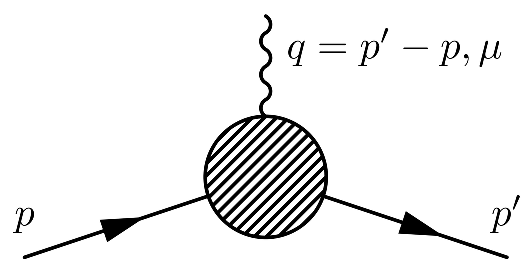
Lattice Calculation of the Hadronic Light by Light Contributions to the Muon Anomalous Magnetic Moment

Luchang Jin (Columbia University)

Tom Blum (UConn) Norman Christ (Columbia) Masahi Hayakawa (Nagoya) Taku Izubuchi (BNL/RBRC) Eigo Shintani (Mainz) Norikazu Yamada (KEK)

Muon Anomalous Magnetic Moment

$$\boldsymbol{\mu}_\mu = -g_\mu \frac{e}{2m_\mu}\mathbf{s}_\mu$$



$$= \bar{u}(p')\left[F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{4m}[\gamma_\mu, \gamma_\nu]q_\nu\right]u(p)$$

Figure : Muon Vertex Function Diagram

$$F_2(0) = \frac{g_\mu - 2}{2} \equiv a_\mu$$

BNL E821 (0.54 ppm) and Standard Model Prediction

Contribution	Value ± Error	Reference
HVP LO	6949 ± 43	Hagiwara et al. 2011
Hadronic Light by Light	105 ± 26	Glasgow Consensus, 2007
Standard Model	116591828 ± 50	
Experiment (0.54 ppm)	116592089 ± 63	E821,The $g-2$ Collab. 2006
Difference (Exp − SM)	261 ± 78	

Table : Standard model theory and experiment comparison [in units 10^{-11}]

There is a 3.3σ deviation between current measurement and Standard Model prediction.

Future Fermilab E989 (0.14 ppm)

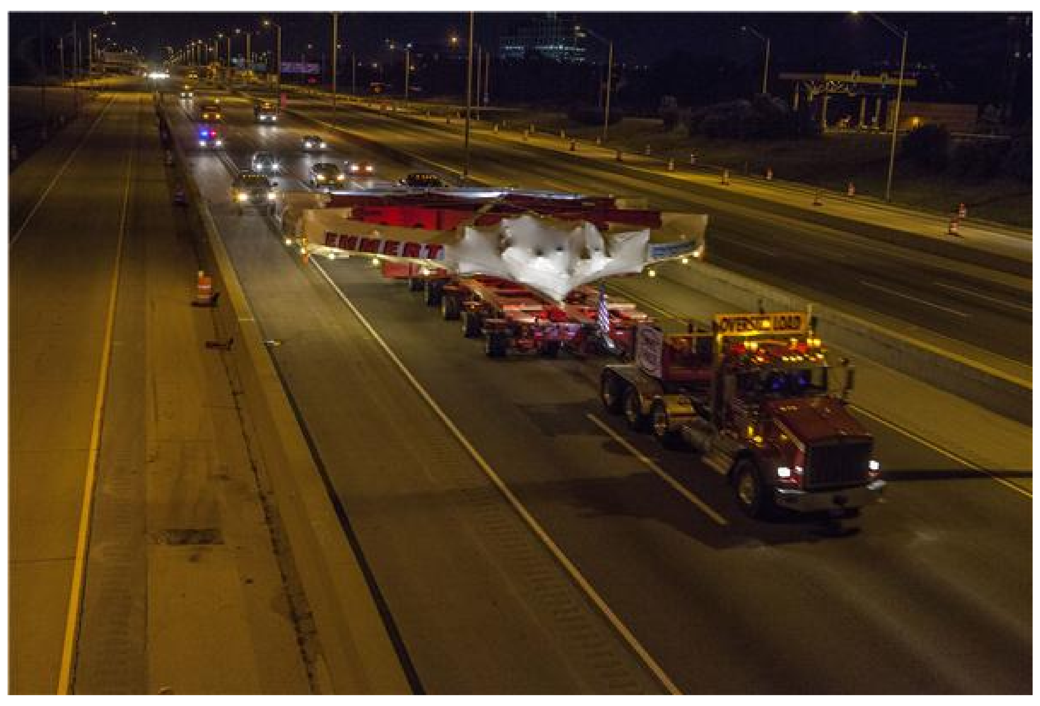


Figure : The 50-foot-wide Muon g-2 electromagnet being driven north on I-355 between Lemont and Downers Grove, Illinois, shortly after midnight on Thursday, July 25, 2013. *Credit: Fermilab.*

Lattice QED - Schwinger Term

$$\mathcal{M}_\mu^{\text{1-loop}} = (-ie)^2 \sum_{x,x'} S(x_{\text{src}}; x)\gamma_\nu S(x; x_{\text{op}})\gamma_\mu S(x_{\text{op}}; x')\gamma_{\nu'} S(x'; x_{\text{snk}}) G_{\nu\nu'}(x; x')$$

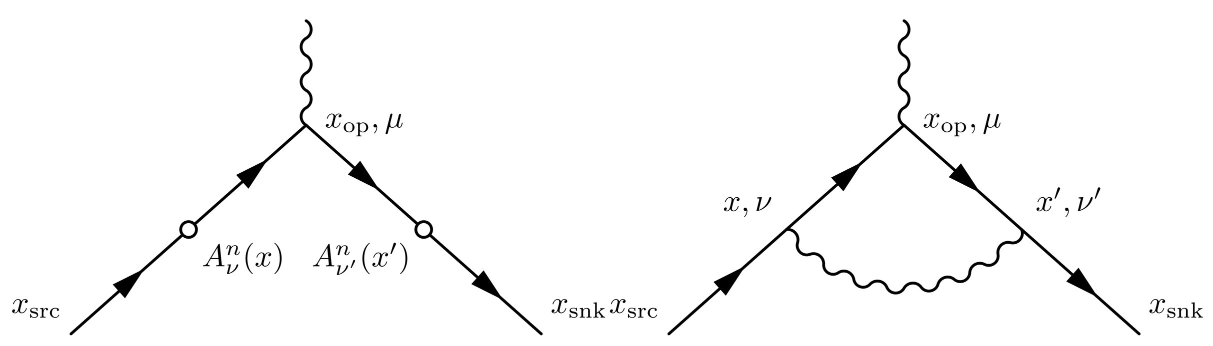


Figure : Schwinger term diagram calculated with (L) stochastic photon (R) exact photon.

Lattice QED - Stochastic Photon

$$G_{\mu\nu}(x; y) \approx \frac{1}{N}\sum_{n=1}^{N} A^n_\nu(x) A^n_{\nu'}(y)$$

$$A^n_\mu(x) = \frac{1}{\sqrt{V}}\sqrt{2}\,\text{Re}\sum_k \frac{\epsilon^n_\mu(k)}{\sqrt{|k^2|}} e^{ik\cdot x}$$

$$\frac{1}{N}\sum_{n=1}^{N} \epsilon^n_\mu(k)\epsilon^{n*}_\nu(k') \approx \delta_{\mu\nu}\delta_{kk'}\ , \ \frac{1}{N}\sum_{n=1}^{N} \epsilon^n_\mu(k)\epsilon^n_\nu(k') \approx 0$$

$$\mathcal{M}_\mu^{\text{1-loop}} = \frac{-e^2}{N}\sum_{n=1}^{N}\left[\sum_x S(x_{\text{src}}; x)\gamma_\nu A^n_\nu(x) S(x; x_{\text{op}})\right]\gamma_\mu\left[\sum_{x'} S(x_{\text{op}}; x)\gamma_{\nu'} A^n_{\nu'}(x') S(x'; x_{\text{snk}})\right]$$

Lattice QED - Exact Photon

$$G_{\mu\nu}(x; y) = \frac{1}{V}\sum_k \frac{\delta_{\mu\nu}}{k^2} e^{ik\cdot(x-y)}$$

$$\mathcal{M}_\mu^{\text{1-loop}} = \frac{-e^2}{V}\sum_k \frac{\delta_{\nu\nu'}}{k^2}\left[\sum_x S(x_{\text{src}}; x)\gamma_\nu e^{ik\cdot x} S(x; x_{\text{op}})\right]\gamma_\mu\left[\sum_{x'} S(x_{\text{op}}; x')\gamma_{\nu'} e^{-ik\cdot x'} S(x'; x_{\text{snk}})\right]$$

Finite Volume Effect and Discretization Errors in Schwinger Term

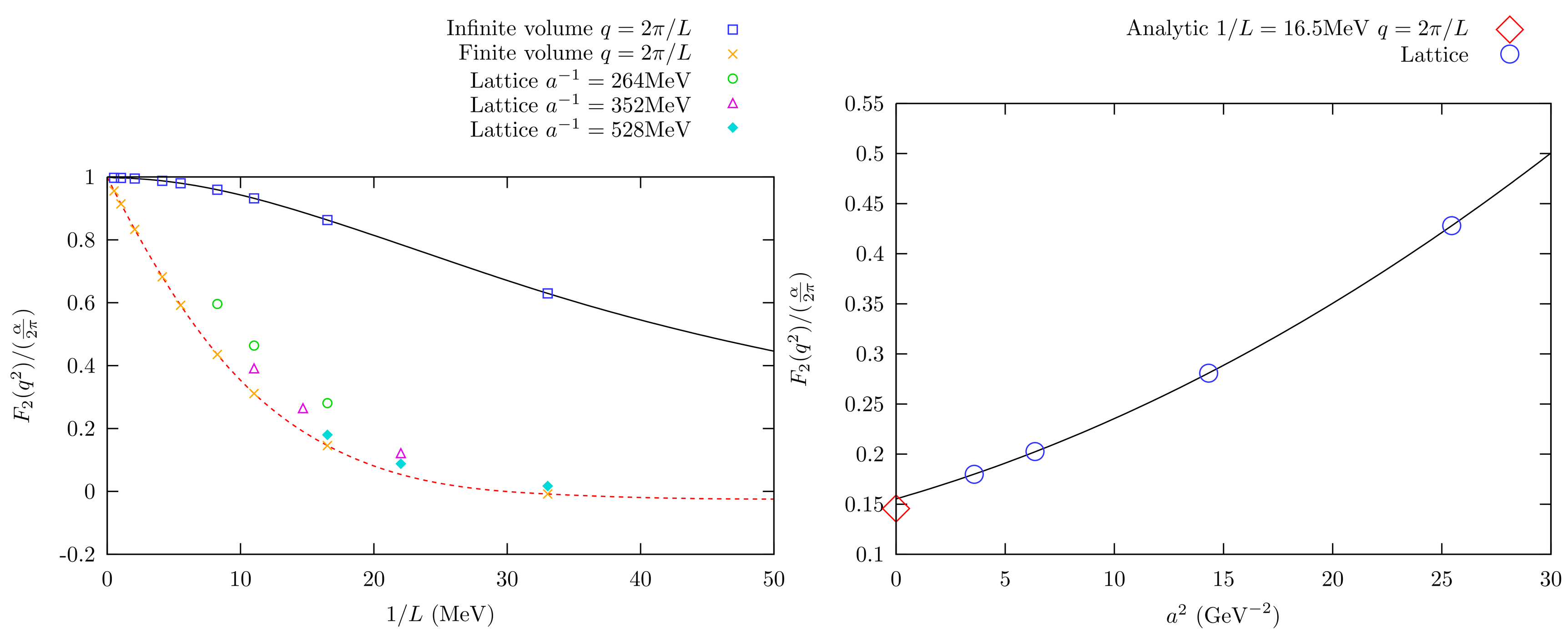


Figure : $L_s = 8$ and $t_{\text{snk}} - t_{\text{op}} = t_{\text{op}} - t_{\text{src}} = T/4$. External photon momentum transfer is $q = 2\pi/L$. a is the lattice spacing. (L) Finite volume effect on F_2: Lattice sizes are $16^3 \times 64$, $24^3 \times 96$, $32^3 \times 128$. The dashed line is the analytic result in finite volume and the solid line is the analytic result in infinite volume but same non-zero momentum transfer. (R) Discretization errors for F_2: Lattice sizes are $12^3 \times 48$, $16^3 \times 64$, $24^3 \times 96$, $32^3 \times 128$. The line is 2nd order polynomial obtained by fitting the results from lattice calculations. An a^4 term is visable.

Light by Light Evaluation Strategy

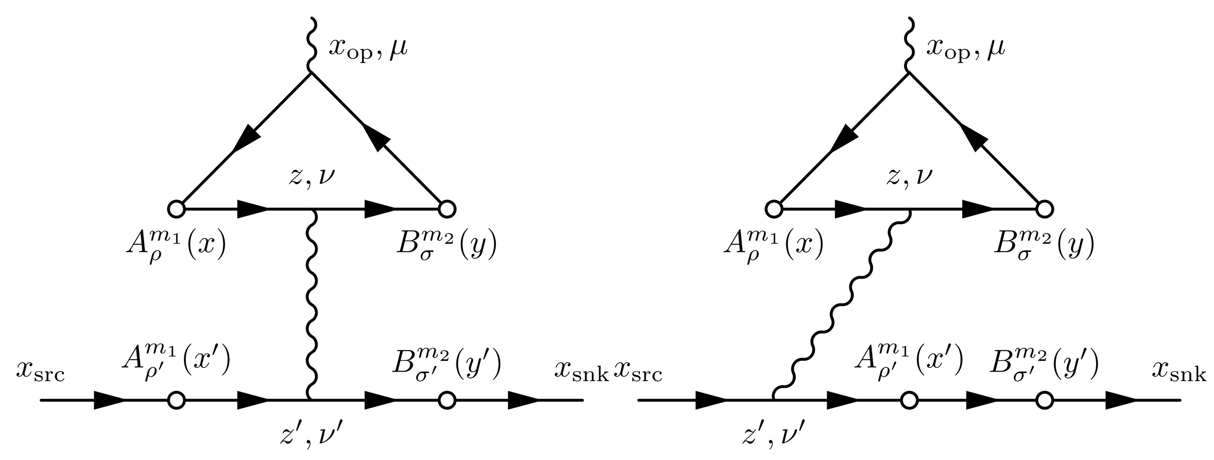


Figure : Light by Light diagrams calculated with one exact photon and two stochastic photon. There are 4 other possible permutations.

Leptonic Light by Light on Lattice

Analytic (loop mass equals the muon mass): $0.371 \times \left(\frac{\alpha}{\pi}\right)^3$

Lattice Size	m_μ	$\frac{\text{Result} \pm \text{Err}}{(\alpha/\pi)^3}$	$N \times S \times M^2$ confs	$\frac{\text{Var}}{(\alpha/\pi)^3}$
$4^3 \times 32$	0.4	0.402 ± 0.082	$1000 \times 1 \times 1^2$	2.6
		0.295 ± 0.011	$5457 \times 1 \times 3^2$	2.4
		0.293 ± 0.010	$365 \times 1 \times 12^2$	2.3
		0.298 ± 0.003	$787 \times 12 \times 12^2$	2.9
$8^3 \times 64$	0.2	0.204 ± 0.044	$1581 \times 1 \times 3^2$	5.2
$8^3 \times 64$	0.4	0.206 ± 0.012	$643 \times 12 \times 12^2$	12.6
$16^3 \times 64$	0.4	0.183 ± 0.173	$326 \times 12 \times 6^2$	64.9

Figure : M stands for the number of stochastic A, B fields, S stands for the number of point sources x_{op} that we use to calculate the external current. The calculation is repeated N times. $\text{Var} = \text{Err} \times \sqrt{N \times S \times M^2}$ stands for the projected variance according to the uncertainty of the result and the total number of confs.

Hadronic Light by Light on Lattice

The Glasgow consensus value: $(0.084 \pm 0.021) \times \left(\frac{\alpha}{\pi}\right)^3$

More realistic ensemble (RBC/UKQC DWF)

- Larger lattice size, 24^3 ($(2.7\text{ fm})^3$)
- Pion mass is smaller too, $m_\pi = 329$ MeV
- Same muon mass (190 MeV)
- $0.11 \lesssim Q^2 \lesssim 0.31\ \text{GeV}^2$
- Use All Mode Averaging (AMA)
  - 6^3 (5^3) point sources/configuration = 216 (125)
  - AMA approximation: "sloppy CG", $r_{\text{stop}} = 10^{-4}$

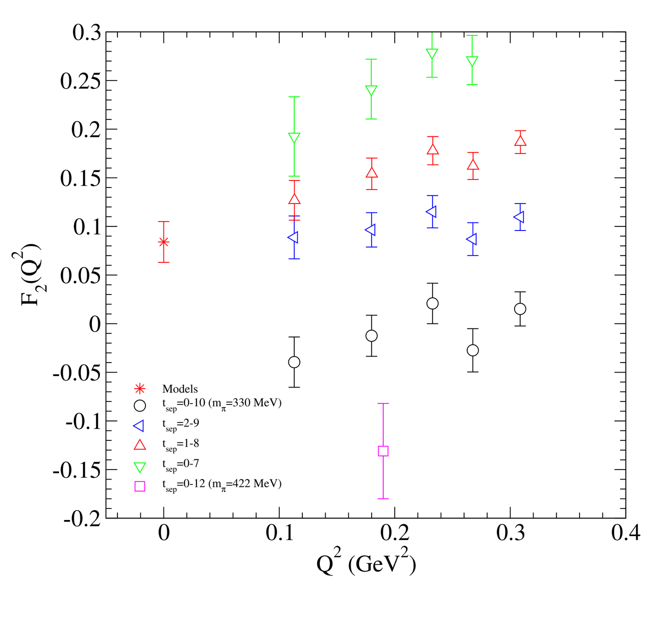


Figure : Results presented on March 3rd 2014 at Fermilab by Tom Blum.