Deconfinement transition as black hole formation by the condensation of QCD strings

(and toward real-time simulation)

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M.H.-Maltz-Susskind, 1405.****[hep-th] tonight 8pm

Berkowitz-M.H.-Hayden-Susskind, in progress

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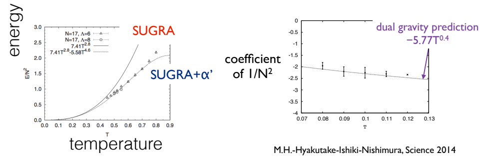
gauge/gravity duality claims deconfinement phase = Black hole

Quantitative test: D0-brane matrix model vs. black 0-brane



looks correct, even at quantum gravity level.

…. but why??

Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

$$H=\frac{\lambda N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}(E^\alpha_{\mu,\vec{x}})^2+\frac{N}{\lambda}\sum_{\vec{x}}\sum_{\mu<\nu}\left(N-\mathrm{Tr}(U_{\mu,\vec{x}}U_{\nu,\vec{x}+\hat{\mu}}U^\dagger_{\mu,\vec{x}+\hat{\nu}}U^\dagger_{\nu,\vec{x}})\right)$$

$$[E^\alpha_{\mu,\vec{x}},U_{\nu,\vec{y}}]=\delta_{\mu\nu}\delta_{\vec{x}\vec{y}}\cdot\tau^\alpha U_{\nu,\vec{y}}$$

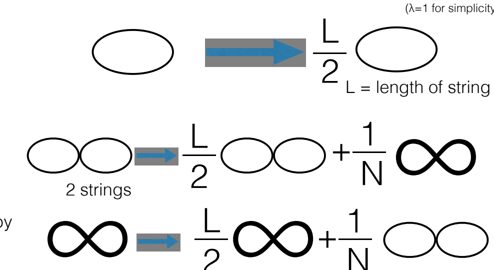
$$\sum_{\alpha=1}^{N^2}\tau^\alpha_{ij}\tau^\alpha_{kl}=\frac{\delta_{il}\delta_{jk}}{N^2}$$

Hilbert space is expressed by Wilson loops.

strong coupling limit

$$H=\frac{N}{2}\sum_{\vec{x}}\sum_{\mu}\sum_{\alpha=1}^{N^2}(E^\alpha_{\mu,\vec{x}})^2$$

(λ=1 for simplicity)



- interaction (joining/splitting) is 1/N-suppressed

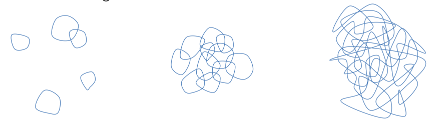
"large-N limit is the theory of free string"

- It is true when L is $O(N^0)$. (→confining phase)
- In deconfinement phase, total length of the strings is $O(N^2)$ → number of intersections is $O(N^2)$→interaction is **not** negligible

large-N limit is still very dynamical!

confining phase = gas of short strings

long and winding string, which is interpreted as BH, appears



as the density of strings increase, interaction between strings becomes important,and…

D-dim square lattice at string coupling

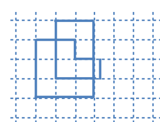
deconfining phase = long string

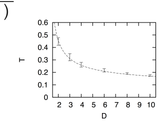
huge mass and entropy are packed in a small region → BH

E = L(T)/2, S = L(T)×log(2D−1)

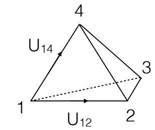
F = E−TS = L(T)×(1/2 - T×log(2D−1))

$$T_c=\frac{1}{2\log(2D-1)}$$

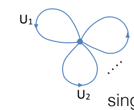




matrix models at string coupling



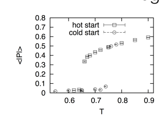
tetrahedron $T_c=\frac{1}{2\log 2}$=0.72…

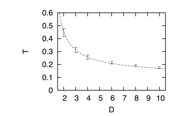


$T_c=\frac{1}{2\log(2D-1)}$

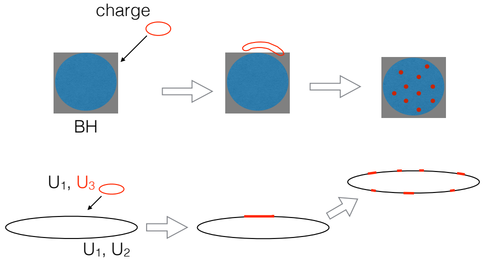
single-site with D-links (Eguchi-Kawai model)

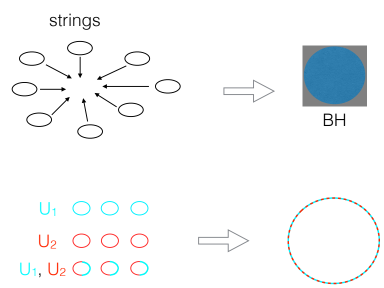
(Equivalent to large-volume lattice via Eguchi-Kawai equivalence)





Real-time study for BH thermalization





thermalization= formation of *generic* string

- (# of U_i) − (# of $U_i^\dagger$) is fixed.
- In the strong coupling limit, number of link variables does not increase. The Hilbert space can be truncated to finite dimensions.
- When there is no $U_i^\dagger$ in the initial condition, simply the combinatorics of fixed numbers of $U_1,\dots,U_d$.

time evolution can be studied!

the simplest case: 1-link model

$\vec{v}_1$=(Tr U^L) → 1-string state

$\vec{v}_2$=(Tr U)(Tr U^{L-1})
$\vec{v}_3$=(Tr U^2)(Tr U^{L-2})
$\vec{v}_4$=(Tr U^3)(Tr U^{L-3})
} → 2-string states

⋮

$\vec{v}_d$=(Tr U)^L → L-string state

d = # of string states with total length L

$H\vec{e}_i=E_i\vec{e}_i$ $\vec{e}_i=c_{ij}\vec{v}_j$

eigenstate ~ "thermal state"

$H\vec{e}_i=E_i\vec{e}_i$

$\vec{e}_i=c_{ij}\vec{v}_j$

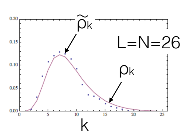
$$\tilde{\rho}_k=\sum_{j\,=\,(k\text{-string state})}|c_{ij}|^2$$

probability that k-string state is observed

$$E_i=\frac{L}{2}+\frac{2}{N}\times(\text{integer})$$

"thermal distribution"

$$\rho_k=\frac{\#\text{ of k-string states}}{\#\text{ of all states }(=d)}$$



- generic state is indistinguishable from "thermal state"
- Does it satisfy Sekino-Susskind's fast scrambling bound?