Computational Issues in BSM Theories
-- Past, Present and Future

Meifeng Lin

Computational Science Center

Brookhaven National Laboratory

Field Theoretic Computer Simulations for Particle Physics And Condensed Matter
Boston University, May 8 – 10, 2014

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Outline

- Introduction
  - Standard Model of Particle Physics
  - Beyond the Standard Model
  - The role of Lattice

- Computational Issues: Past, Present and Future
  - General issues
  - Past: viability of technicolor theories
  - Present: searching for Higgs imposters
  - Future: ?

- Summary and Outlook

Standard Model of Particle Physics

- Standard Model of particle physics describes the strong and electroweak interactions of the elementary particles.

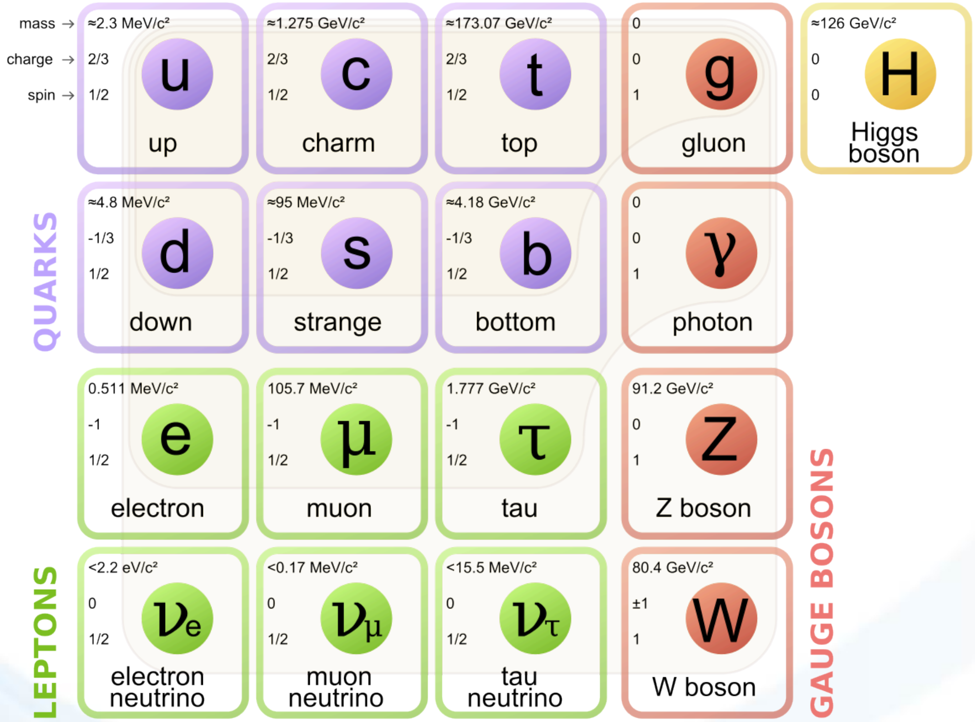


Image from Wikipedia

- Quantum chromodyanmics (QCD) is the theory of the strong interactions between quarks and gluons. SU(3) gauge symmetry.
- The electroweak sector is described by $SU(2)_L \times U(1)_Y$ symmetry.
- Inputs to the SM: particle masses and the gauge couplings ➔ 19 parameters.

The SM Higgs Mechanism

- What gives fermions and gauge bosons mass? In the Standard Model, this is achieved by the Higgs mechanism.
- A complex scalar SU(2)$_L$ doublet is introduced *by hand*,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with the potential ($\lambda > 0$)

$$V(\Phi) = \mu^2 \mid \Phi^\dagger \Phi \mid + \lambda \left(\mid \Phi^\dagger \Phi \mid \right)^2$$

- When $\mu^2 < 0$, a non-zero vacuum expectation value (*vev*) develops and spontaneously breaks the electroweak symmetry.

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v^2 = -\frac{\mu^2}{2\lambda} = (246\ GeV)^2$$

- The scalar doublet can be written in terms of a physical Higgs field *h*

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

The SM Higgs Mechanism (cont'd)

- The gauge bosons acquire mass through this vev,

$$\begin{aligned} M_W^2 &= \frac{1}{4} g^2 v^2 \\ M_Z^2 &= \frac{1}{4}(g^2 + g'^{\,2}) v^2 \\ M_A &= 0. \end{aligned}$$

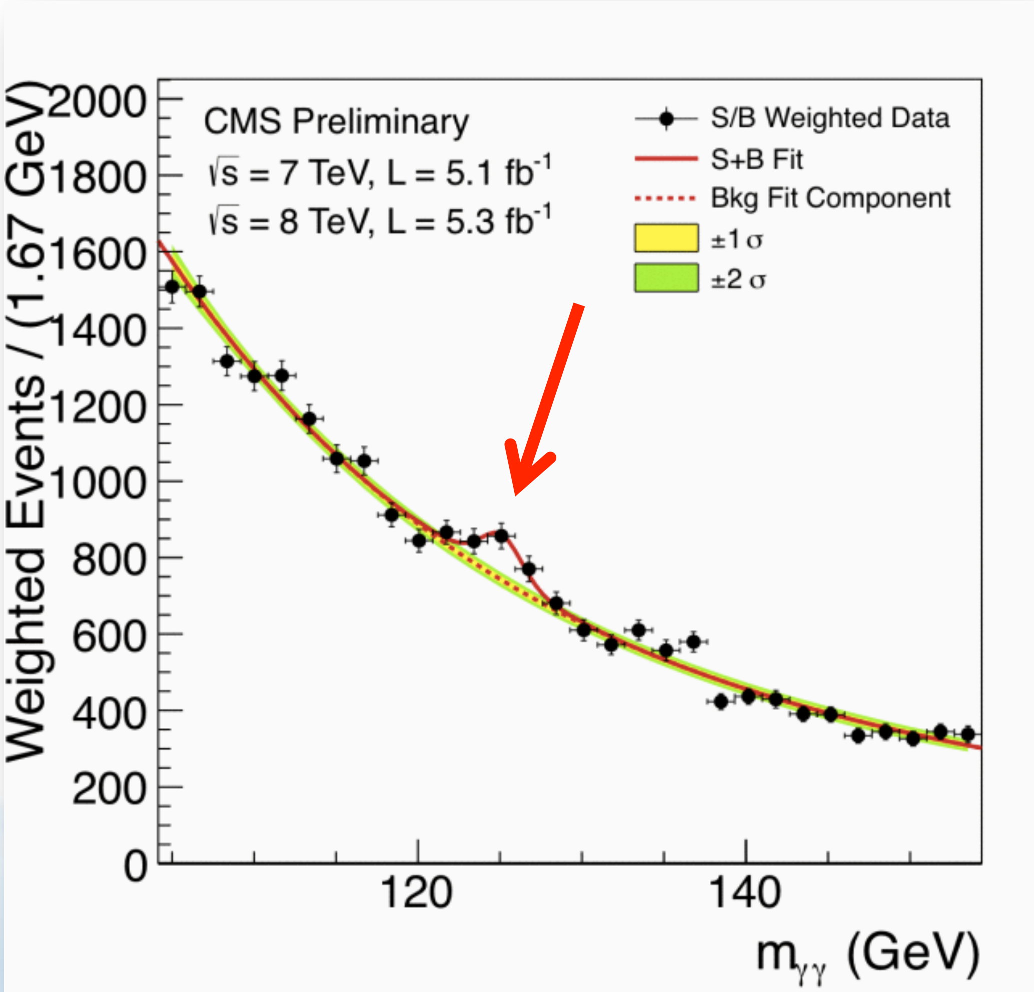
- A consequence of the SM Higgs mechanism is the existence of a scalar Higgs boson, with the mass determined by the Higgs self coupling

$$M_h^2 = 2v^2\lambda$$

- Higgs mass is not known *a priori*.
- Prior to the LHC discovery, various precision experiments put bounds on the range of allowed Higgs mass.

LHC Discovery

- On July 4, 2012, two teams at LHC announced the discovery of a new particle consistent with the SM Higgs boson.



- Both CMS and Atlas observed a new particle state with mass ~126 GeV.
- Other properties of the "Higgs" boson need to be established.
- More work is needed to confirm this is indeed the SM Higgs boson.
- New direction for BSM theories: can this "Higgs" boson be produced by some BSM models.

The need for BSM theories

- Higgs has been found. Why are we still interested in BSM theories?
  - Standard Model doesn't incorporate gravity.
  - Standard Model cannot explain many experimental observations:
    - Neutrino masses, dark matter and dark energy, etc.
  - Hierarchy problem:
    - Why is the Higgs mass (~126 GeV) so much lighter than the Planck scale?
    - Requires delicate fine-tuning ➔ Unnatural.
- The SM Higgs mechanism is a parameterization. It doesn't explain the dynamical origin of the electroweak symmetry breaking
- Technicolor theories: EW symmetry is broken dynamically via new strong dynamics at TeV scale and above.

Technicolor in a nutshell

- Introduce new gauge interactions SU(N_{TC}) at Λ_{TC}, with N_{TF} flavors of technifermions.

- Technifermions possess chiral symmetry, similar to the QCD fermions.

- This chiral symmetry is spontaneously broken ➔ massless Techni-Goldstone bosons.
  - Three of these Goldstone bosons provide mass for the W and Z gauge bosons.
  - Others (if any) remain massive – model dependent.

- Technifermion condensate at the Extended Technicolor scale Λ_{ETC} provides mass for the SM fermions. mechanism for the generation of fermion masses.

$$m_{q,l} \simeq \frac{\langle \overline{Q}Q \rangle_{ETC}}{\Lambda^2_{ETC}}$$

- Λ_{ETC} has to be large to suppress flavor-neutral changing current (FCNC) to be consistent with experimental bounds ➔ $\Lambda_{ETC} \sim 10^3$ TeV

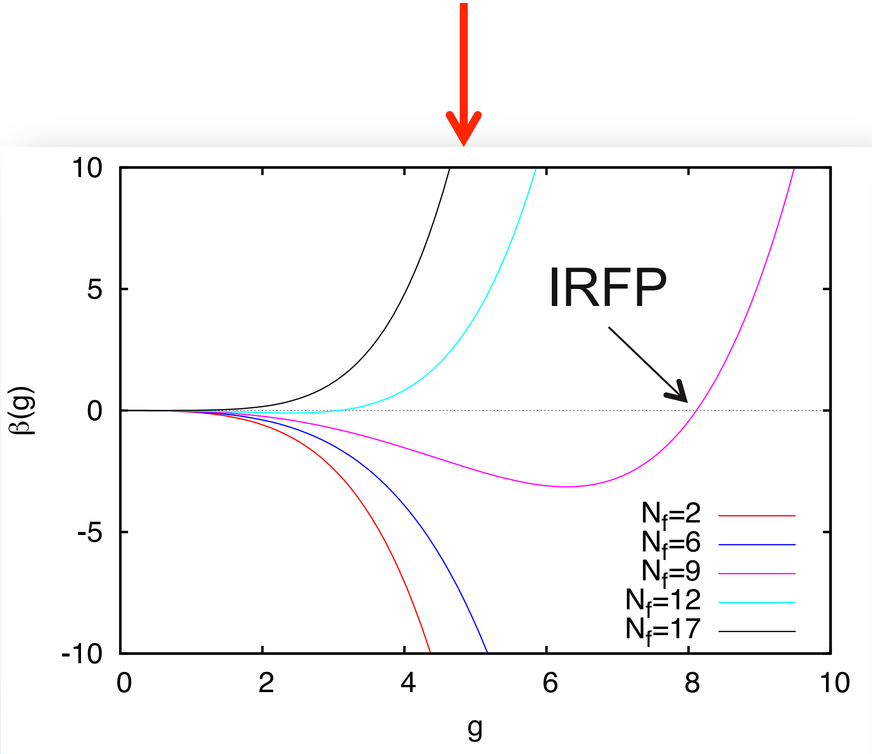
Constraints for Technicolor Theories

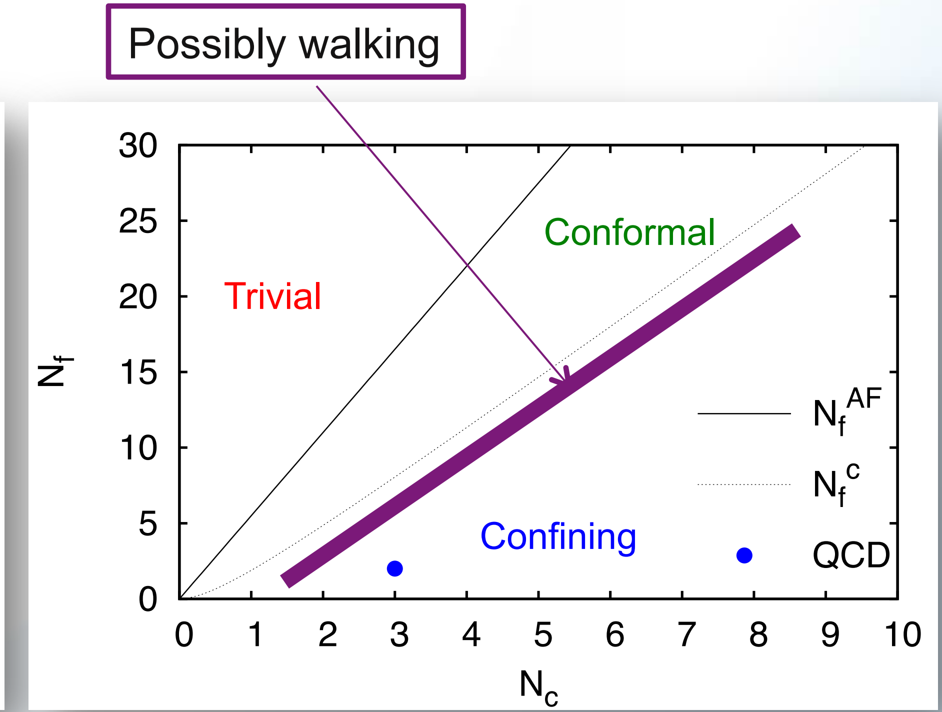
- Technifermion condensate needs to be enhanced to provide large enough mass for the SM fermions.
  - One consequence of this is that the anomalous dimension γ has to be large, O(1).
- The electroweak S parameter needs to be small to satisfy the LEP experimental constraint: S ≈ 0.
  - Naïve scaled-up QCD would violate this constraint.
- After Higgs discovery, a viable BSM theory must have a light scalar boson with a mass consistent with the LHC finding.
- All these would require that the new strong interactions are non-QCD like, probably with near-conformal behaviors. ➔ Walking Technicolor.
- Walking technicolor theories may be able to produce light Higgs in the form of a light dilation, or a composite pseudo-Nambu-Goldstone boson (Little Higgs).

What Walking Technicolor Theory May Look Like

- Perturbative 2-loop beta function for an SU(N) gauge theory with N_f fundamental fermions.

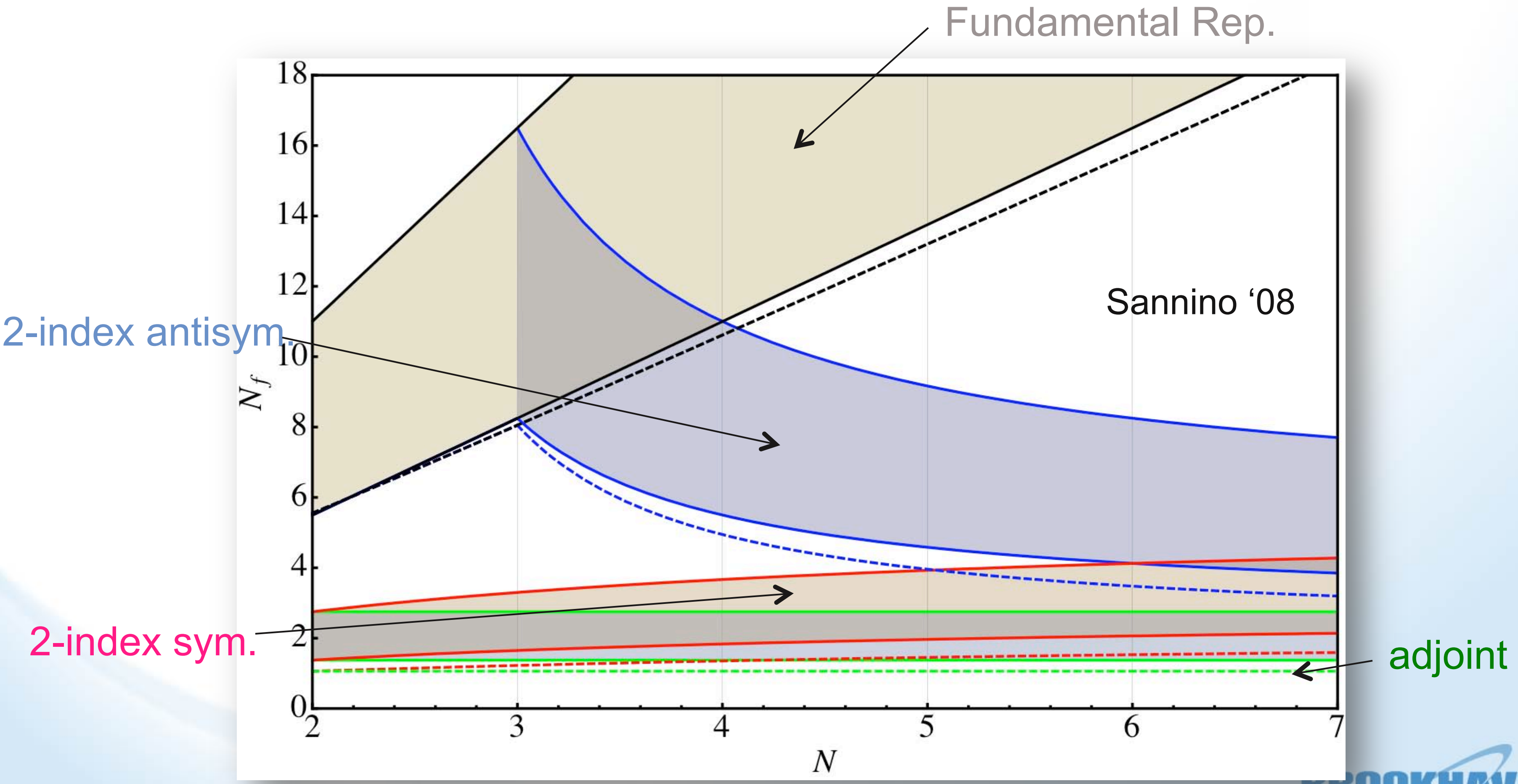
- The phase space can be separated into three different regions.





Needle in a Haystack

- There are a lot of candidate strongly interacting theories.
- Non-perturbative calculations are needed. → Comes the lattice.



The role of Lattice

- Such theories are strongly interacting and intrinsically non-perturbative.
- Lattice gauge theory can calculate a lot of non-perturbative quantities from first principles.
- BSM model builders need us to verify that technicolor theories can indeed satisfy the constraints.
- I will focus on SU(3) gauge theories with N_f fermions in the fundamental representation.

LGT Basics

- The core of LQCD simulations is Hybrid Monte Carlo (Molecular Dynamics + Monte Carlo) in the Euclidean space:

$$\langle O \rangle = \frac{\int [dU] O[U] \det(\not{D} + m) e^{-S_g[U]}}{Z}$$

- We don't really calculate the determinants directly. Instead, pseudofermion fields (bosonic fields) are introduced. For $N_f = 1$,

$$Z = \int [dU][d\phi^\dagger][d\phi] e^{-\phi^\dagger \frac{1}{(D^\dagger(m_f) D(m_f))^{1/2}} \phi - S_g[U]}$$

- Most computation-intensive part is to solve the Dirac equations:

$$D[U]\psi = b, \quad \text{or} \quad D^\dagger[U] D[U]\phi = b',$$

- D is a large, sparse, diagonally dominant matrix. Iterative solvers, such as conjugate gradient (CG), are typically used to solve the equations. Its condition number worsens as the quark mass m_f gets smaller ➔ it takes longer to converge.

Computational Complexity for BSM Theories

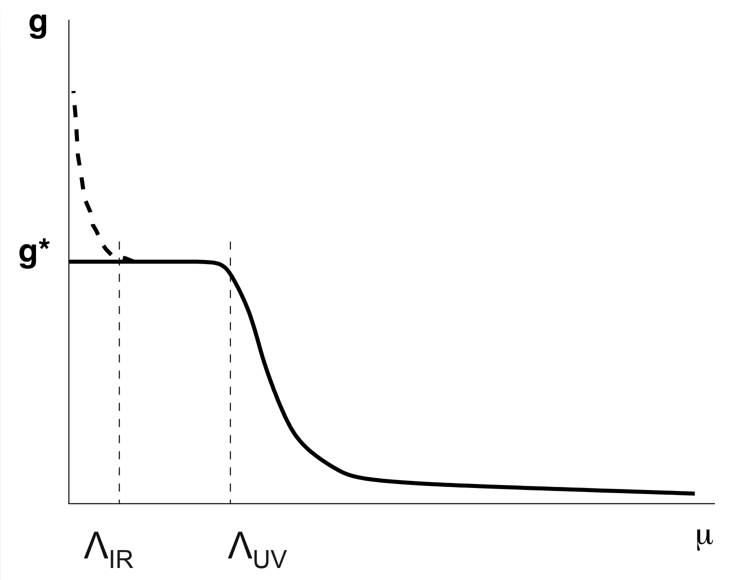
- For SU(3) simulations with large N_f, cost increases quickly as N_f is increased.

$$\langle O\rangle = \frac{\int [dU]O[U][\det(\not{D}+m)]^{N_f} e^{-S_g[U]}}{Z}$$

- Naïve cost per molecular dynamics trajectory

$$\sim N_f(\#\text{ of flavors}) \times N_f^{1/2}(\#\text{ of steps per trajectory}).$$

- Multi-scale problem:
  - Need $\Lambda_{UV} >> \Lambda_{IR}$ to separate UV physics from IR.
  - $\Lambda_{UV} \sim 1/a$, $\Lambda_{IR} \sim$ confinement scale, M_V
  - ➔ Need fine lattice spacing.
  - ➔➔➔ large number of lattice sites.



- C.f. cost for QCD $\left(\frac{L}{\text{fm}}\right)^5 \frac{\text{MeV}}{m_\pi}\left(\frac{\text{fm}}{a}\right)^6$

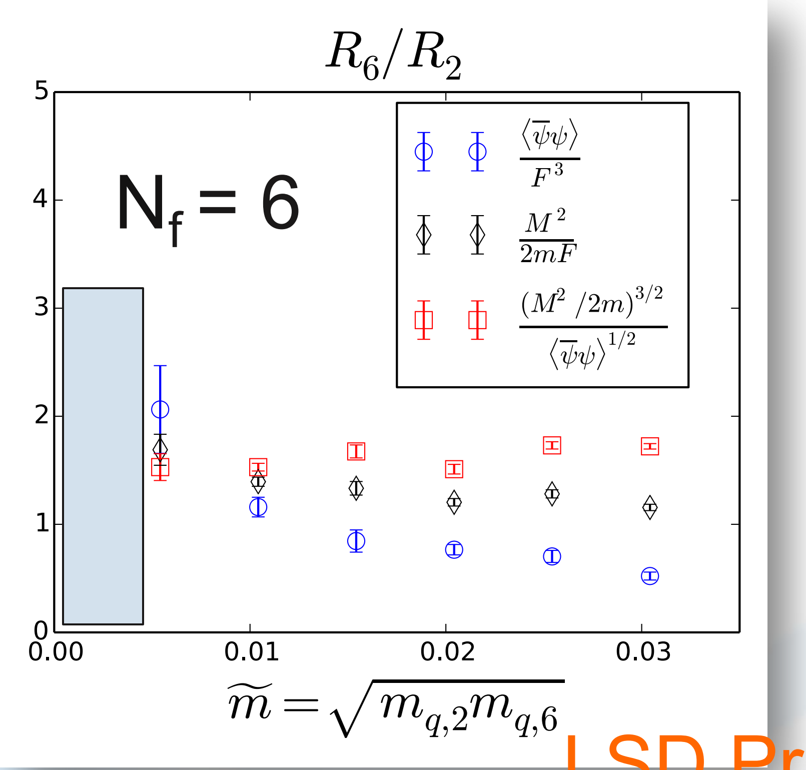
Christ and Jung, 2007

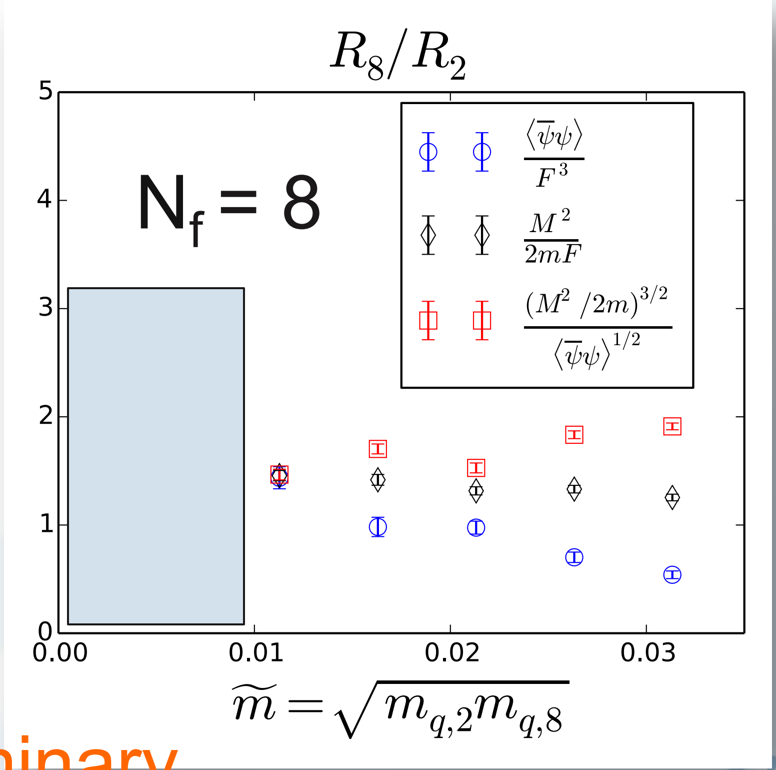
Approaching the Chiral Limit

- Lattice simulations are performed at several finite quark masses due to the formidable numerical cost at small m_f, and rely on chiral perturbation theory to extrapolate to
  - The physical limit for QCD.
  - The chiral limit for BSM theories.
- The converge of ChPT becomes worse or even questionable as N_f is increased.
  - In the region accessible with current computing power, next-to-leading-order chiral perturbation theory is not applicable. Going to higher orders requires too many unknown parameters.
- For QCD, simulations directly at the physical limit are becoming available.
- For BSM, going to lighter quark masses is much harder, and requires more computing power, and/or better algorithms.

Example: Condensate Enhancement

- We want to know if increasing N_f can increase the chiral condensate in the chiral limit.
- Three equivalent ways (in the chiral limit) to determine via Gell-Mann-Oakes-Renner relation.
- We'd like to know what the enhancement is at the chiral limit.

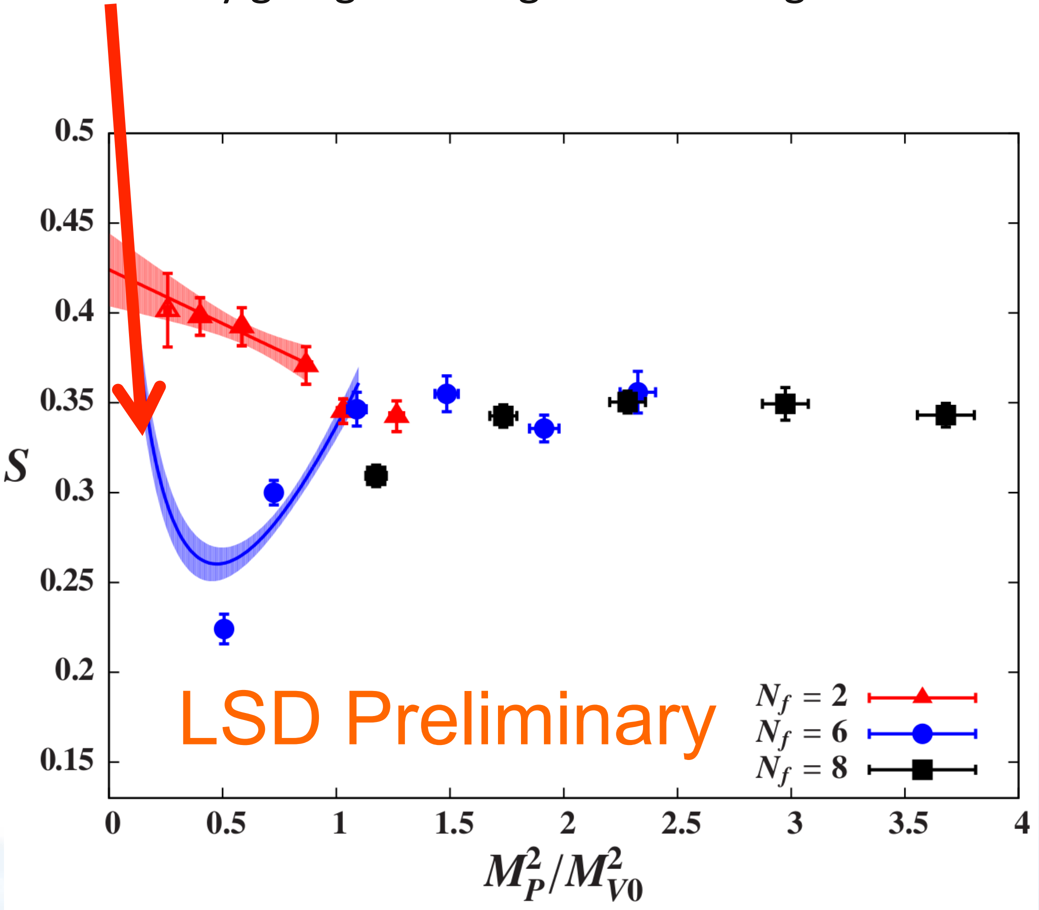




LSD Preliminary

Example: S Parameter

- We’d like to see a reduction of the S parameter compared to QCD.
- At simulated mass region, there is some evidence of reduction. But there is still some uncertainty going to the lighter mass region.



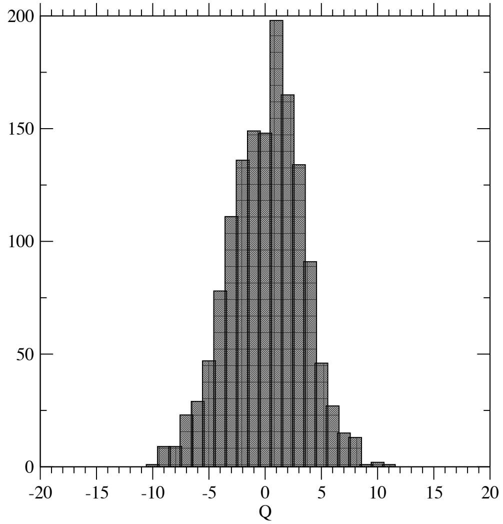
Plot by D. Schaich

Topological Charge Freeze

- A topological charge, or the winding number, of a give gauge configuration is defined as

$$
\begin{aligned}
Q[A] &= \frac{1}{32\pi^2}\int d^4x\epsilon_{\mu\nu\gamma\rho}\mathrm{Tr}F_{\mu\nu}(x)F_{\gamma\rho}(x), \\
F_{\mu\nu}(x) &= \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + [A_\mu(x), A_\nu(x)] \,.
\end{aligned}
$$

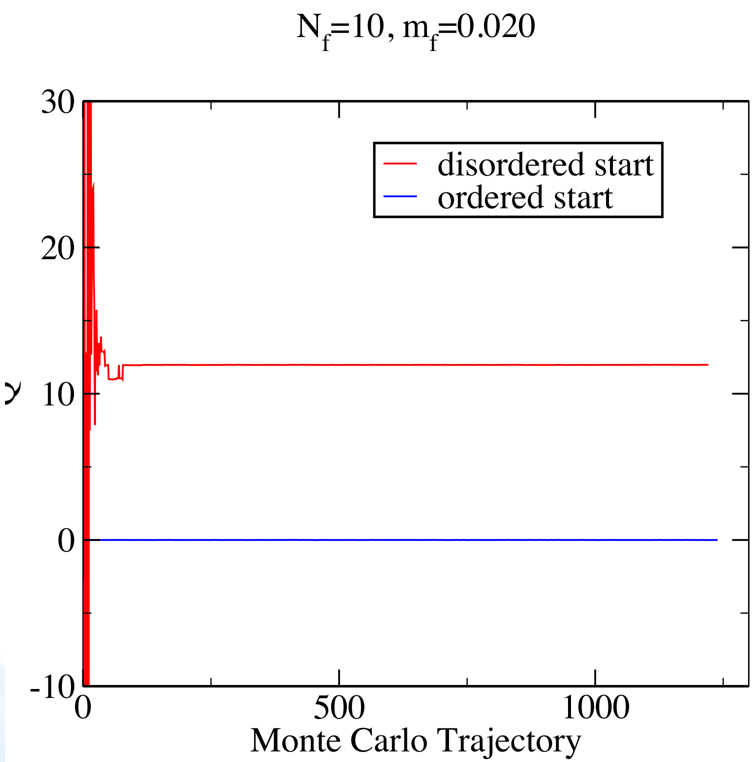
- For SU(3) gauge theories with N_f fundamental fermions, Q takes integer values.
- In a finite volume, Q can take all the possible integer values, which would follow a Gaussian distribution

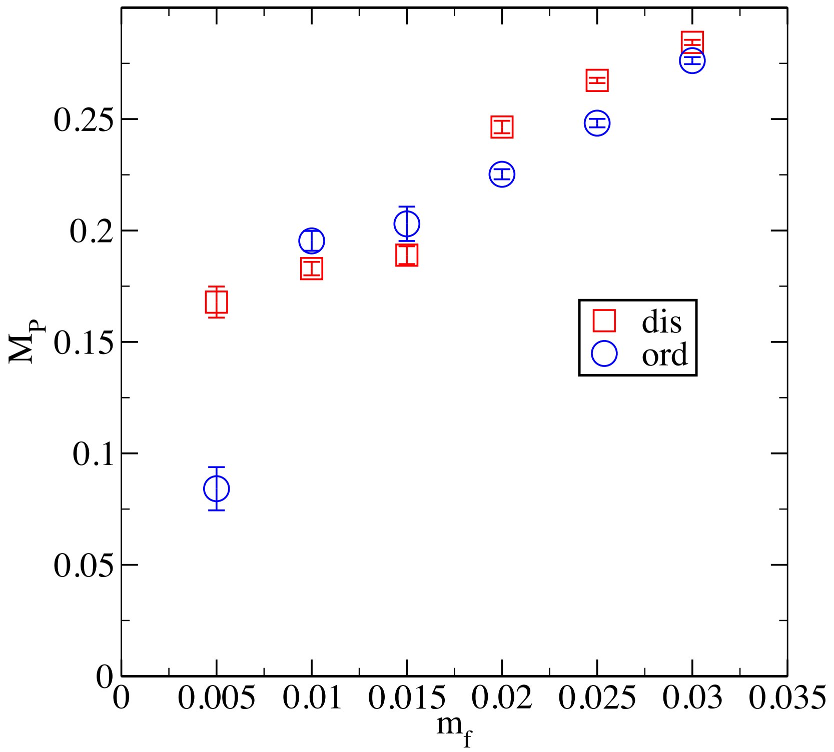


$N_f = 2+1$ QCD case, Aoki et al., PRD83, 074508(2011)

Topological Charge Freeze

- As N_f is increased, it becomes harder for topological charge to tunnel, especially at a small a.
- QCD simulations have similar issues at fine lattice spacings.





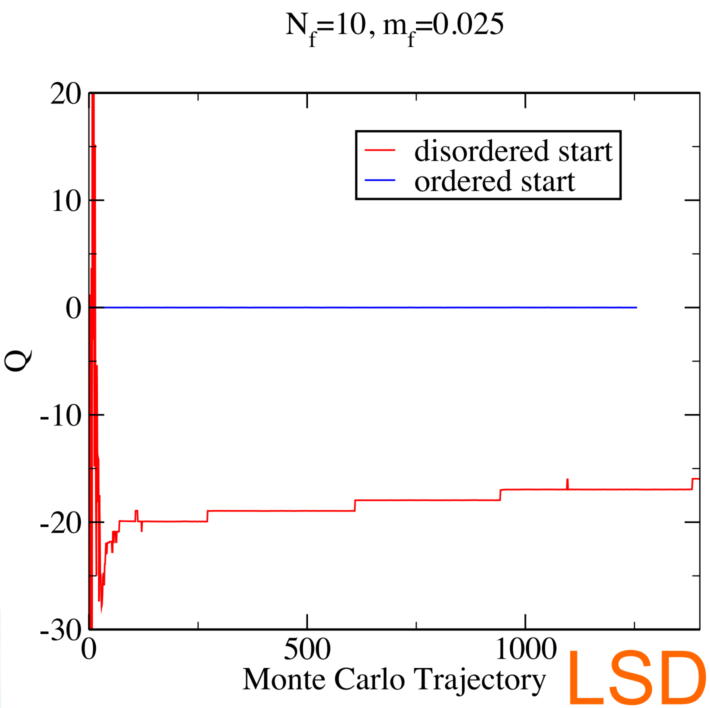
LSD Preliminary

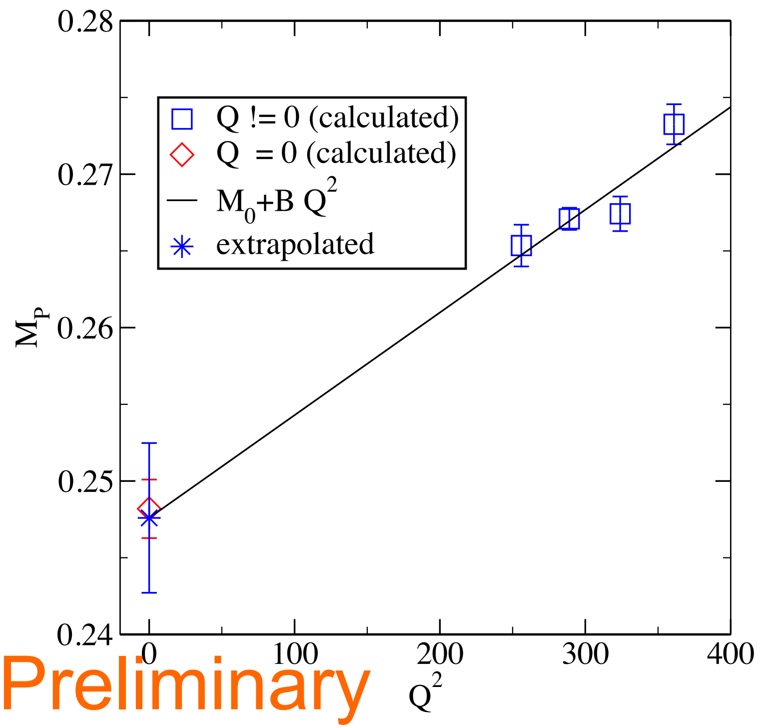
Effects of Frozen Topology

- At fixed Q, the hadron masses depends linearly on Q^2 to first order.

Brower, Chandrasekharan, Negele, and Wiese, PLB560, 64(2003)

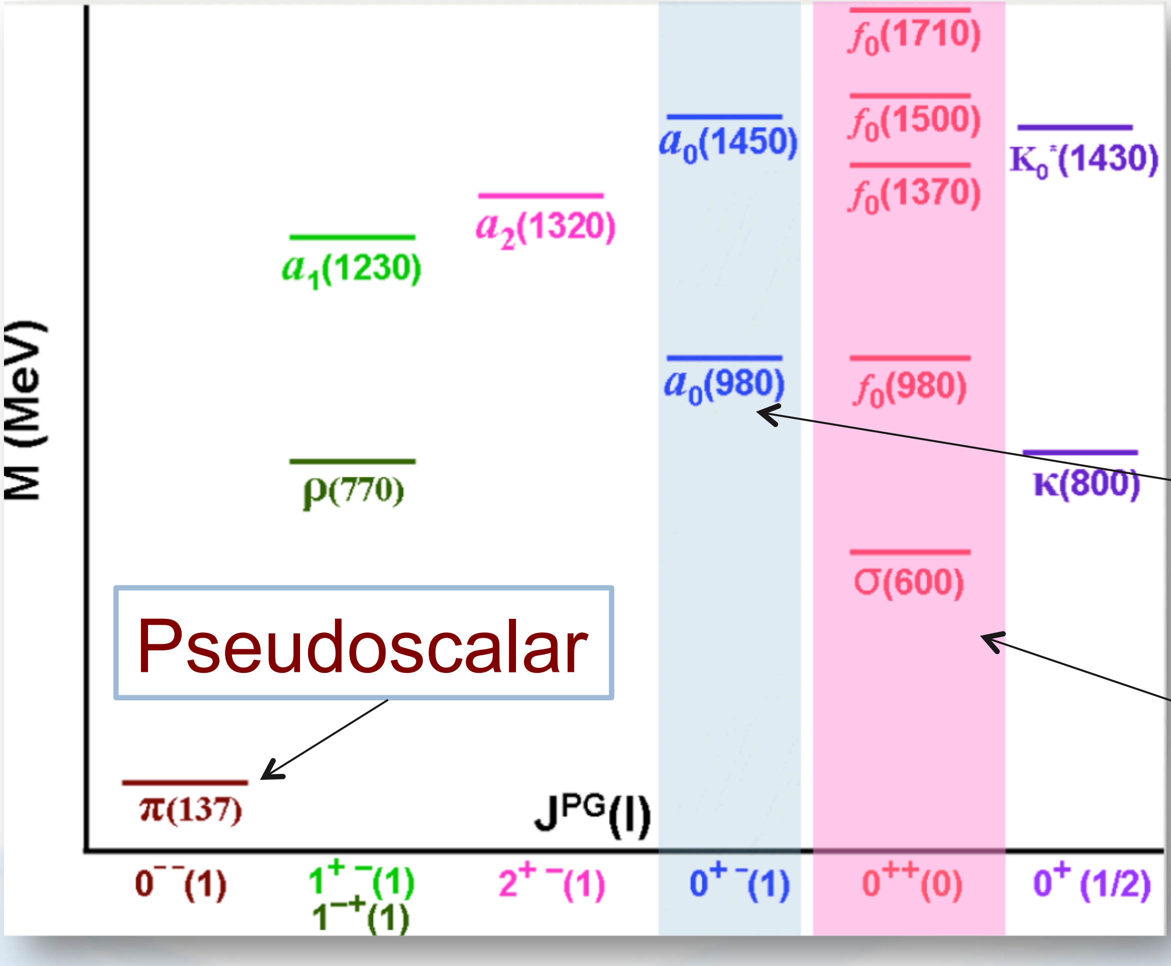
$$M_Q = M(0) + \frac{1}{2} M^{(2)}(0) \frac{1}{V\chi_t}\left(1 - \frac{Q^2}{V\chi_t}\right) + O\left(\frac{1}{V^3}\right),$$





LSD Preliminary

Scalar in QCD



Pseudoscalar

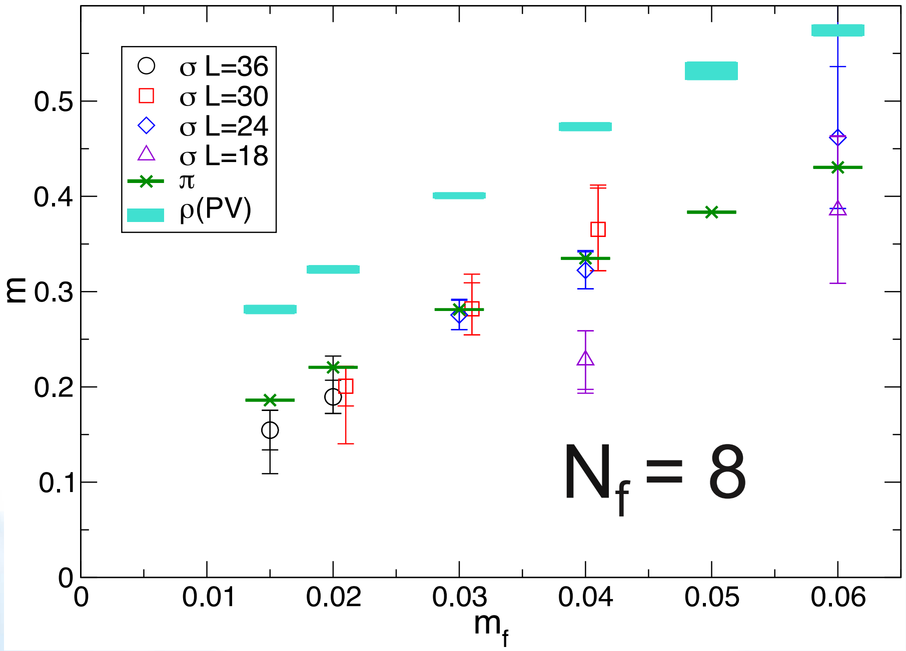
Flavor triplet

Flavor singlet

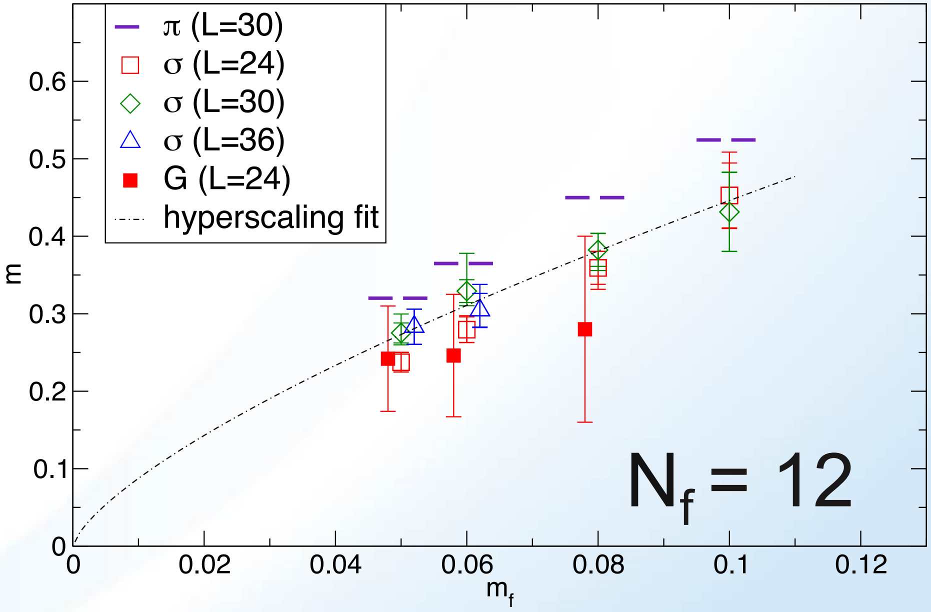
(Plot from N. Mathur et al. 2007.)

Light Composite Scalar from Near-Conformal Gauge Theories?

- Now that a new scalar boson has been discovered, can we produce such a light scalar from technicolor theories?
- First evidence appeared in near-conformal or conformal theories.



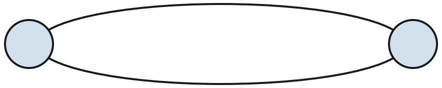
Y. Aoki et al. (LatKMI Coll.) 2014



Y.Aoki et al. (LatKMI Coll.) 2013

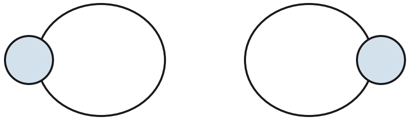
Not a cheap calculation

- For non-flavor singlet states, such as the pion, the correlation functions involve only the "connected" diagrams.



$$C(t) \quad = \quad \sum_{\vec{x}} \langle P(\vec{x},t) P^{\dagger}(\vec{0},0) \rangle$$

- For flavor-singlet states, such as the flavor-singlet scalar, disconnected diagrams are needed in the calculation.



➔ needs all-to-all propagators. Direct cost proportional to lattice volume

- Typically stochastic estimators are used ➔ noise is introduced, and requires a large number of noise estimators to get a signal.

- In the chiral limit, the pseudoscalar mass goes to 0, but the scalar mass should remain finite. ➔ Can we see this crossover with lighter fermion masses?

Summary and Outlook

- Lattice BSM simulations can provide valuable input for BSM model building.
- Such simulations are numerically expensive, and more challenging that QCD simulations.
- Prominent issues right now are the difficulty in getting to the chiral limit, and obtaining the flavor-singlet scalar boson mass with good precision.
- Outlook:
  - Increasing computing power and algorithmic innovations (multigrid?) will help us perform simulations at lighter masses and larger volumes.
  - Evolving theoretical understanding may pinpoint or rule out some BSM theories that are inconsistent with experimental observations.
  - Perhaps we will finally find the right BSM theory, or concede that standard model is the perfect theory and there's nothing more?