Universal energy transport in quantum critical systems in any dimension

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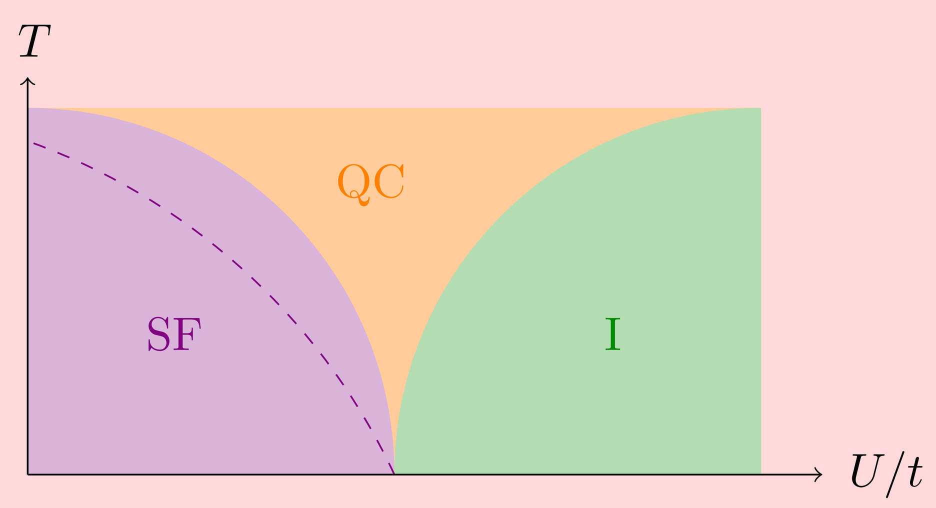
Transport without Quasiparticles

The classical picture of transport in a QFT in d spatial dimensions by long-lived quasiparticles breaks down at a *quantum critical point*, often described with the language of conformal field theory (CFT). A famous example is the 2d Bose-Hubbard superfluid-insulator transition [1], whose low energy theory is described by the $d = 2$ O(2) Wilson-Fisher fixed point.

$$H = -t\sum_{i\sim j} b_i^\dagger b_j + U\sum_i b_i^\dagger b_i(b_i^\dagger b_i - 1)$$

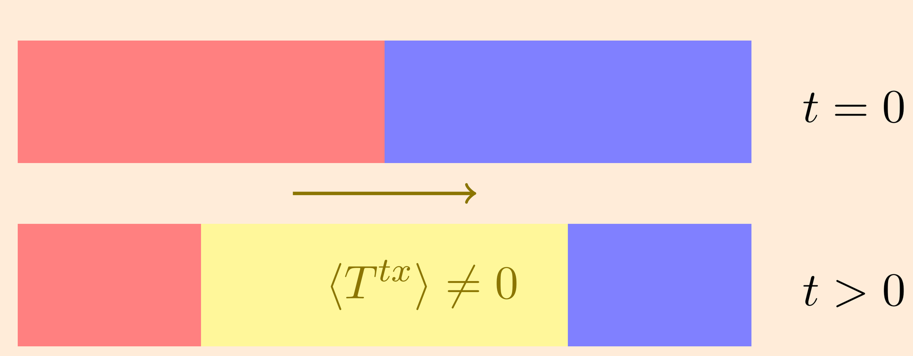


Although field theory at finite temperature T is accessible by compactifying imaginary time with period T^{-1}, only a discrete set of Matsubara frequencies are accessible, and determining transport coefficients and real-time dynamics at all frequencies is a major challenge. Novel techniques such as AdS/CFT are finally allowing for dynamical computations consistent with conformal symmetry.

A "Thermalization Quench"

Consider placing two semi-infinite CFTs in $d = 1$ of central charge c together at temperatures T_L, T_R. As they interact, a universal steady state develops, with energy flow: [2]

$$\langle T^{tx}\rangle = \frac{c\pi}{12}\left(T_\mathrm{L}^2 - T_\mathrm{R}^2\right).$$



This equation is often understood in linear response as $\langle T^{tx}\rangle = cgT_\mathrm{av}\Delta T$ where g is the *quantum of thermal conductance*, which can be measured experimentally [3], though it is more general. This universal formula has been observed numerically and is approximately valid even when conformal symmetry is broken [4]. It is easy to understand where it comes from; holomorphic factorization implies that left/right-moving sectors do not interact. The steady state is formed by left movers from right bath, and right movers from left bath, overlapping. This steady state forms instantaneously: the ends of the steady-state region propagate at the speed of light. The full probability distribution of energy transfer can be obtained from effective fluctuation relations on $\langle T^{tx}\rangle$.

Higher Dimensions: We generalize this set-up to higher dimensions, which is unexplored in the literature. The theory is strongly interacting and there is no integrability, meaning that previous techniques are not applicable. Momentum conservation suggests that the energy current has a ballistic component; so we postulate steady-states do *emerge dynamically.* We will show this naturally follows from hydrodynamics.

Emergent Steady States

A CFT in d spatial dimensions at finite temperature T breaks both Lorentz invariance by picking out a preferred rest frame (denote by time-like vector u^μ) and scale invariance. The resulting stress-energy tensor is then completely constrained by symmetry to have the form

$$\langle T^{\mu\nu}\rangle = a_d T^{d+1}\left(\eta^{\mu\nu} + (d+1)u^\mu u^\nu\right).$$

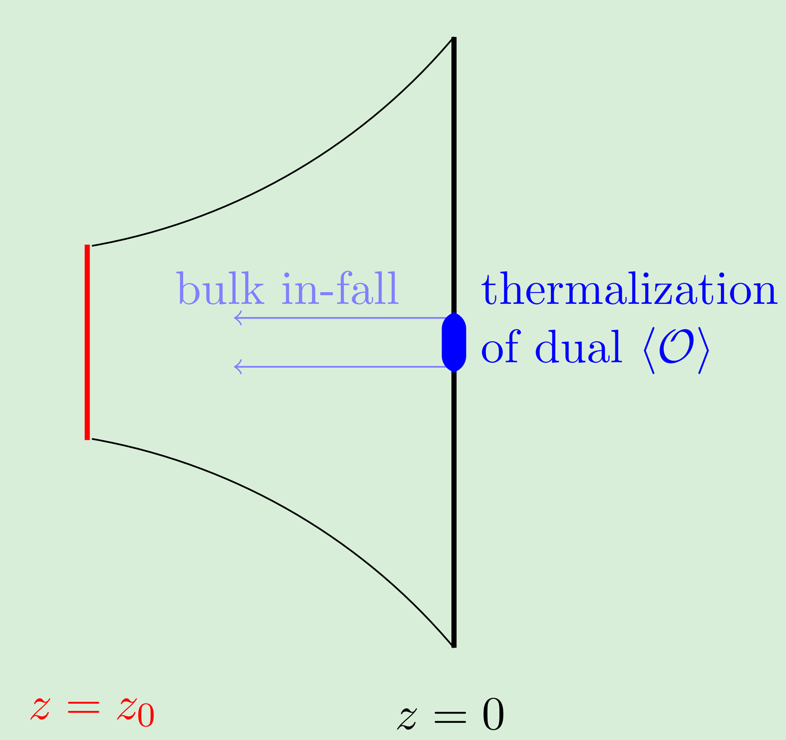
a_d is proportional to the degrees of freedom of the CFT; e.g. $a_1 = c\pi/12$. A strongly-coupled CFT should have *no other steady states* (up to additional conservation laws).

Thermodynamics: This can be understood by noting that the only generic conserved quantity is P^μ. The most generalized Gibbs ensemble is $\exp[\beta_\mu P^\mu]$, where $\beta_\mu = u_\mu T^{-1}$.

AdS/CFT: More generally, we expect to encode dynamics of energy-momentum sector in Einstein-Hilbert gravity in AdS_{d+2}, if $a_d \sim L^d/G_\mathrm{N} \gg 1$. Gravity can derive this thermodynamic insight – the only regular t, x-independent solutions to Einstein's equations are black branes dual to finite T states:

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[\frac{\mathrm{d}z^2}{f(z)} - f(z)u_\mu \mathrm{d}x^\mu u_\nu \mathrm{d}x^\nu + (\eta^{\mu\nu} + u^\mu u^\nu)\mathrm{d}x^\mu \mathrm{d}x^\nu\right]$$

where $f = 1 - (4\pi Tz/(d+1))^{d+1}$. Anisotropic geometries are not regular: this is a *geometric encoding* of the fact that perfect conformal fluids do not support shear stress. [5] The possibility for excitations to fall past the event horizon of this black brane at $z = z_0$ allows for a dual description of thermalizing systems. The metric above is an approximately a solution of Einstein's equations with u^μ, T allowed arbitrary variations in x^μ that obey the equations of perfect hydrodynamics; correcting this solution leads to higher orders (e.g., viscosity at first order) in a hydrodynamic gradient expansion.



Steady-state energy transport is universally governed by Lorentz-boosted thermal states. Exceptions are theories with pathologically many conserved quantities: e.g., free particles or de-coupled sectors. Adding conserved charges adds chemical potentials that characterize emergent steady-states.

References

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[2] D. Bernard and B. Doyon. *Journal of Physics* **A45** 362001 (2012).
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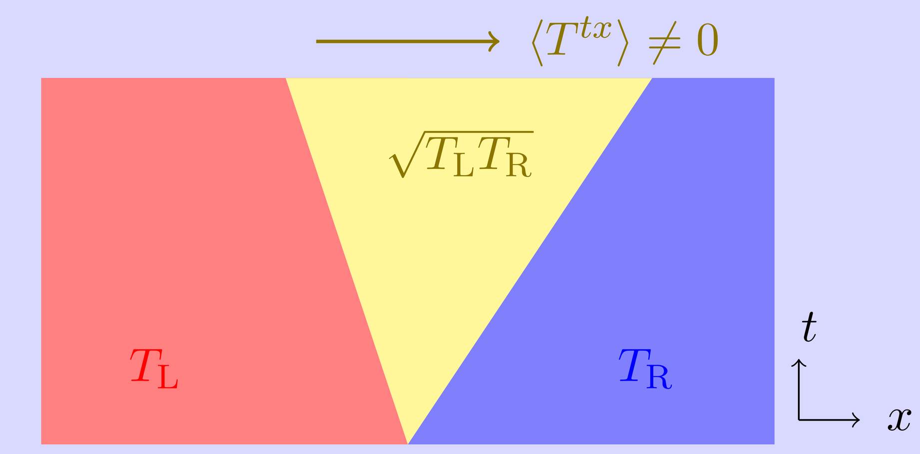
Dynamics of Interacting Heat Baths

We extend the $d = 1$ set-up to arbitrary d. For times $t \gtrsim \lambda/(T_\mathrm{L} + T_\mathrm{R})$, with $\lambda = \eta/a_d T^d$ the dimensionless viscosity, perfect conformal hydrodynamics is a good approximation for an interacting field theory. The equations of motion are simply $\partial_\mu\langle T^{\mu\nu}\rangle = 0$. Domain walls propagate at known, non-symmetric, velocities into heat baths leaving behind a universal steady state with Lorentz boost and temperature fixed by $T_\mathrm{L}, T_\mathrm{R}$. [6]



The *full counting statistics* of the energy transfer $\mathcal{Q}$ from left to right across interface area A in a time span $t \gg \lambda/(T_\mathrm{L} + T_\mathrm{R})$ are also expected to follow an *extended fluctuation relation*:

$$\langle \mathcal{Q}^n\rangle = At\left(\frac{\mathrm{d}}{\mathrm{d}Z}\right)^{n-1}\langle T^{tx}(\beta_\mathrm{L} + Z, \beta_\mathrm{R} - Z)\rangle.$$

$$\langle T^{tx}\rangle = a_d(T_\mathrm{L}T_\mathrm{R})^{(d+1)/2}\frac{\chi - 1}{\chi}\sqrt{\frac{(1 + d\chi)(d + \chi)}{d}}$$

with $\chi = (T_\mathrm{L}/T_\mathrm{R})^{(d+1)/2}$. We derive this relation by appealing to the rapid appearance, and PT-symmetry, of the emergent steady-state. This is the *first result* for heat transfer between baths out of equilbrium in a model in $d > 1$, where theories are not integrable. This result passes non-trivial consistency checks.

***d* = 1:** Although hydrodynamics is not applicable, the above results still hold and can be derived in full rigor from CFT technology [2], or a solution of Einstein's equations. The geometry of merging black branes, dual to interacting heat baths, is well-defined and the black hole partition function recovers the result above for $\langle \mathcal{Q}^n\rangle$ [5].

Stability: The fate of the steady-state to possible instabilities such as turbulence is unknown. In $d = 2$, zeroth-order conservation of enstrophy $\int \mathrm{d}x\mathrm{d}y\ u^t(\epsilon^{\mu\nu\rho}u_\mu\partial_\nu u_\rho)^2$ rules out the possibility of turbulence; the steady-state should be robust.

Momentum Relaxation

Impurities or lattices break microscopic translation (and Lorentz) invariance and momentum is not conserved. Modifying momentum conservation to $\partial_\mu\langle T^{\mu i}\rangle = -\tau^{-1}\langle T^{ti}\rangle$, for $t \gg \tau$, $\langle T^{tx}\rangle$ is suppressed and energy diffuses: $\partial_t\langle T^{\mu\nu}\rangle \approx (\tau/d)\partial_x^2\langle T^{\mu\nu}\rangle$. [5]