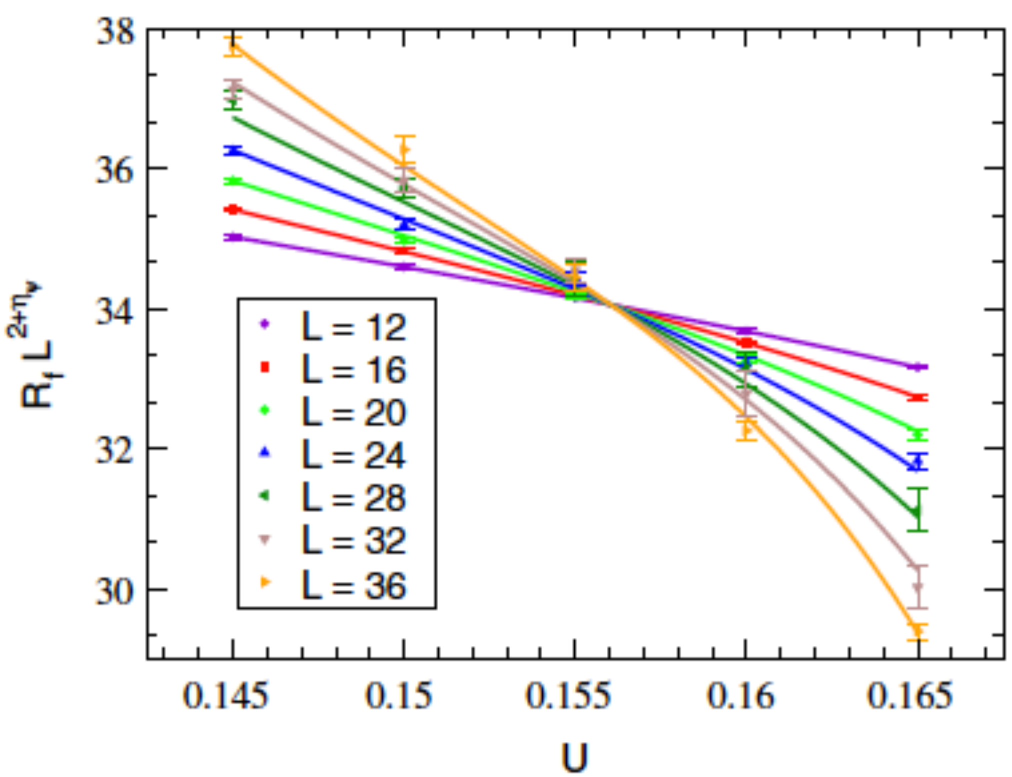

Combined fit results

(PRD 88, 021701, 2013)

$U_c = 0.1560(4)$

$\nu = 0.82(2)$

$\eta = 0.62(2)$

$\eta_\psi = 0.37(1)$

SU(2) x Z_2 Gross-Neveu model results

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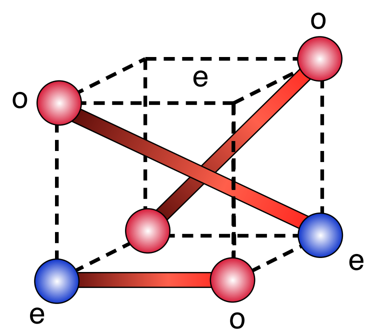

Z_2 Gross-Neveu

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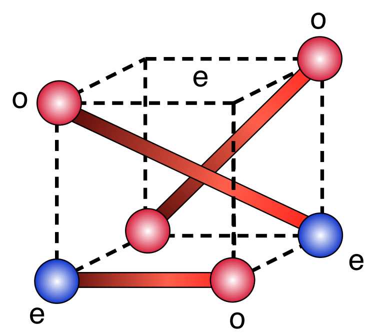

Z_2 Gross-Neveu

Combined fit results

(PRD 88, 021701, 2013)

$U_c = 0.0893(1)$

$\nu = 0.83(1)$

$\eta = 0.62(1)$

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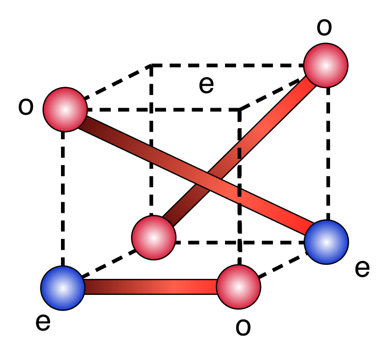

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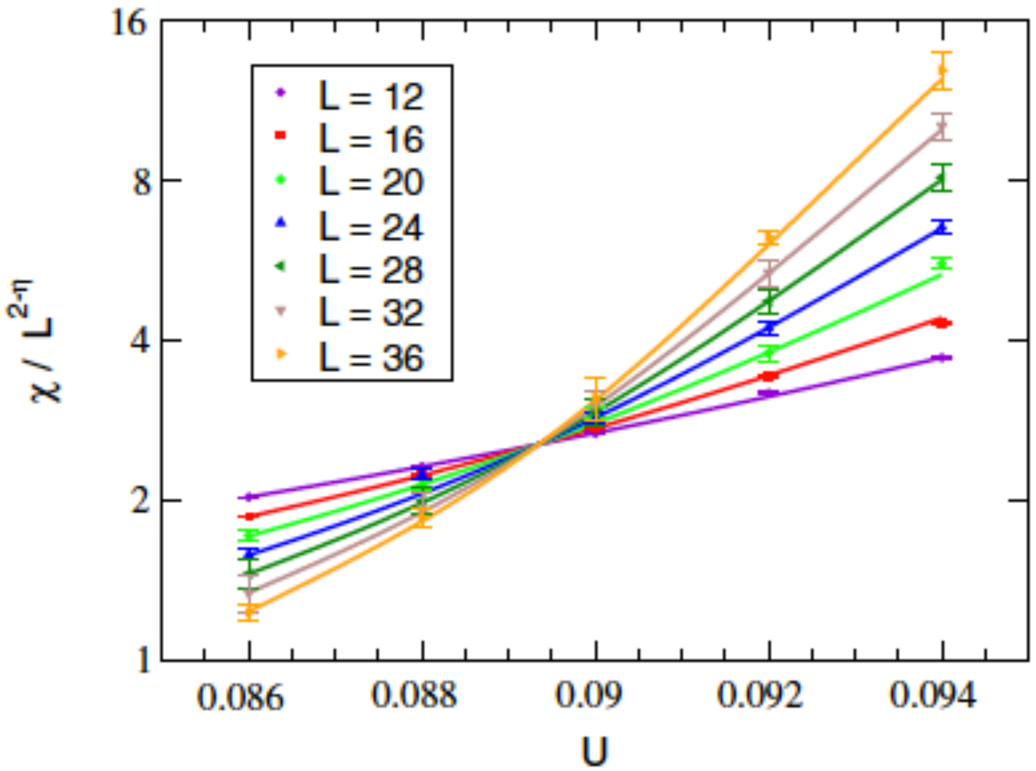

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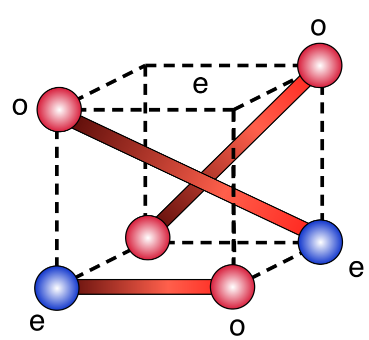

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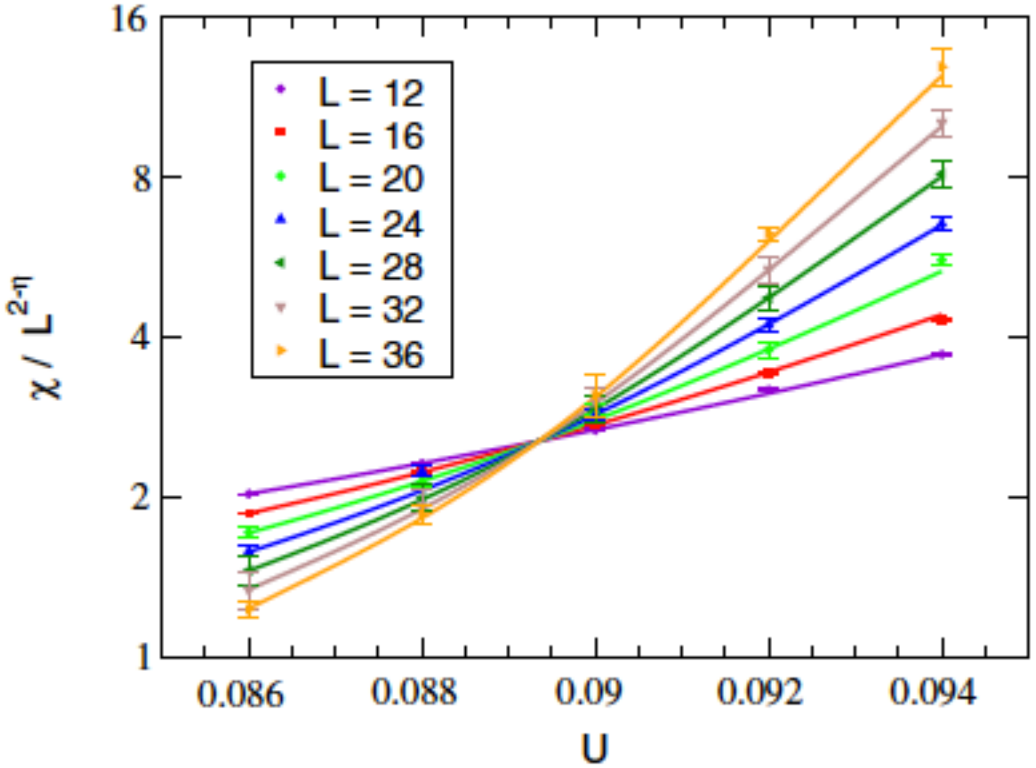

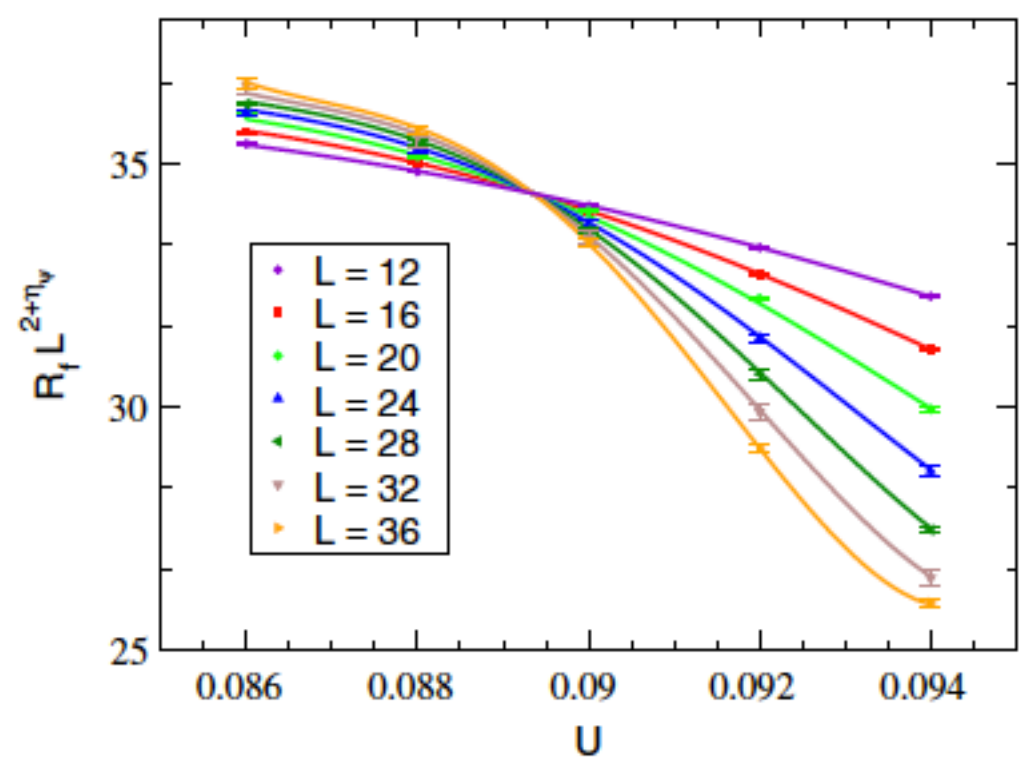

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Comparison with Fermion Bag Results

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Staggered Fermion Model	Symmetry	Work	ν	η	η_ψ
N=1 Lattice-GN	SU(2) x Z_2	Karkkainen,et.al. (1994)	1.00(4)	0.756(8)	-
N=1 Lattice GN	SU(2) x Z_2	SC & Li (2012)	0.83(1)	0.62(1)	0.38(1)
N = 1 Lattice-Th	SU(2)x U(1)	Debbio, et.al., (1997)	0.80(15)	0.70(15)	-
N = 1 Lattice-Th	SU(2)x U(1)	Barbour et. al., (1998)	0.80(20)	0.4(2)	-
N=1 Lattice-(GN/Th)	SU(2) x U(1)	SC & Li (2013)	0.849(8)	0.633(8)	0.373(3)

Backup Slide
Bosonic k-point correlation function
in the non-linear xy model

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Bosonic k-point correlation function in the non-linear xy model

Using the identity

$$\mathrm{e}^{\beta\cos(\theta_{\mathbf{x}}-\theta_{\mathbf{x}+\alpha})} = \sum_{\mathbf{k}_{\mathbf{x},\alpha}} \mathbf{I}_{\mathbf{k}_{\mathbf{x},\alpha}}(\beta)\mathrm{e}^{\mathbf{i}(\theta_{\mathbf{x}}-\theta_{\mathbf{x}+\alpha})}$$

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can show **D.Banerjee, S.C PRD(2010)**

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$$\int [\mathbf{d}\theta] \Bigg(\prod_{\langle \mathbf{x},\alpha \rangle} e^{\beta \cos(\theta_{\mathbf{x}} - \theta_{\mathbf{x}+\alpha})} e^{\mathbf{i}\varepsilon_{\mathbf{z_1}} \theta_{\mathbf{z_1}}} e^{\mathbf{i}\varepsilon_{\mathbf{z_2}} \theta_{\mathbf{z_2}}} \ldots e^{\mathbf{i}\varepsilon_{\mathbf{z_k}} \theta_{\mathbf{z_k}}} \Bigg)$$

$$= \sum_{[\mathbf{k}]} \Bigg(\prod_{\langle \mathbf{x},\alpha \rangle} \mathbf{I}_{\mathbf{k}_{\mathbf{x},\alpha}} \Bigg) \left\{ \prod_{\mathbf{x}} \delta\Big(\varepsilon_{\mathbf{x}} \mathbf{n}_{\mathbf{x}} + \sum_{\alpha} (\mathbf{k}_{\mathbf{x},\alpha} - \mathbf{k}_{\mathbf{x}-\alpha,\alpha}) \Big) \right\}$$

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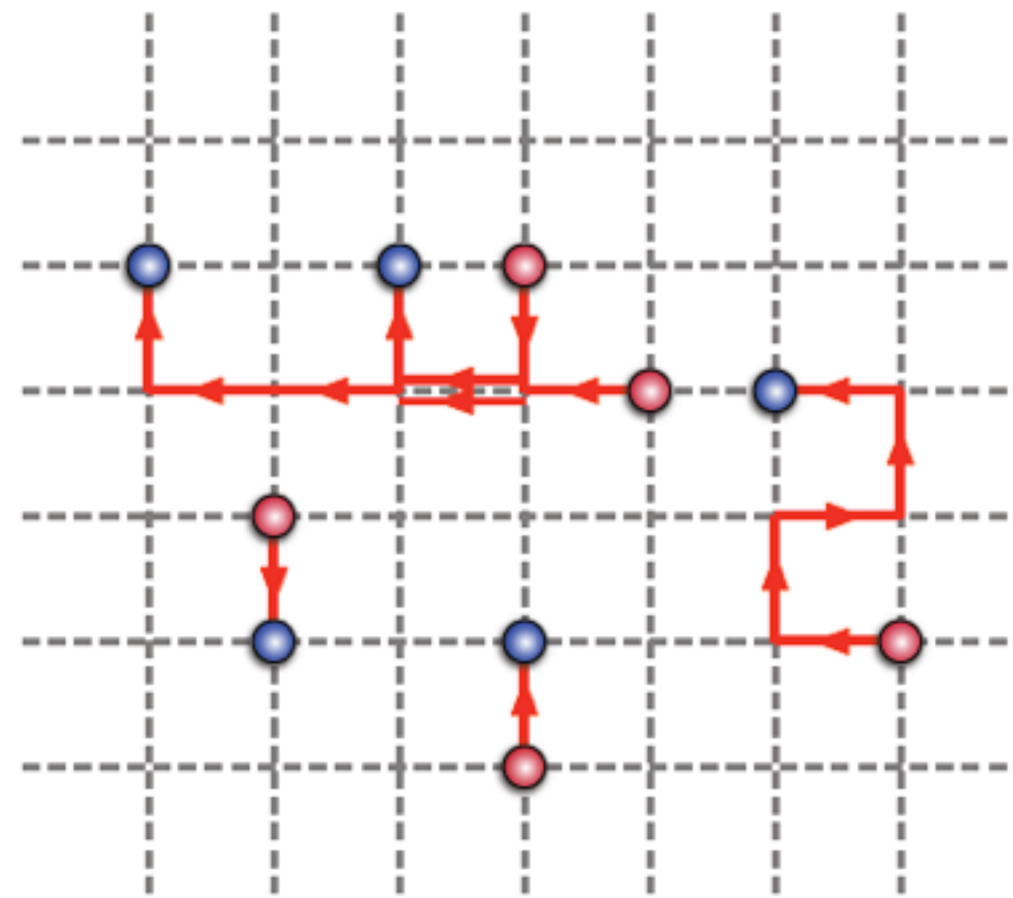

example of a [k] configuration consistent with [n]