European Union
European Regional Development Fund
Spanish DGI : FIS 2012-38206
FPU Grant : AP 2009-0668

Loop Models

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Introduction

Loop models are statitstical mechanics problems whose degrees of freedom are loops or random walks. Concretely, loops appear in many areas in physics ranging from Anderson localization [1] to frustated magnetism [2]. They also are elements of loop algorithms for Monte Carlo in quantum magnetic systems [3-4]. Convenient discretization of the field theories involved in these models are interesting to avoid some difficulties inherent to these methods.

The models we present here belong to a family of completely-packed loop models in three dimension and in two dimensions [5-8]. These models have two phases, one with short loops and another with some extended loops. Between the two phases we find phase transitions of novel types.

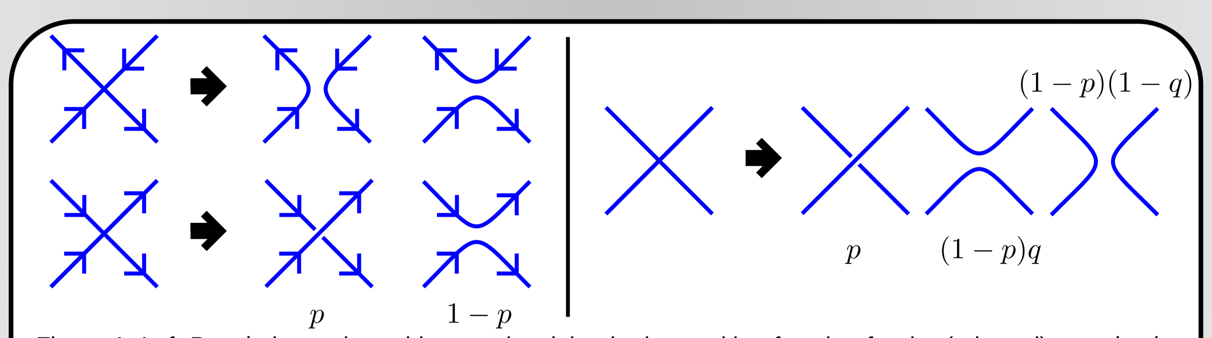

Figure 1. Left Panel shows the pairings and weights in the partition function for the (oriented) completely-packed loop model, for two different kind of lattices. Right panel shows them for the 'completely-packed loop model with crossings'

We studied two families of models, both defined on a four-fold coordination lattice. While in the first case the links are oriented, and there are 2 incoming and two outgoing links at each node, in the second case, the links are unoriented. Loops are composed by tiling appropiately the pairings of Fig. 1 and choosing one of n possible colors for each loop. Then, the partition function is the sum to all configurations of the products of the weights of each node.

Loop Model in 2D

In two dimensions, we studied the 'completely-packed loop model' (CPLC) [6]. Loops in this model are constructed with pairings at Fig. 1 (right panel) and a sample configuration is shown at Fig. 2 (left panel). The partition function is then

$$Z_{CPLC} = \sum_C p^{N_p}[(1-p)q]^{N_q}[(1-p)(1-q)]^{N_{1-q}} .$$

This partition function allows numerical tricks to have access to exceptionally large systems.

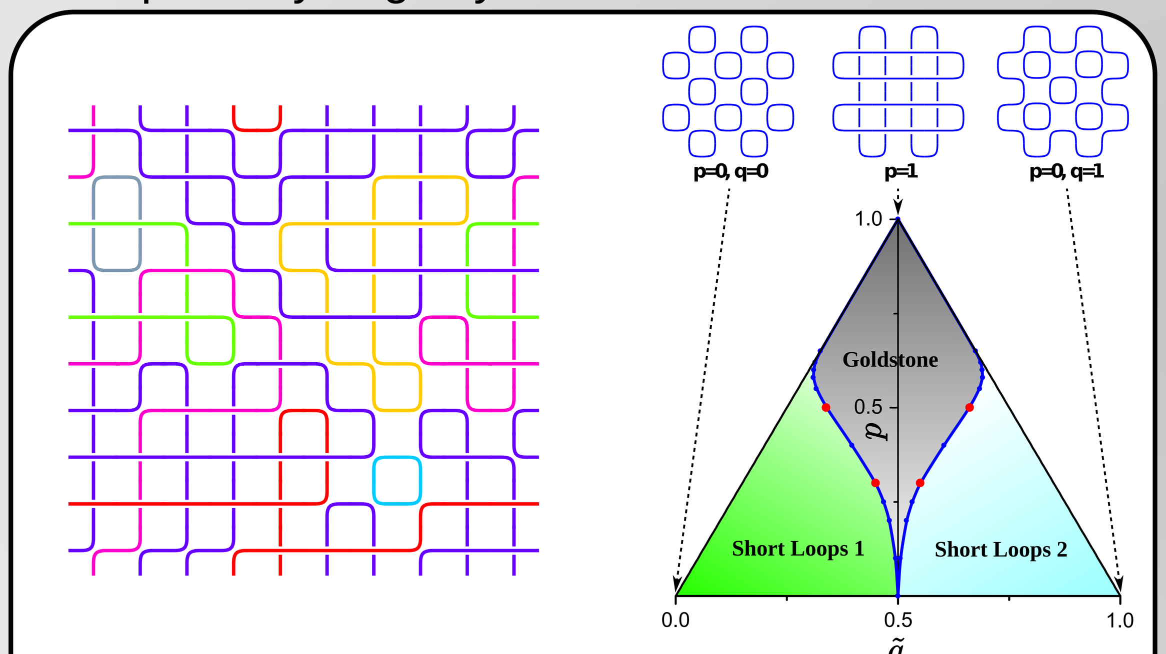

Figure 2. Left panel shows a 10x10 configuration of the CPLC in two dimensions. Right panel shows the phase diagram for the CPLC, where $\tilde{q} = (1-p)q$. It is shown a caricature of the configuration for the three special points: p=1, (p=0,q=1) and (p=0, q=0).

The loop model is a discretization of the RP^{n-1} sigma field theory, in the replica-limit of n→1. The phase diagram (Fig. 2, right panel) includes:

- **Goldstone Phase:** The loops are 'almost' brownian, however there are unusual logarithmic corrections.
- **Short loop Phase:** The disordered phase of the field theory.
- **Phase diagram boundary:** The phase diagrams are those of the 'completely packed loop model' on the L (p=0) or Manhattan lattice (q=0 or q=1) in 2d.
- **Critical Lines:** Show a new universality class of classical phase transitions (intriguing hint of connection with 2D Anderson metal-insulator transition, $\nu = 2.745(19)$)

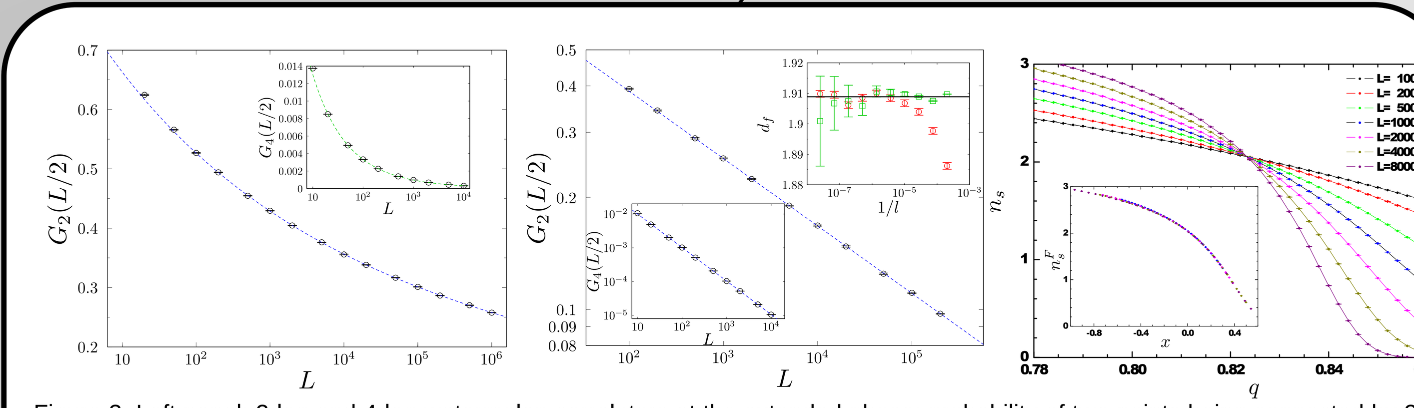

Figure 3. Left panel: 2-leg and 4-leg watermelon correlators at the extended phase: probability of two points being connected by 2 and 4 strands, respectively. The fitting function is to the universal law $G_k(L/2) = C_k/log(L/r_0)^{\alpha_k}$, with $\alpha_2 = 2,\ \alpha_4 = 12$. Center panel: shows the same correlators at the critical point for p=1/2. Fittings give an estimate for the exponents $x_2 = 0.091(1)$ and $x_4 = 0.491(1)$, where $G_k(L/2) \propto L^{-2x_k}$. Right panel: Spanning number (related to stiffness), number of strands which span the system, and its scaling form that gives an estimate for the correlation length critical exponent $\nu = 2.745(19)$.

Loop Models in 3D

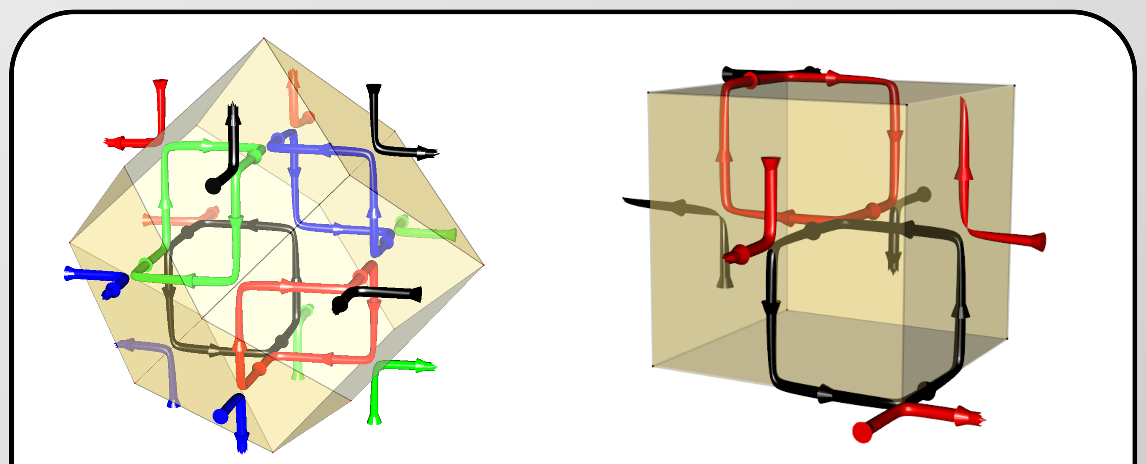

Figure 4. p=0 configurations for the Cardy's 3D L-lattice [1] (left) and K-lattice (right) [5,7], with the Wigner-Seitz cell faces shaded.

We studied 3D loop models that can be mapped to CP^{n-1} sigma models (field theories familiar from 2D quantum magnets with SU(n) symmetry). Loops are formed with pairings at Fig. 1 (left panel). Here, the parameter n is the number of possible colours for each loop. The partition function is

$$Z_{loops} = \sum_C p^{N_p}(1-p)^{N_{1-p}} n^{\text{number of loops}} .$$

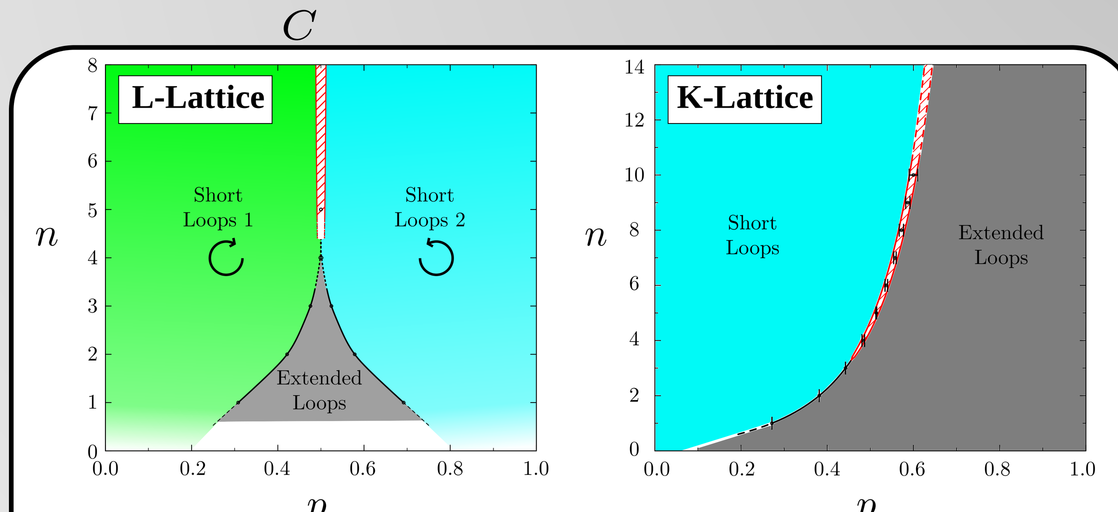

Figure 5. Phase diagrams for the two lattices studied.

Deconfined Criticality [12] in classical loop model

- p=1/2 gives a Z_4 symmetry between four possible short loop phases.
- Add new coupling J that favours short loops while preserving symmetry.

Increasing J leads to a new phase transition spontaneously breaking the symmetry of the lattice and with an emergent U(1) symmetry [13]. This is the deconfined transition, discussed at length in (2+1)D magnets: the loop model provides a more convenient platform for studying its properties.

Phase diagram (Fig. 5):

- **Extended loops phase:** The ordered phase of the field theory. Corresponds to the Neel state in (2+1d) SU(n) magnets.
- **Short loops phase:** The massive phase of the field theory. Corresponds to a dimerised phase in SU(n) magnets.
- **Critical line:** Phase transitions of the (compact) CP^{n-1} sigma model or AF - VBL transition in SU(n) magnets.

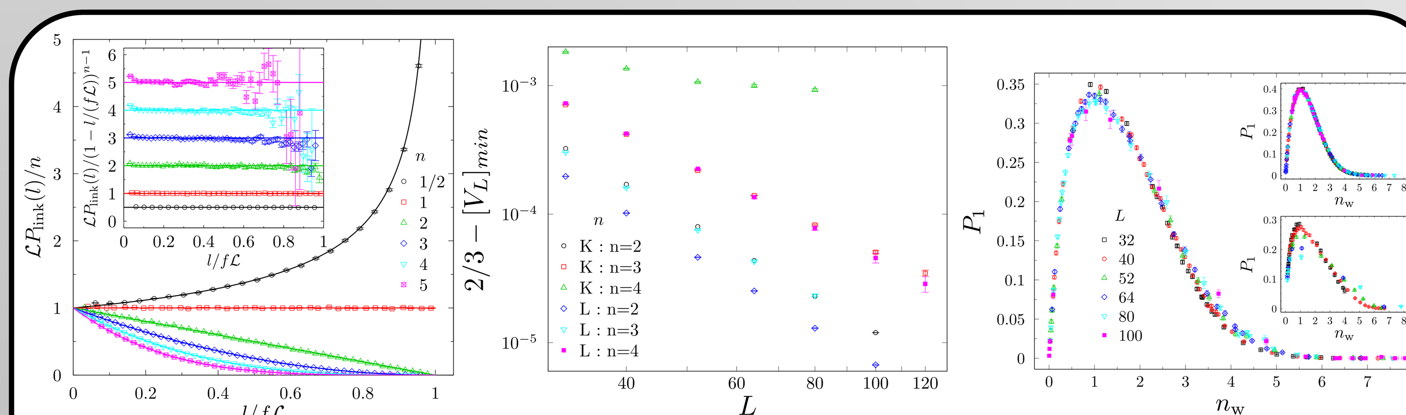

Figure 6. Left panel: Probability of a chosen link to belong to a loop of length l, where $\mathcal{L}$ is the number of links of the system and f is the fraction of them occupied by extended loops [8]. Center panel: Binder cumulant of the energy for several n and both lattices, when the transition is continuous it goes to zero with a power-law. Right panel: Probability of having just one extended strand in the system versus the average of the number of spanning strands (n=3 main panel, n=2 top inset, and n=4 lower inset) .

Critical Points.

- *n=1.* Exact mapping to the class C model of the Anderson transitions [9-11]. Critical exponents: $\nu = 0.997(2),\ \eta = -0.06(2)$.
- *n=2.* Critical exponents compatible with O(3) universality class: $\nu = 0.708(5),\ \eta = 0.04(3)$.
- *n=3.* Results suggest novel critical point that is unexpected from the point of view of Landau Ginsburg theory. Critical exponents: $\nu = 0.536(13),\ \eta = 0.23(2)$.
- *n≥4.* Discontinuos phase transitions.

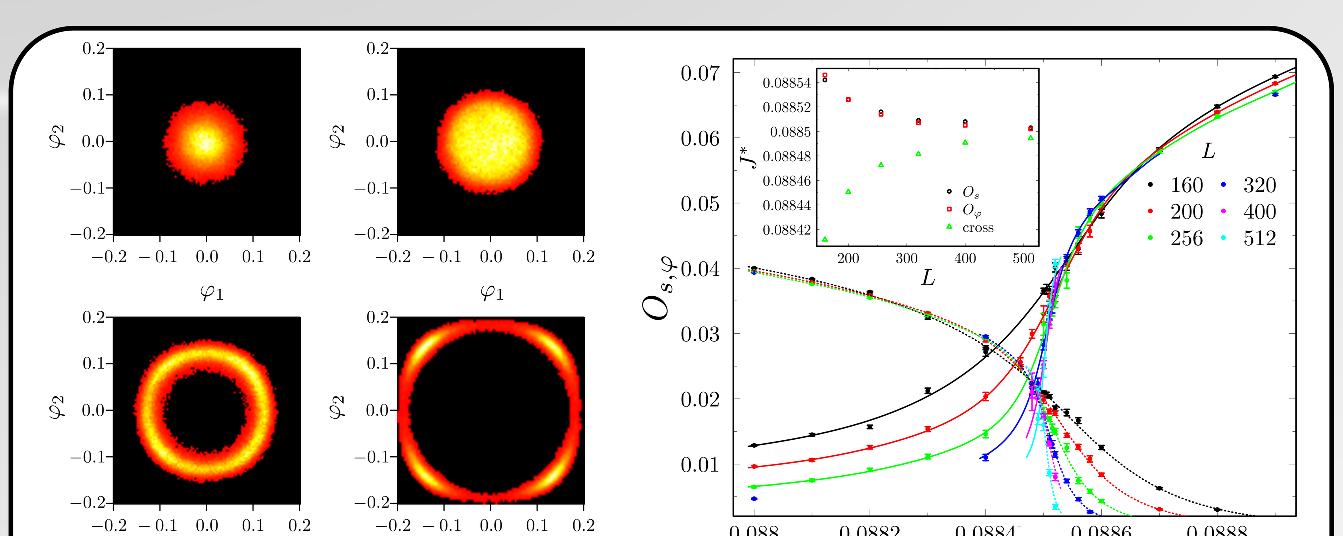

Figure 6. Left Panel: Probability distribution of one of the order parameters $\vec{\varphi}$, as the coupling constant J varies and the system cross the phase transition. Right Panel: Two order parameters simultaneously critical (O_s , $O_\varphi = |\vec{\varphi}|$).

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