

Quantum mechanics and the geometry of spacetime

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Outline

- Brief review of the gauge/gravity duality
- Role of strong coupling in the emergence of the interior
- Role of entanglement in the shape of the geometry. Wormholes and entanglement.
- Vague similarity between tensor networks and spacetime geometry.

Gauge/Gravity Duality

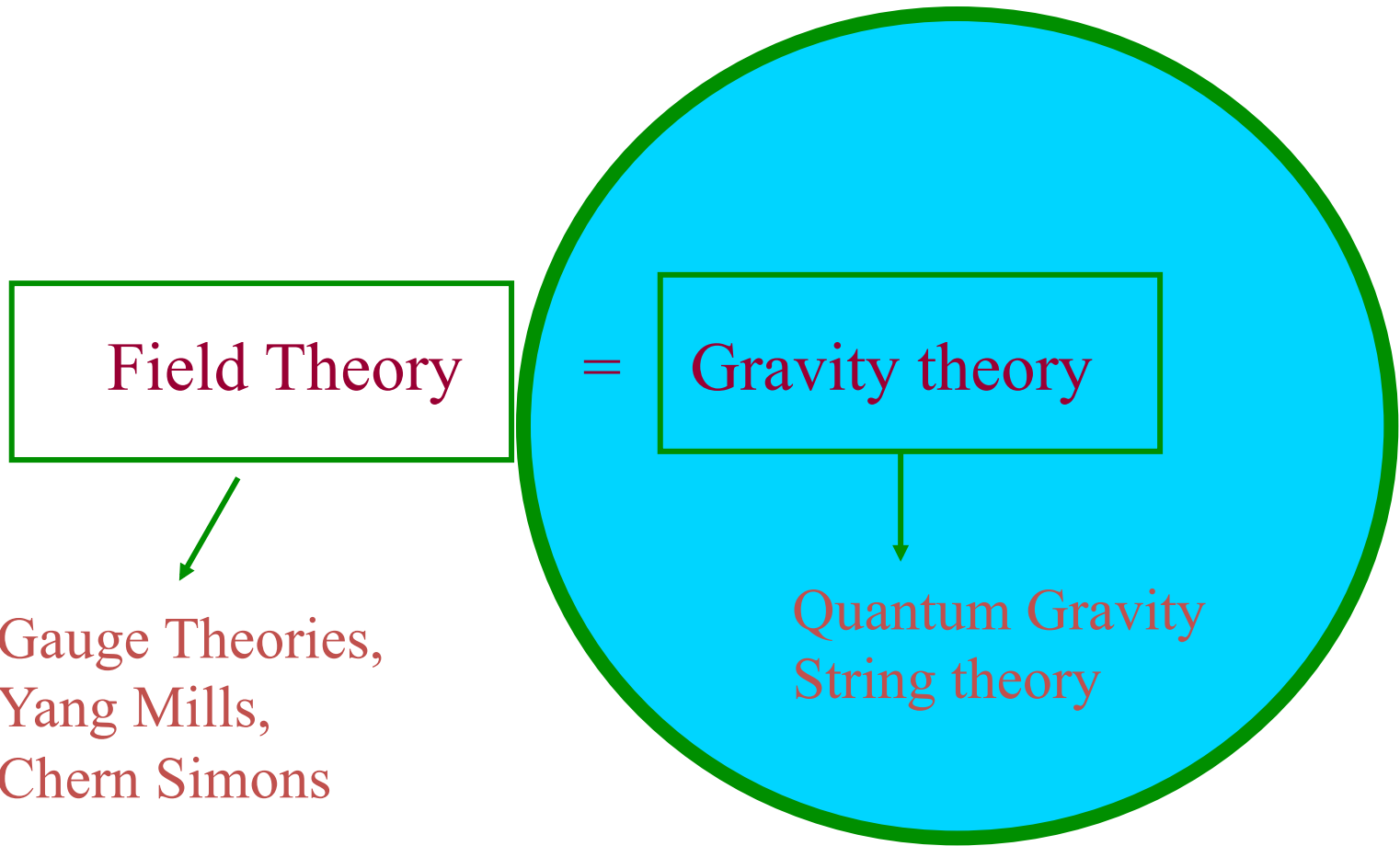
(or gauge/string duality, AdS/CFT, holography)

Quantum Field
Theory

Theories of quantum
interacting particles



Dynamical
Space-time
(General relativity)
string theory



Example

- Maximally supersymmetric SU(N) Yang Mills theory at large N. (Four dimensional gauge theory with some scalars and fermion fields, adjusted to that it has the maximum amount of supersymmetry).

SU(N) Super Yang Mills

=

string theory
on AdS₅ x S⁵

JM

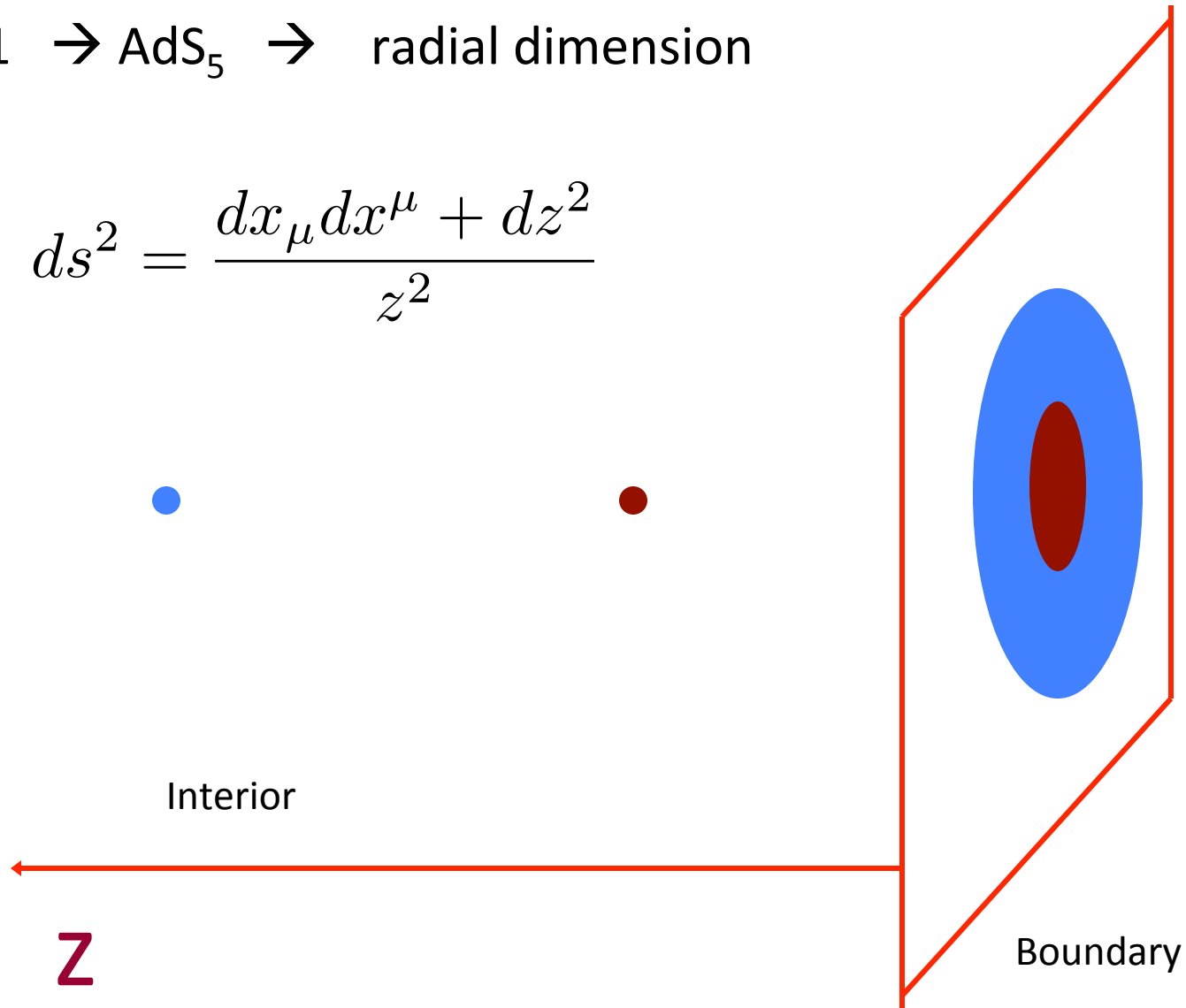
$\frac{R}{l_s}$ increases as the coupling constant of the gauge theory increase

Strong gauge theory coupling \rightarrow large radius space = general relativity description

The extra dimension

3+1 \rightarrow AdS₅ \rightarrow radial dimension

$$ds^2 = \frac{dx_\mu dx^\mu + dz^2}{z^2}$$

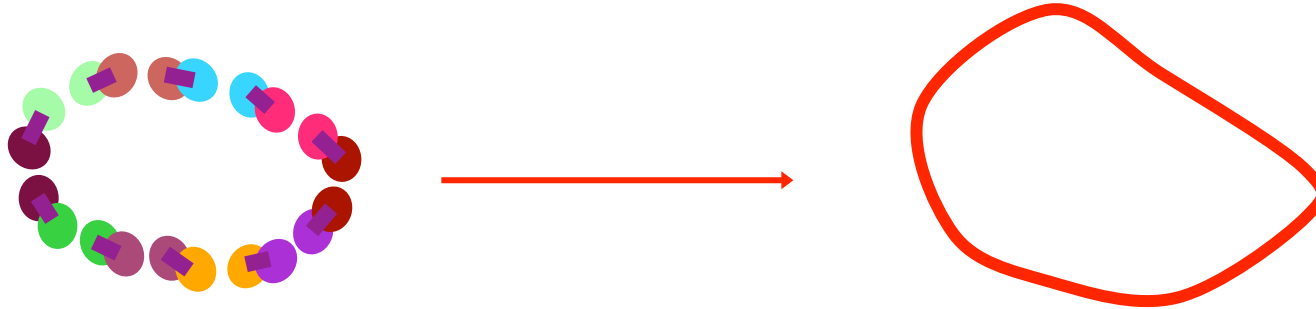


Large N

- Large number of colors \rightarrow semiclassical geometry = small G_N

$$G_N \sim \frac{1}{N^2}$$

Gluon chains \rightarrow strings



String Interactions

$$\sim \frac{1}{N} \sim \sqrt{G_N}$$

Effective gluon interactions = $g^2 N$ = size of quantum effects in the boundary field theory.

$$(g^2 N)^p \sim \frac{\text{Radius of curvature of space}}{\text{Size of graviton}}$$

Effective gauge theory coupling

Large $N \rightarrow$ weakly coupled string theory in the bulk

Ordinary Einstein gravity theory \rightarrow Strongly coupled field theory

The emergence of the bulk spacetime could be understood completely within the planar approximation (weakly coupled string theory, large N).

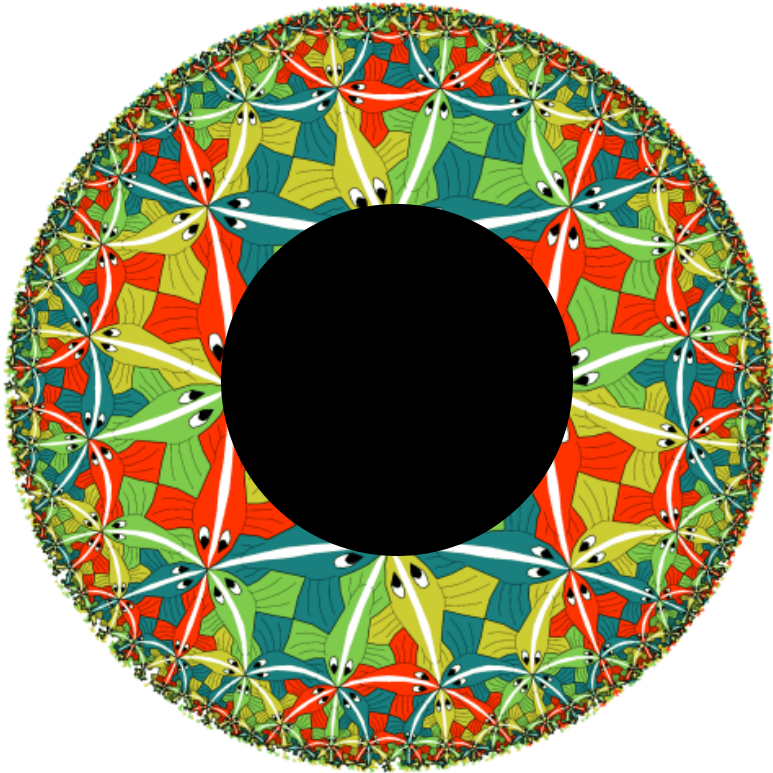
We need to tackle strongly coupled systems:

Approaches

- 1) Integrability: Use very special symmetries of maximally supersymmetric Yang Mills
 \rightarrow Anomalous dimensions, amplitudes, etc. Review: see Beisert et al..
- 2) Use supersymmetry to compute special quantities. (Sphere partition functions,
Wilson loops) Pestun, Kapustin, Jacobs, Willet, etc.
- 3) Computer simulations (matrix quantum mechanics) Hanada et al
- 4) Assume the duality and use it to gain intuition about strongly coupled systems (AdS/QCD,
AdS/CMT)

Black holes in AdS

Thermal configurations in AdS.



Entropy:

$$S_{\text{GRAVITY}} = \text{Area of the horizon} = \\ S_{\text{FIELD THEORY}} = \\ \text{Log[Number of states]}$$

Strominger-Vafa

Evolution: Unitary

Long distance fluctuations of the horizon \rightarrow hydrodynamics

Interior = ?

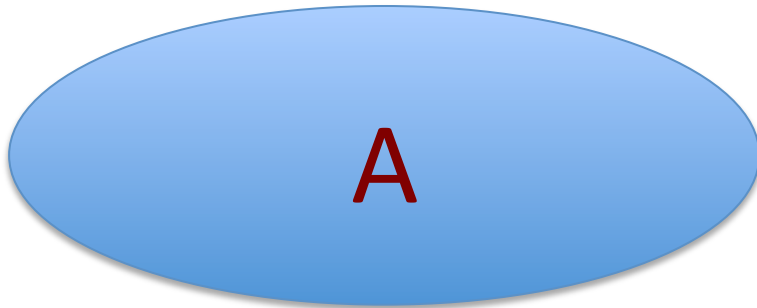
Conclusions

- Numerics can give interesting checks of the duality.
- It could help us understand better the emergence of a local theory in the bulk.
- The duality could also serve as an interesting test case for new numerical techniques.

Entanglement and the gauge/gravity duality

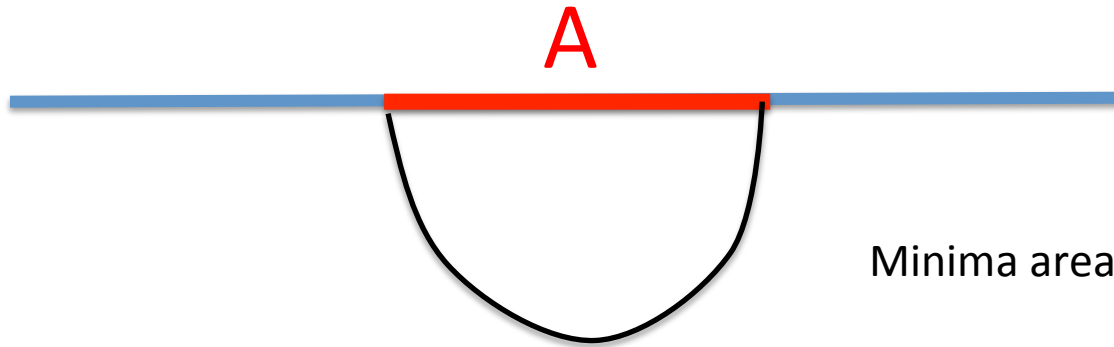
Entanglement in QFT

- In QFT the vacuum is a very entangled state of the fundamental degrees of freedom.
- Most of this entanglement is short range.



$$S_A = \frac{\text{Area}}{\epsilon^2} + \dots$$

- There is interesting QFT information in the dots (c, f theorems)



Minima area surface in the bulk

Ryu-Takayanagi
(Hubeny, Rangamani ..)

Fursaev, Headrick,
Lewkowycz, JM

$$S_A = \frac{A_{\text{minimal}}}{4G_N}$$

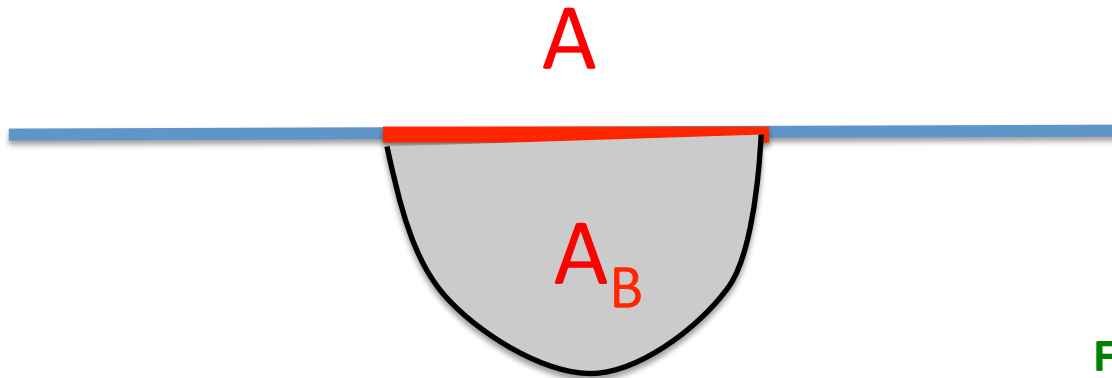
Leading order in G_N expansion

This a generalization of the Bekenstein-Hawking formula for black hole entropy

Quantum correction

$$S_A = \frac{\text{Area} + \alpha' \text{ corrections}}{4G_N} + S_q$$

$$S_q = S_{\text{Bulk entanglement}} + \dots$$



Faulkner, Lewkowycz, JM

Define a bulk region A_B , inside the minimal surface.

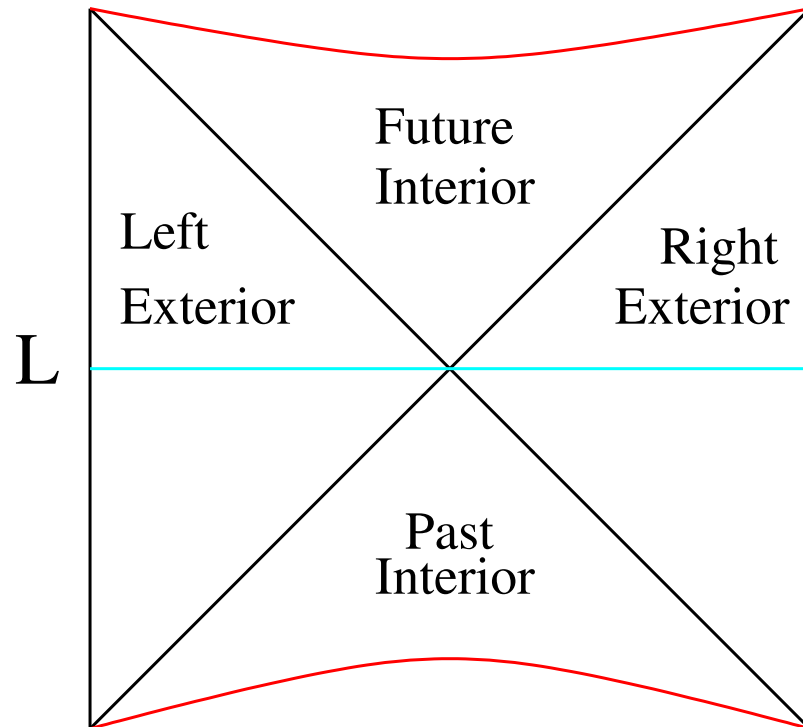
Compute the entanglement of the bulk quantum fields between A_B and the rest of the spacetime.

Some lessons

- Entanglement is computed by a geometric construction.
- But many other observables are also computed by the bulk geometry.
- We think that the bulk geometry reflects more directly the patterns of entanglement of the gauge theory.

A more drastic example

Eternal AdS black hole

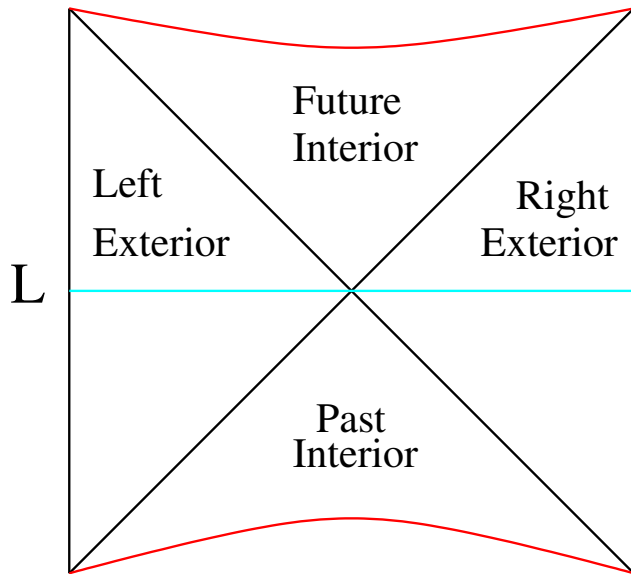


R Entangled state in two non-interacting CFT's.

Israel
JM

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

Eternal AdS black hole



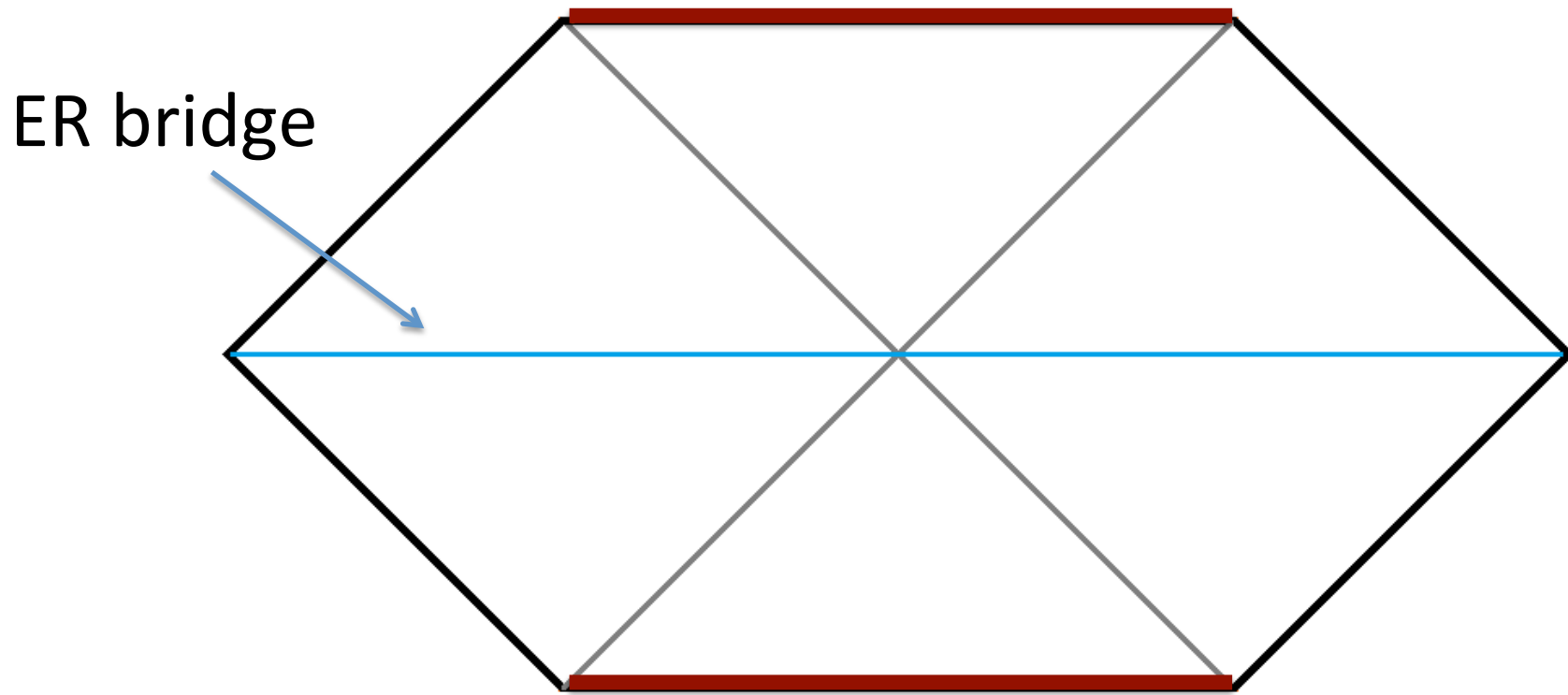
$$R \quad \frac{\text{Area}}{4G_N} = \text{Entanglement entropy}$$

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

- We generate a spatial connection despite the fact that the two quantum field theories are not interacting in any way.
- Just entanglement (of the right type) has created a geometric connection.

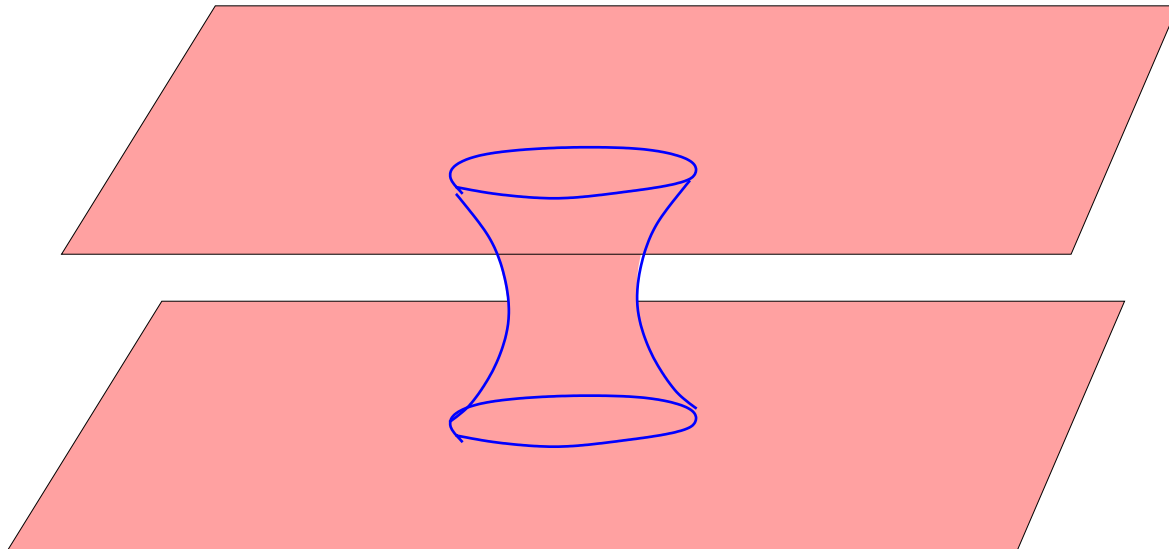
Wormholes

Eternal black hole

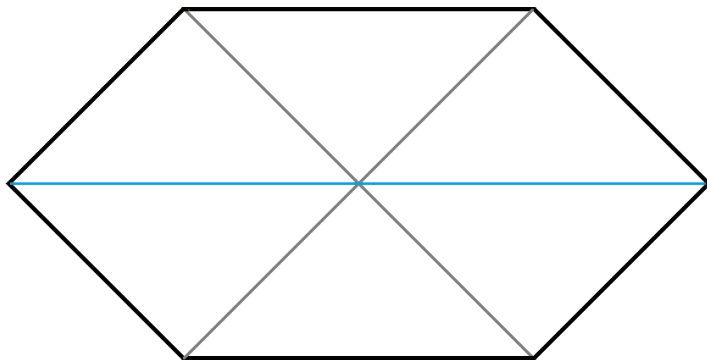
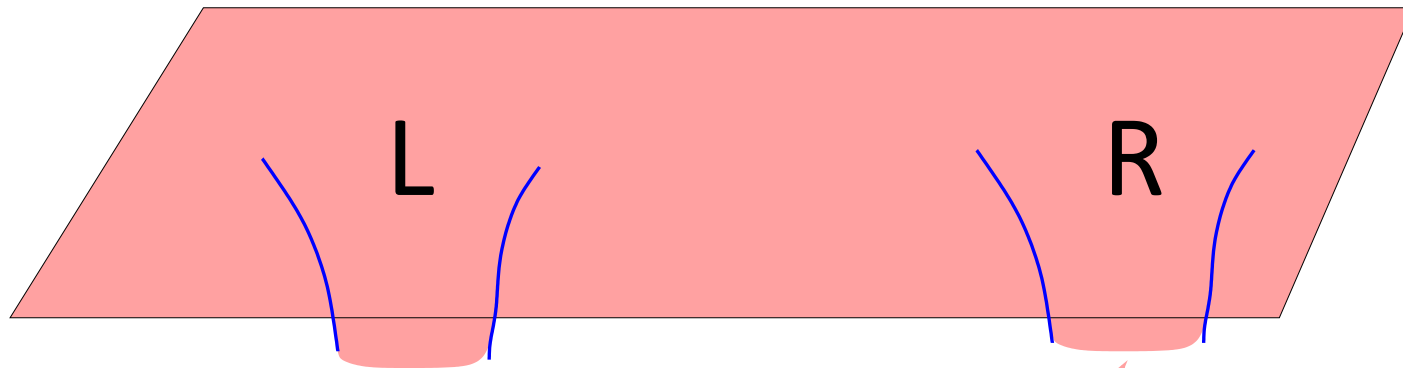


Maximally extended Schwarzschild geometry

Einstein-Rosen bridge



Wormhole interpretation.



Non travesable

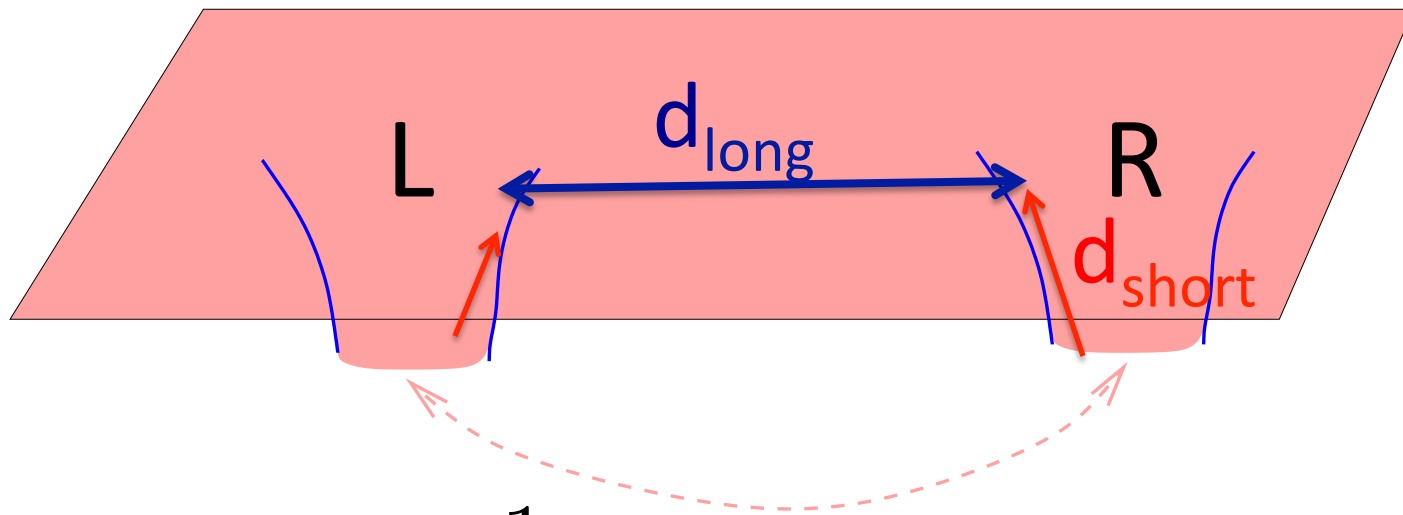
No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

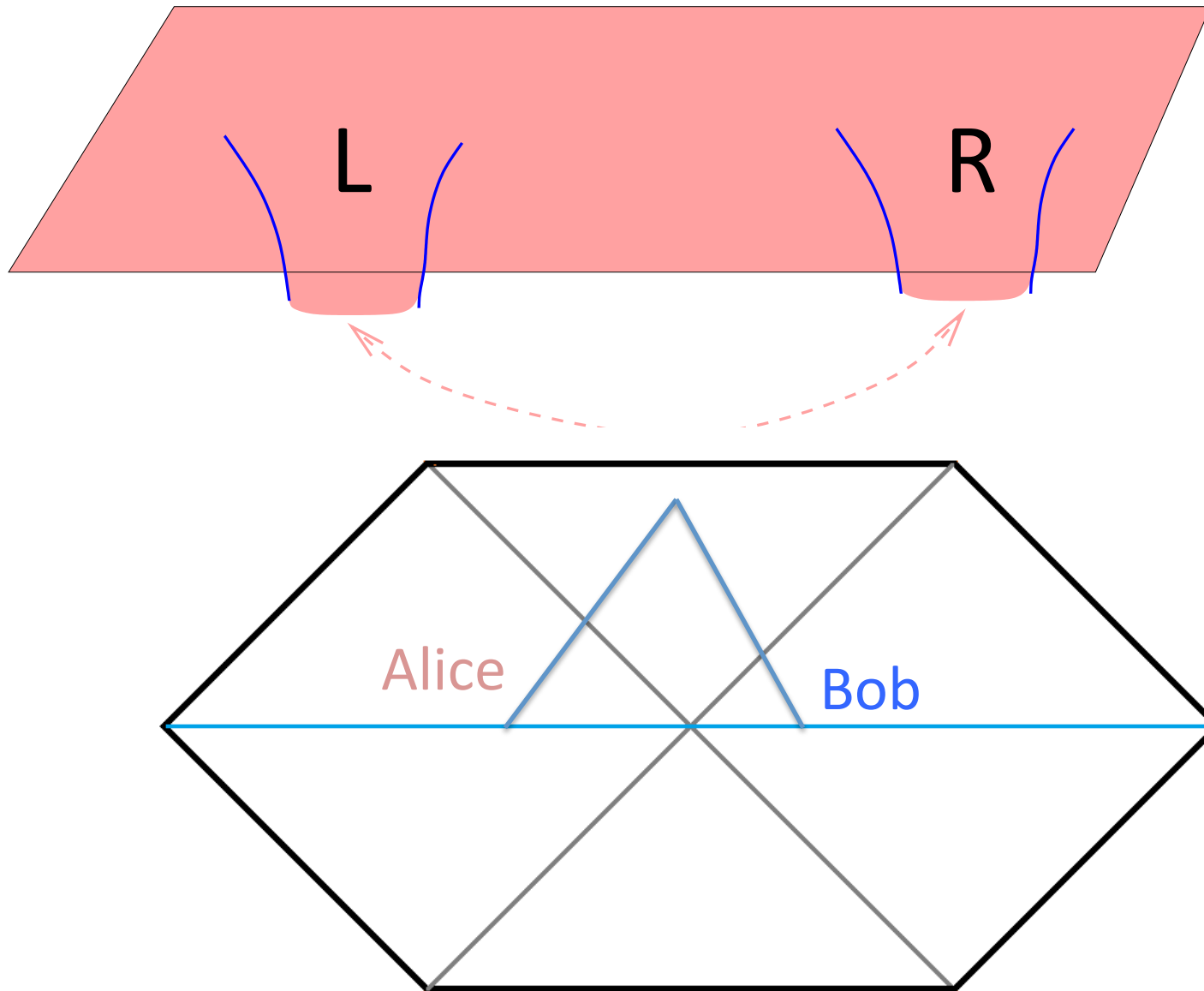
ER = EPR

Entangled black holes in the same spacetime

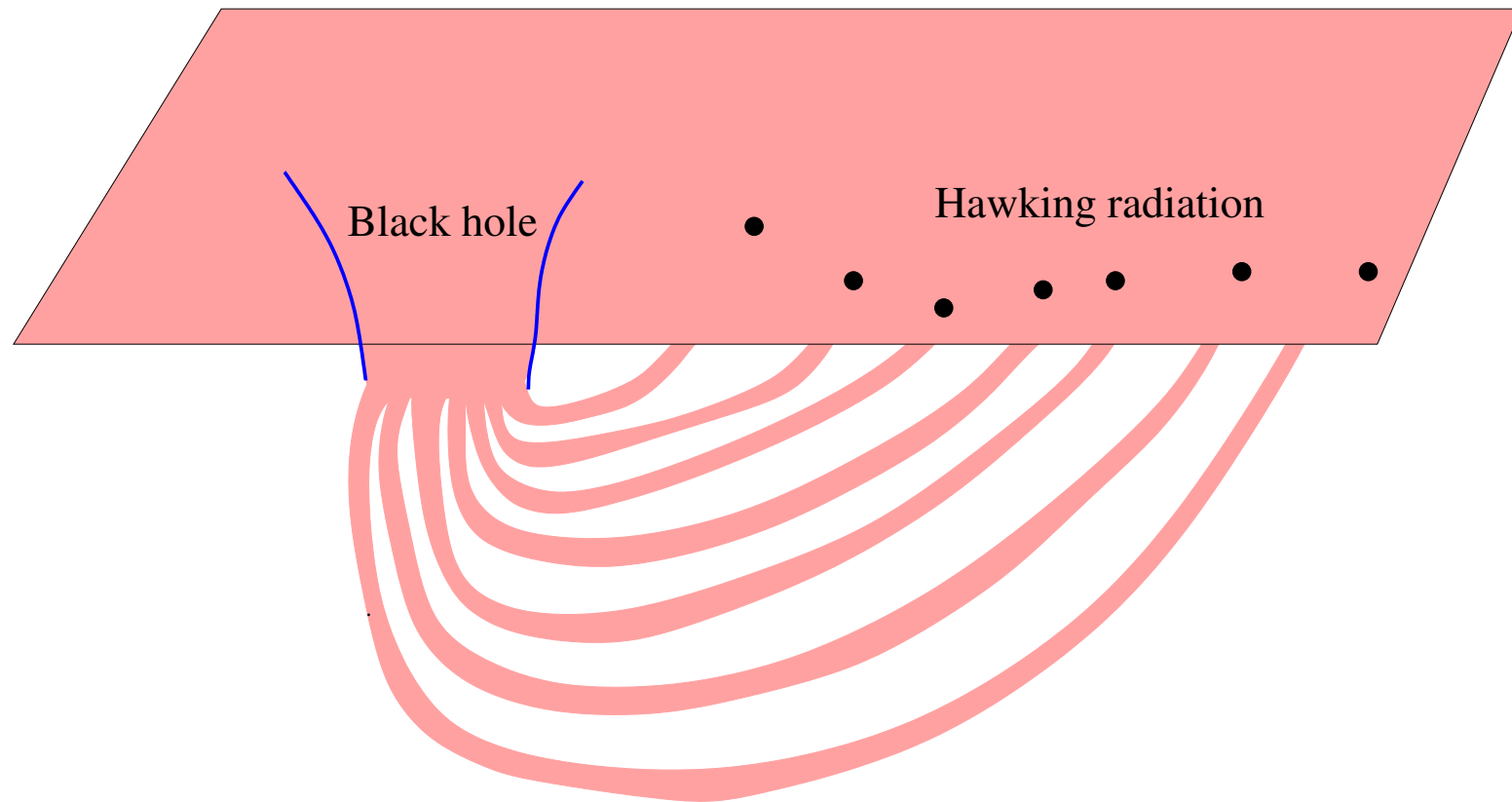


$$\langle \phi \phi \rangle = \frac{1}{d_{\text{short}}^2}$$

How to arrange a forbidden meeting



Black hole + radiation ?



We do not really know what this picture means, except to say that it is possible to send signals behind the horizon by manipulating the radiation.

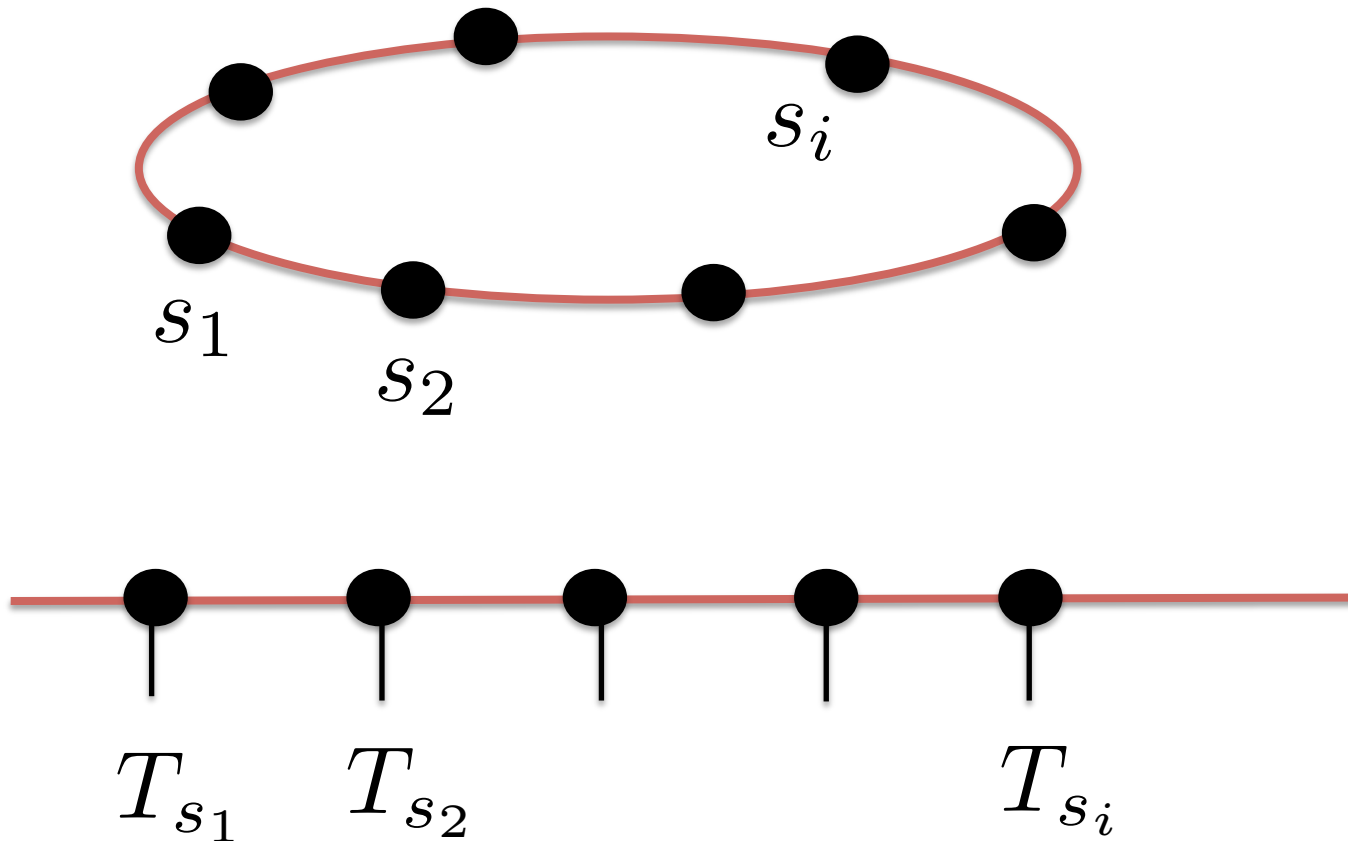
Conclusions

- Entanglement can be computed simply at strong coupling \rightarrow Minimal areas.
- Generalizations of the Hawking Bekenstein black hole entropy formula.
- ER = EPR: Entanglement can produce a geometric connection (view it as a constraint on quantum gravity theories).

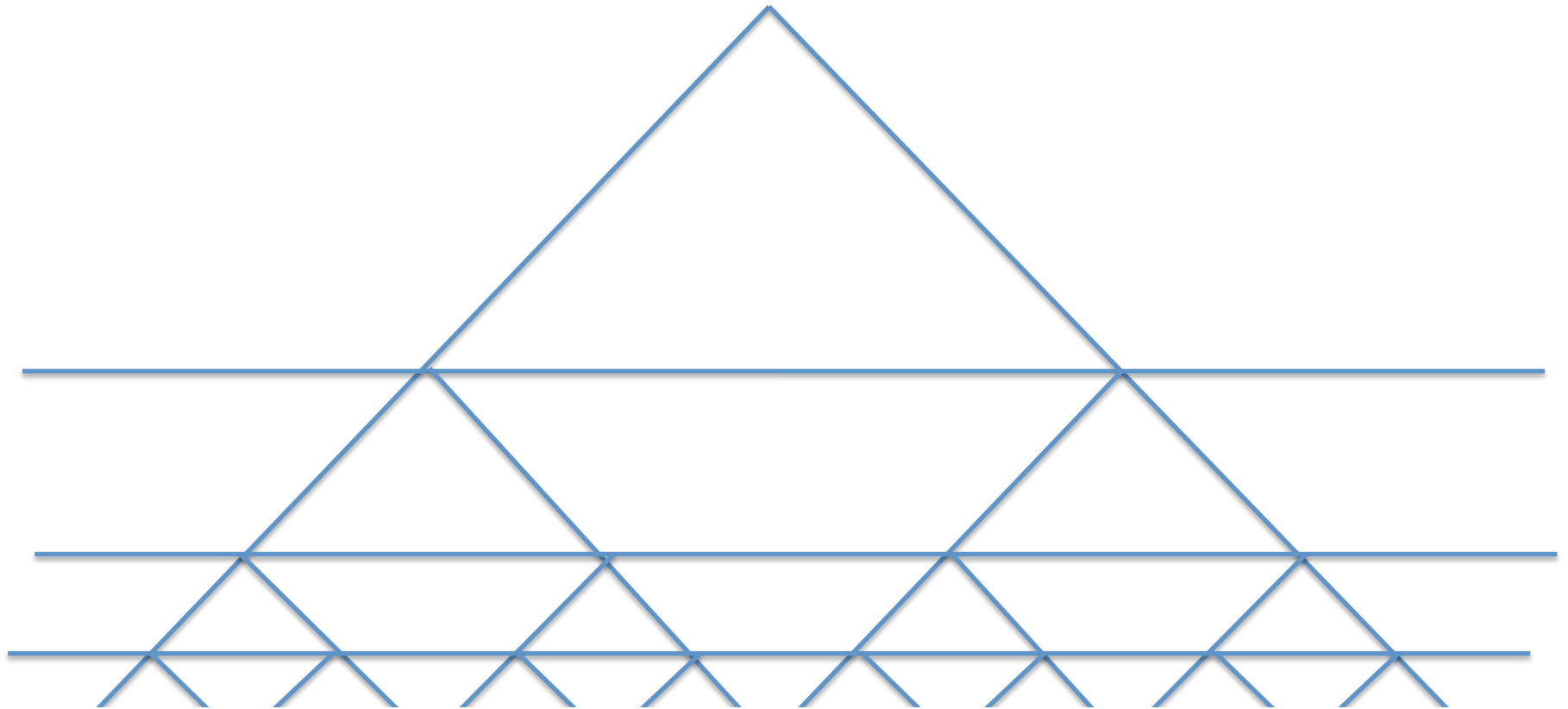
Tensor networks and geometry

Matrix product states

$$\Psi(s_1, \dots, s_n) = \text{Tr}[T_{1,s_1} T_{2,s_2} \cdots T_{n,s_n}]$$



Scale invariant wavefunctions (MERA)



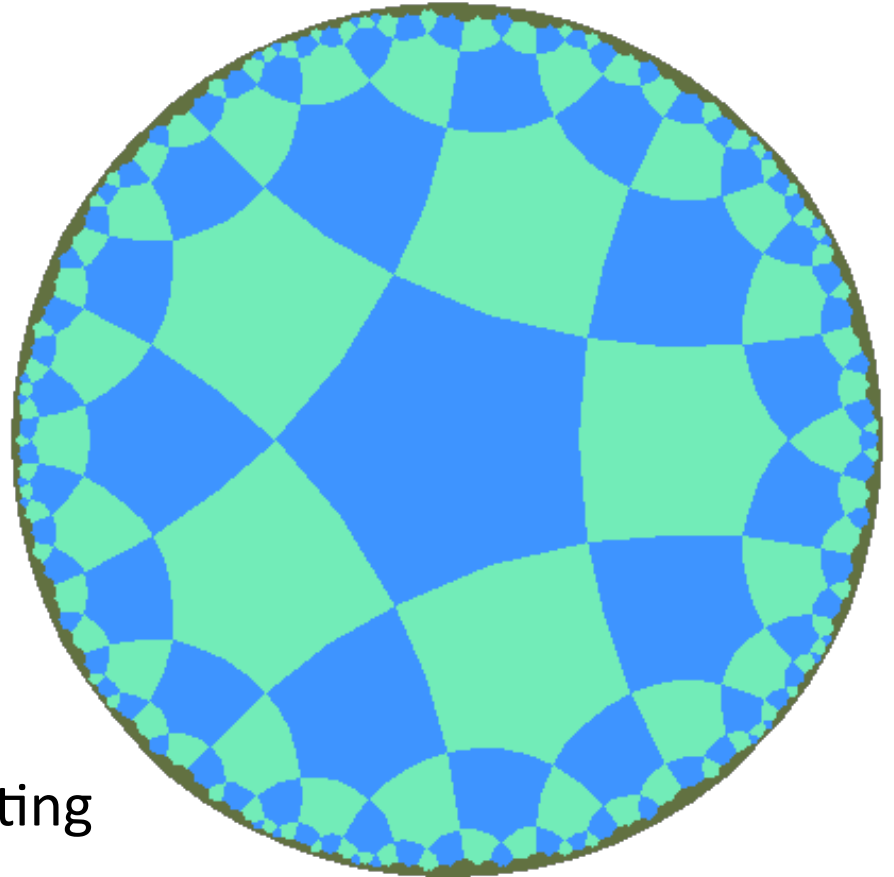
Each vertex is a five index tensor. Each line is an index contraction.

Indices \rightarrow not "real" states.

Vidal

This is similar to the geometry of AdS

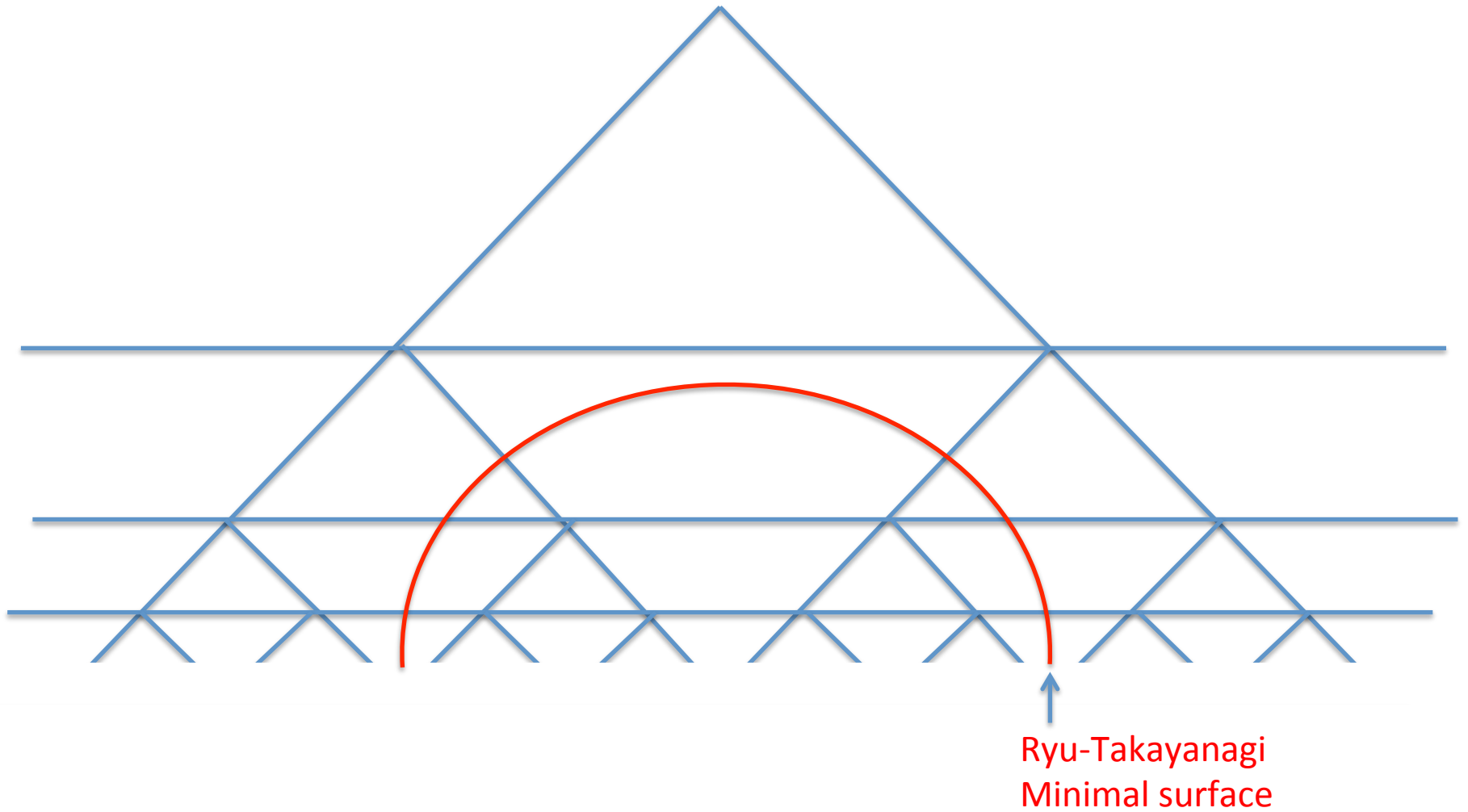
Swingle



Think of the tensors as representing the AdS vacuum wavefunction.

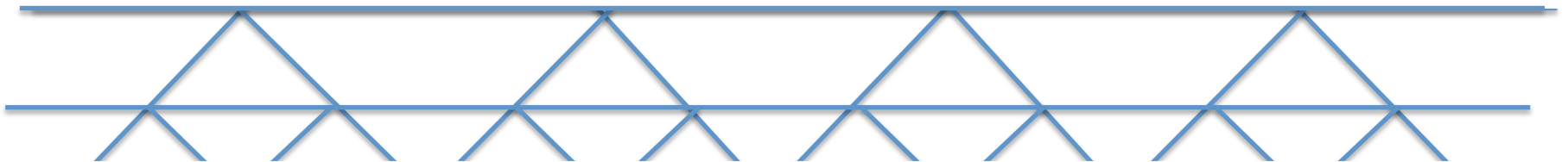
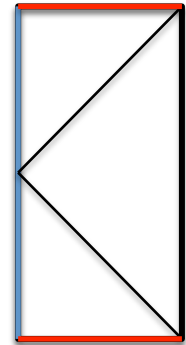
Tensor index contractions \rightarrow entanglement

Entanglement & structure of space



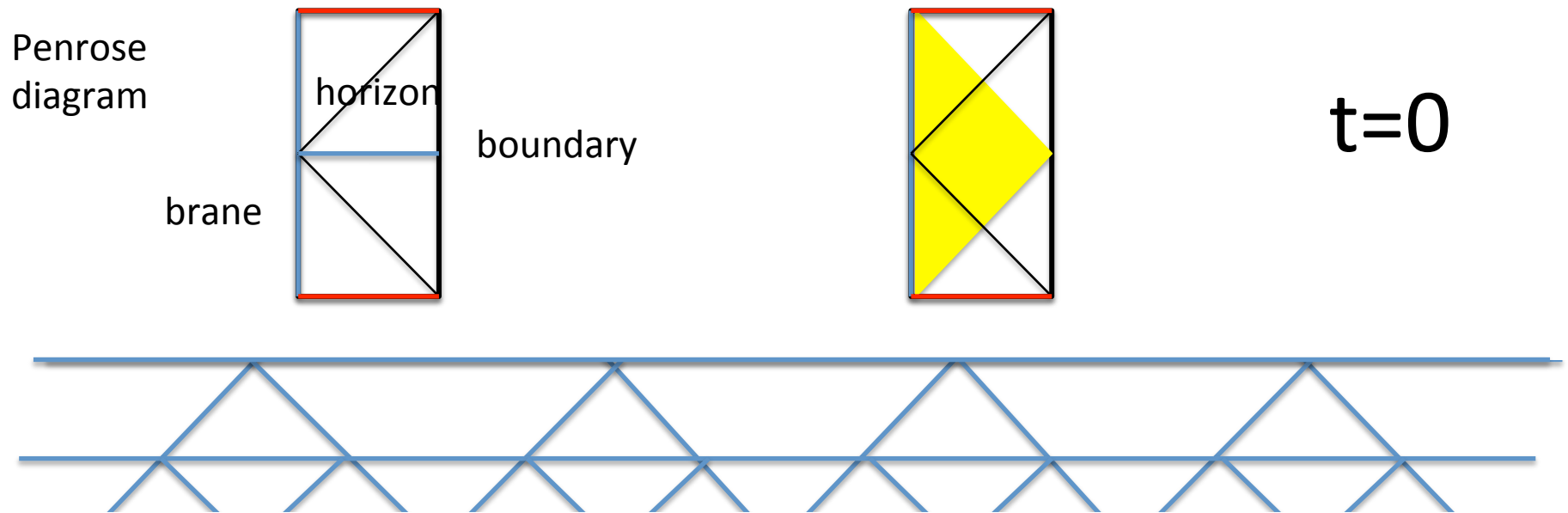
Conformal invariant system in a state with a mass gap.

eg: AdS space with an end of the world brane in the IR



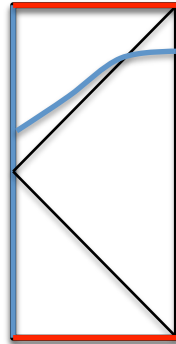
Time dependence

Start with a state with a gap and evolve it.
Eg. Brane in Ads that falls into a black hole

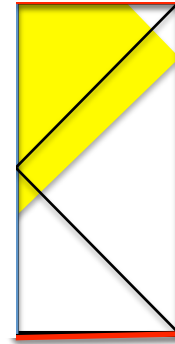


Penrose diagram

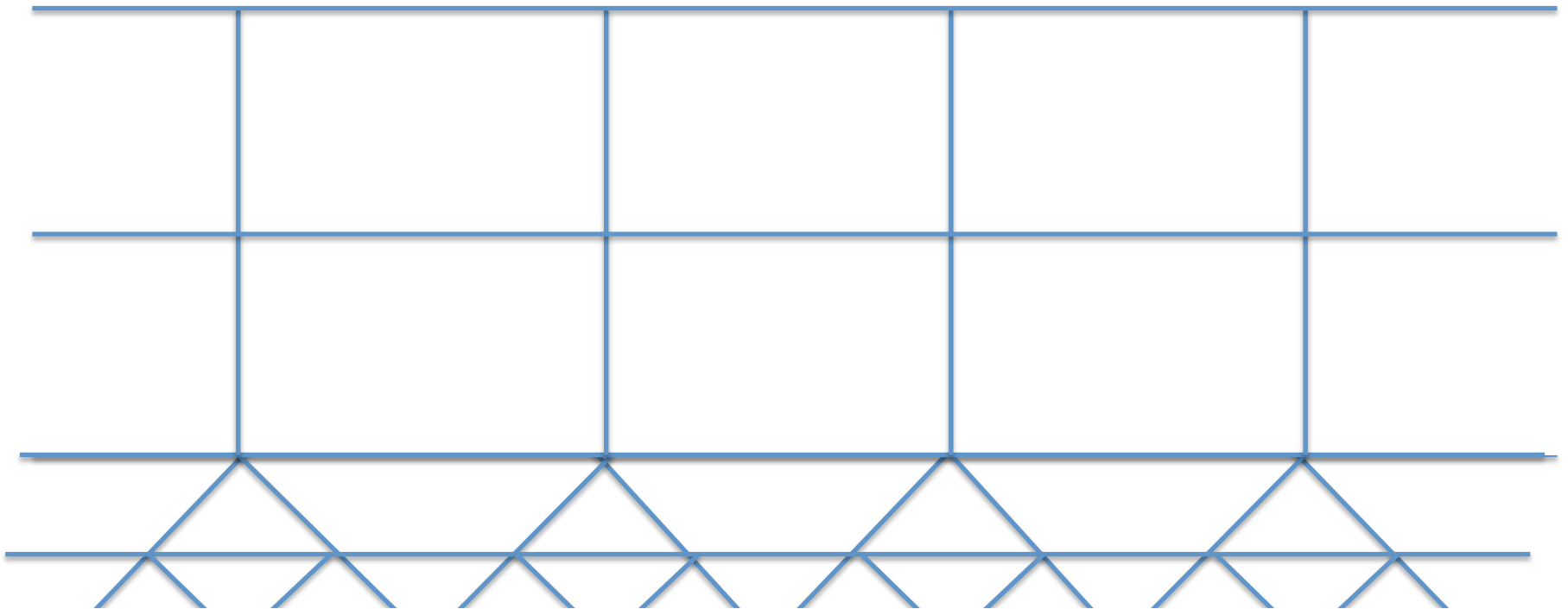
brane



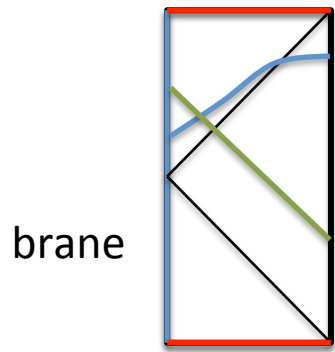
boundary



$t > 0$

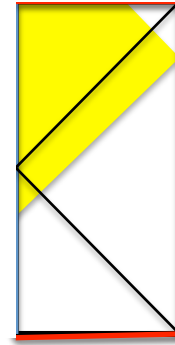


Time evolution produces a wavefunction that can be represented as a geometry which is simply longer.

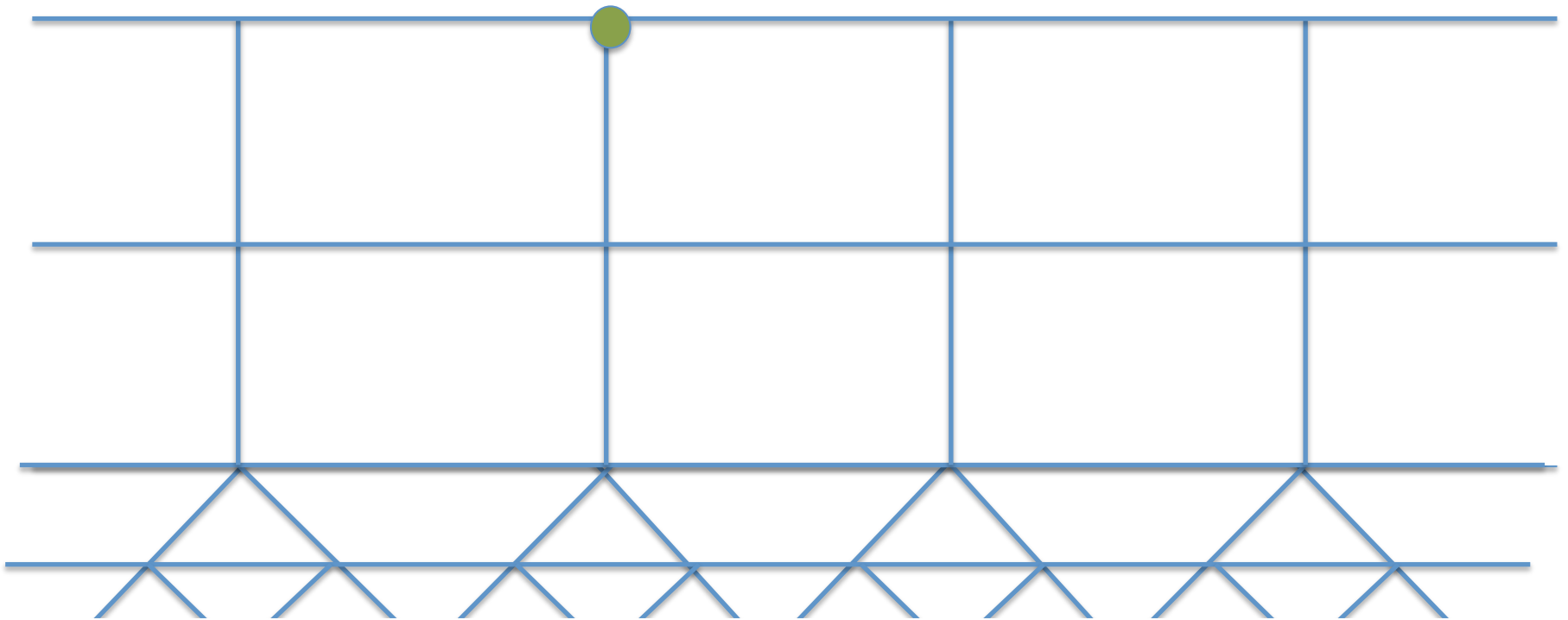


brane

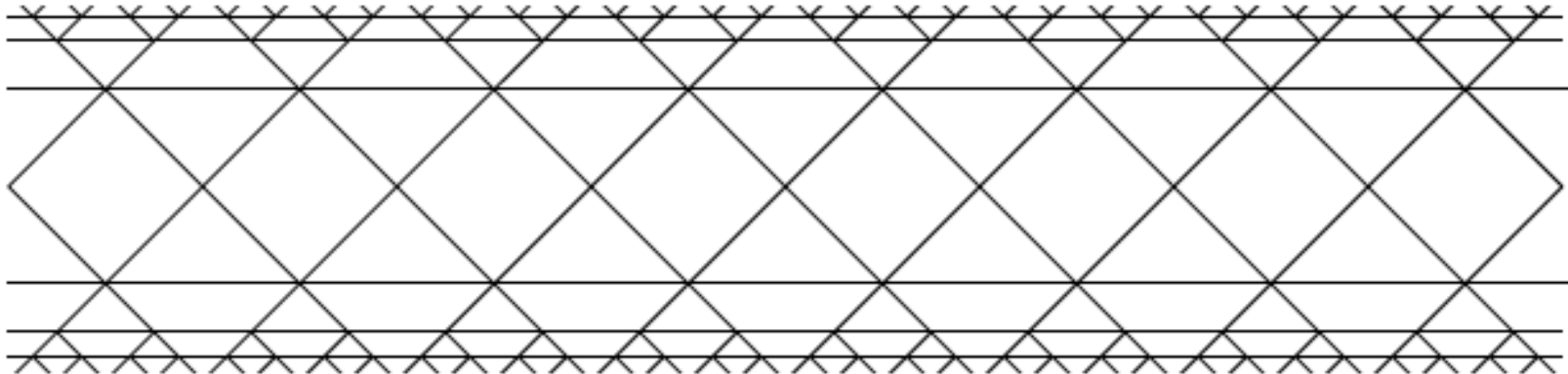
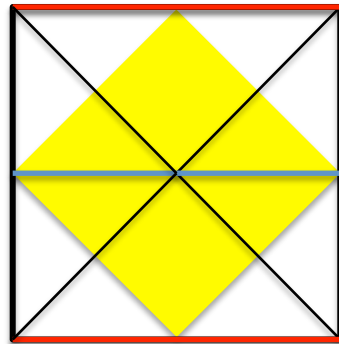
boundary



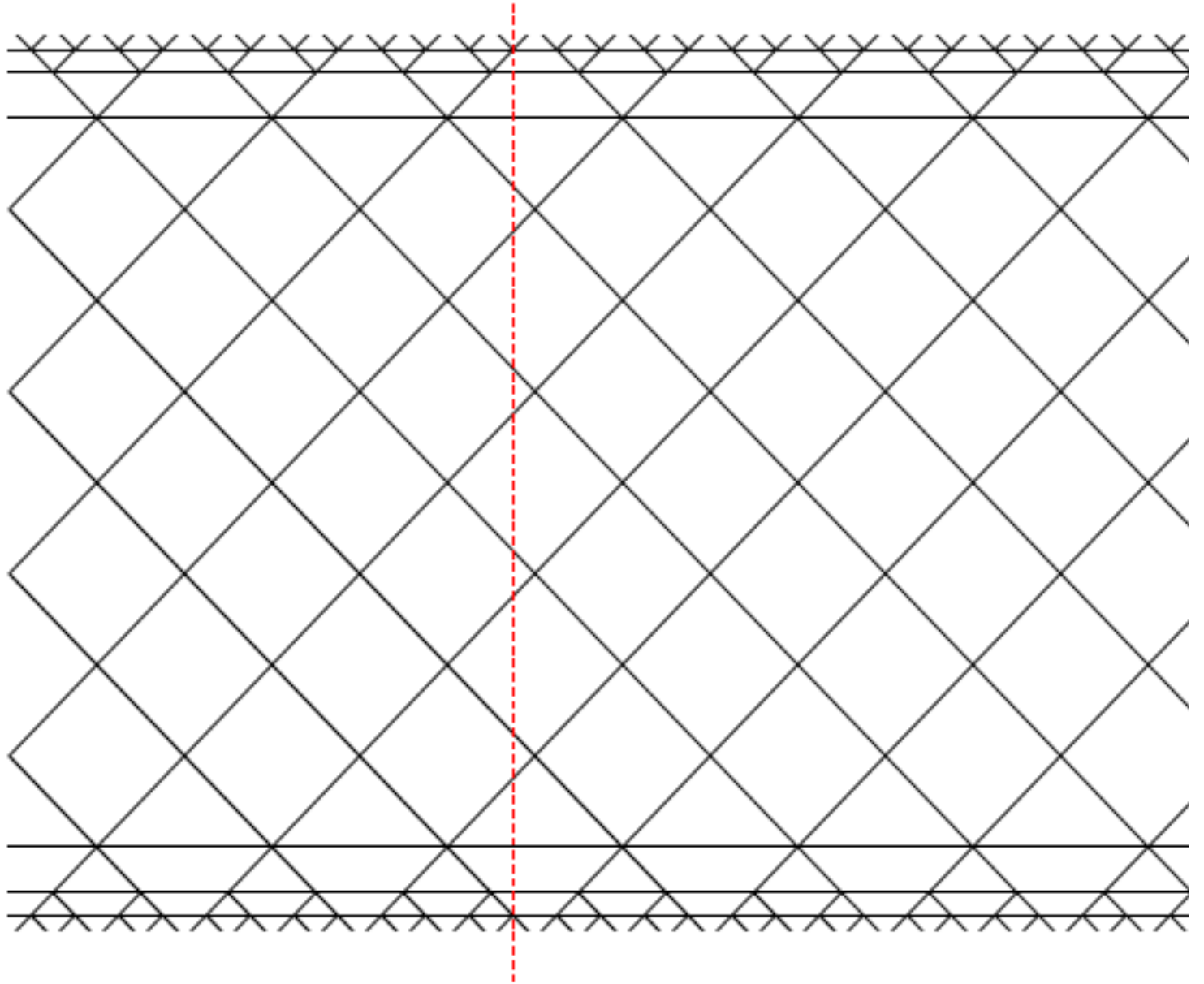
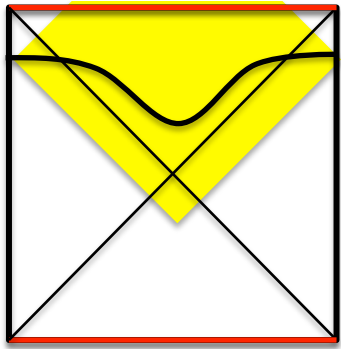
$t > 0$

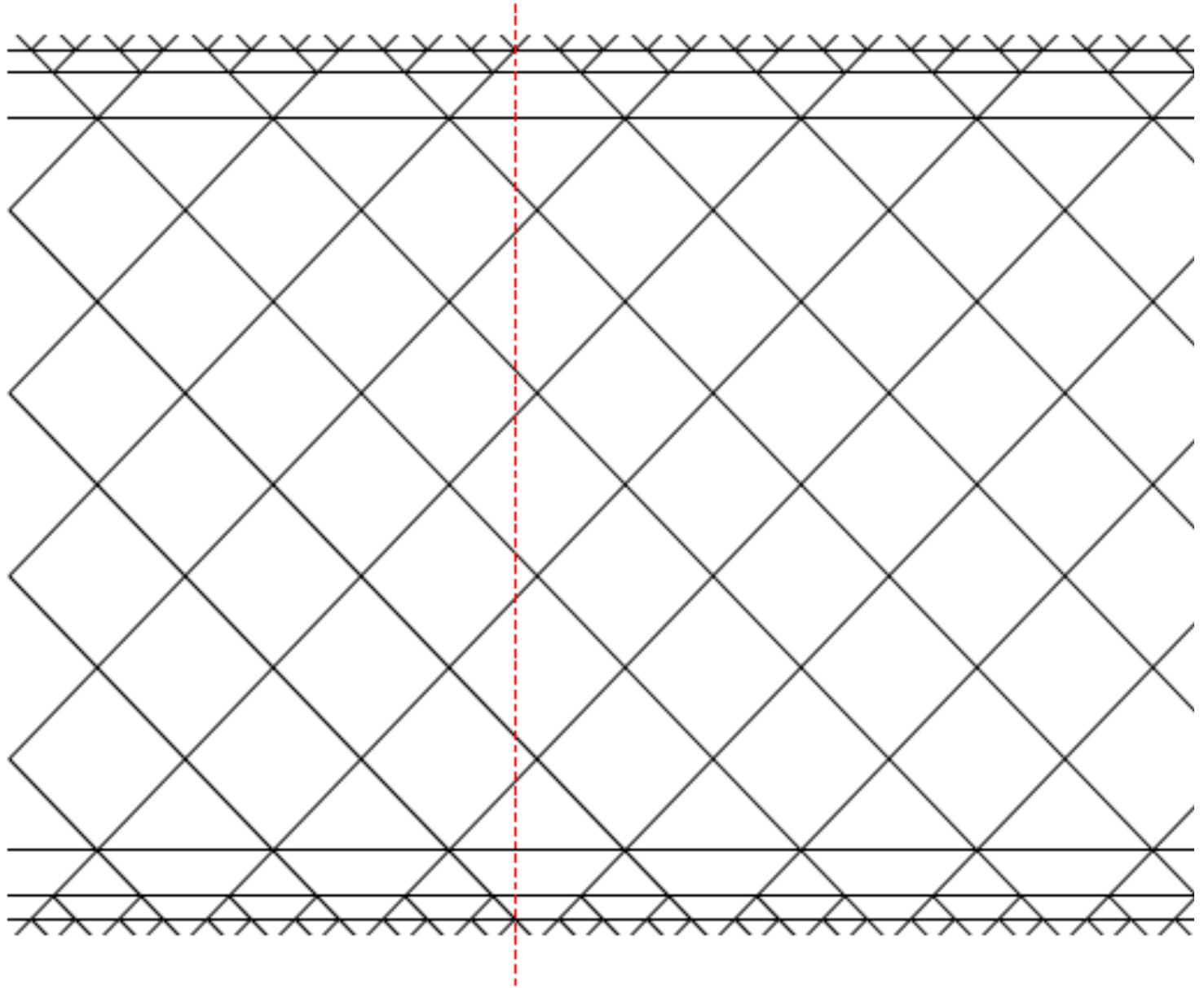
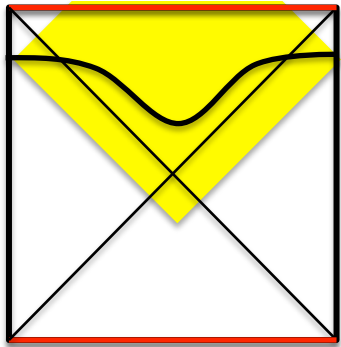


Addition of particles \rightarrow changes in the tensors



→
Spatial direction along horizon





Captures better the entanglement pattern.

Seems more similar to the "nice slices", which expand. The two horizons moving away...

Conclusions

- There are similarities between tensor networks and geometry.
- Both are constructions of the wavefunction and are constrained by the patterns of entanglement.
- In the case of black holes, it might help for understanding the interior.
- Growth of interior \rightarrow need to add more tensors for describing the increasing complexity of the wavefunction.