Classical and Quantum Simulation of Gauge Theories in Particle and Condensed Matter Physics

Uwe-Jens Wiese

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics, Bern University

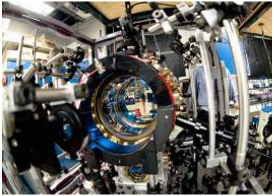
Field Theoretic Computer Simulations
for Particle Physics and Condensed Matter
Boston University, May 9, 2014

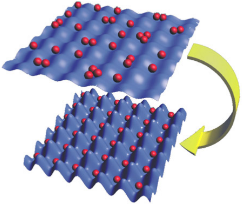
Collaboration:
Debasish Banerjee, Michael Bögli, Pascal Stebler, Philippe Widmer (Bern), Christoph Hofmann (Colima), Fu-Jiun Jiang (Taipei), Mark Kon (BU), Marcello Dalmonte, Peter Zoller (Innsbruck), Enrique Rico Ortega (Strasbourg), Markus Müller (Madrid)

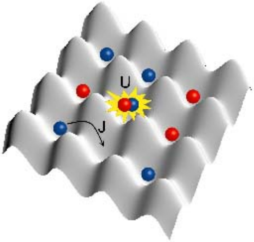
Outline

Outline

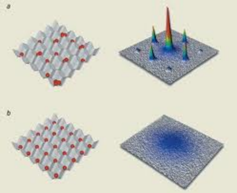
Ultra-cold atoms in optical lattices as analog quantum simulators





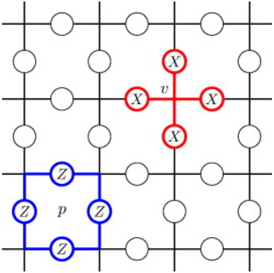


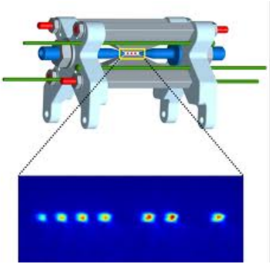
Superfluid-Mott insulator transition in the bosonic Hubbard model



M. Greiner, O. Mandel, T. Esslinger, T. Hänsch, I. Bloch,
Nature 415 (2002) 39.

Digital quantum simulation of Kitaev's toric code (a $\mathbb{Z}(2)$ lattice gauge theory) with trapped ions





• Precisely controllable many-body quantum device, which can execute a prescribed sequence of quantum gate operations.
• State of simulated system encoded as quantum information.
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

A. Y. Kitaev, Ann. Phys. 303 (2003) 2.
B. P. Lanyon et al., Science 334 (2011) 6052.

Outline

Hamiltonian formulation of Wilson's $U(1)$ lattice gauge theory

$$U = \exp(i\varphi), \ U^\dagger = \exp(-i\varphi) \in U(1)$$

Electric field operator E

$$E = -i\partial_\varphi, \ [E, U] = U, \ [E, U^\dagger] = -U^\dagger, \ [U, U^\dagger] = 0$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

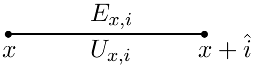
$U(1)$ gauge invariant Hamiltonian

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_\Box (U_\Box + U_\Box^\dagger),$$

$$U_\Box = U_{x,i} U_{x+\hat{i},j} U^\dagger_{x+\hat{j},i} U^\dagger_{x,j}$$

operates in an infinite-dimensional Hilbert space per link

$U(1)$ quantum link model



$$U = S_1 + iS_2 = S_+, \;\; U^\dagger = S_1 - iS_2 = S_-$$

Electric flux operator E

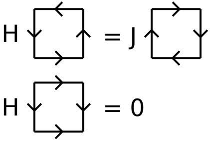
$$E = S_3, \;\; [E, U] = U, \;\; [E, U^\dagger] = -U^\dagger, \;\; [U, U^\dagger] = 2E$$

Generator of $U(1)$ gauge transformations

$$G_x = \sum_i (E_{x-\hat{i},i} - E_{x,i}), \quad [H, G_x] = 0$$

Gauge invariant Hamiltonian for $S = \frac{1}{2}$

$$H = -J \sum_{\square} (U_\square + U_\square^\dagger)$$



defines a gauge theory with a 2-d Hilbert space per link.

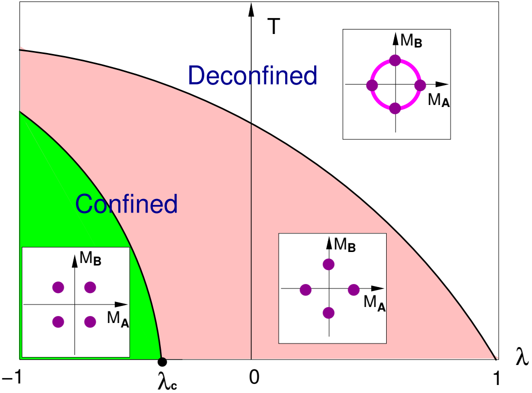
D. Horn, Phys. Lett. B100 (1981) 149
D. S. Rokhsar, S. A. Kivelson, Phys. Rev. Lett. 61 (1988) 2376
P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647
S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455
R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Outline

Hamiltonian with Rokhsar-Kivelson term

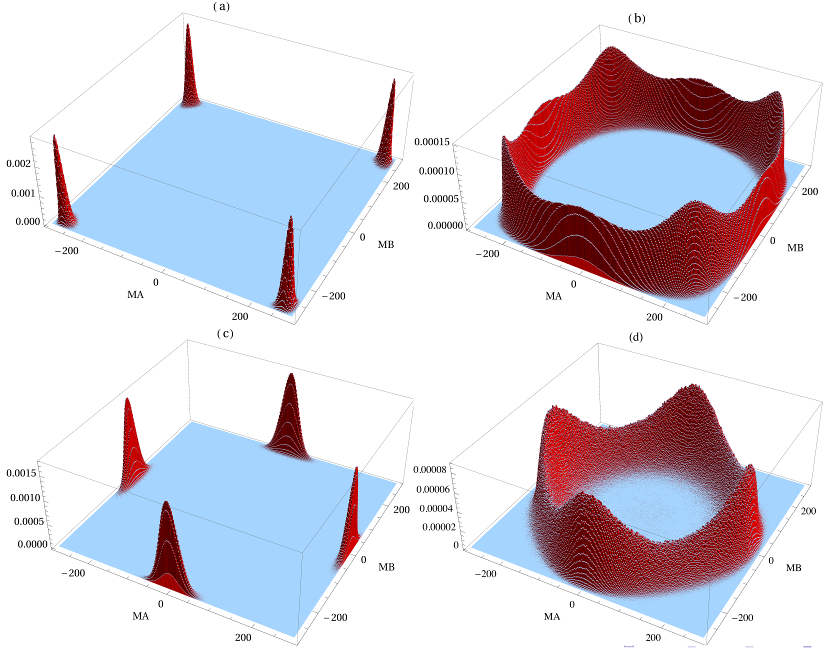
$$H = -J \left[\sum_{\square} (U_{\square} + U_{\square}^{\dagger}) - \lambda \sum_{\square} (U_{\square} + U_{\square}^{\dagger})^2 \right]$$

Phase diagram

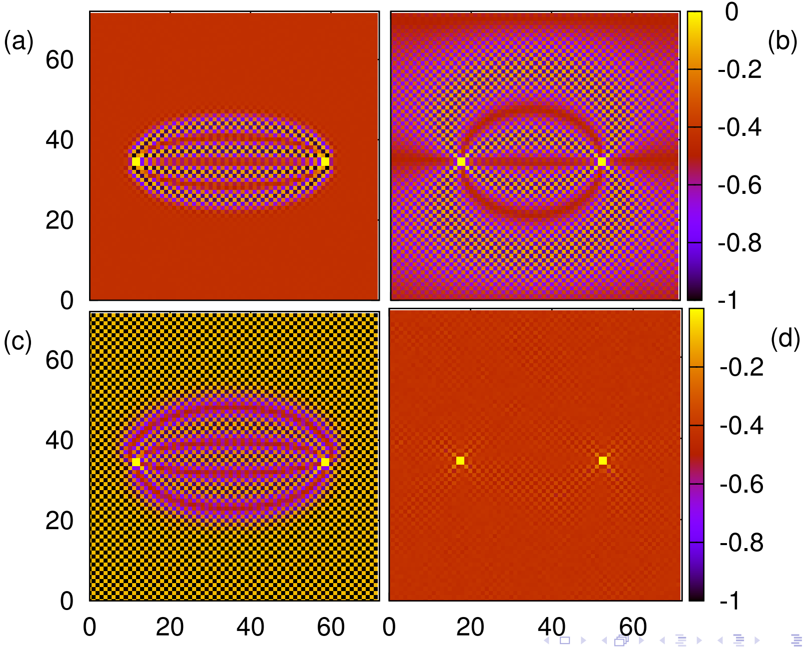


D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

Probability Distribution of the Order Parameters: $L^2 = 24^2$

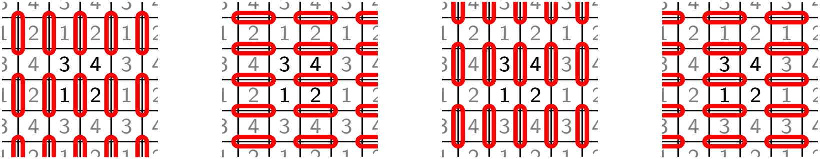


Energy density of charge-anti-charge pair $Q = \pm 2$: $L^2 = 72^2$

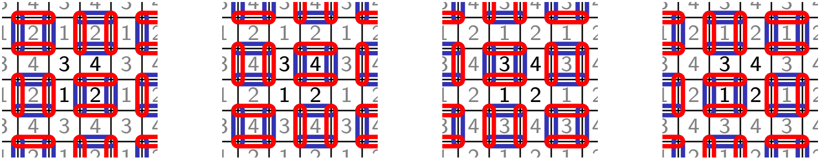


Rokhsar and Kivelson's Quantum Dimer Model

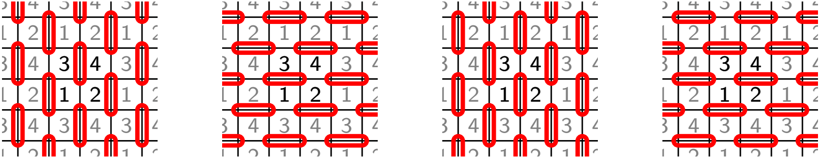
Confining columnar phase:



Confining plaquette phase:



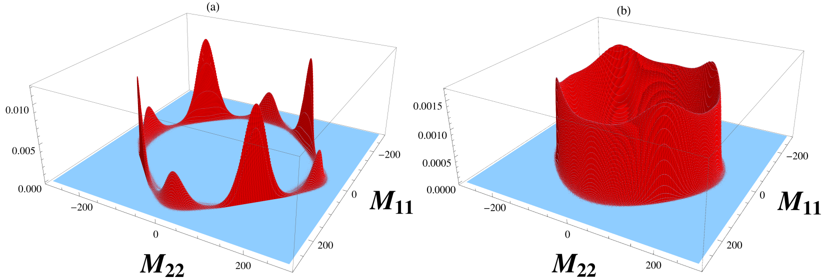
Deconfined staggered phase:



Hamiltonian with Rokhsar-Kivelson term

$$H = -J \left[\sum_{\Box} (U_{\Box} + U_{\Box}^{\dagger}) - \lambda \sum_{\Box} (U_{\Box} + U_{\Box}^{\dagger})^2 \right]$$

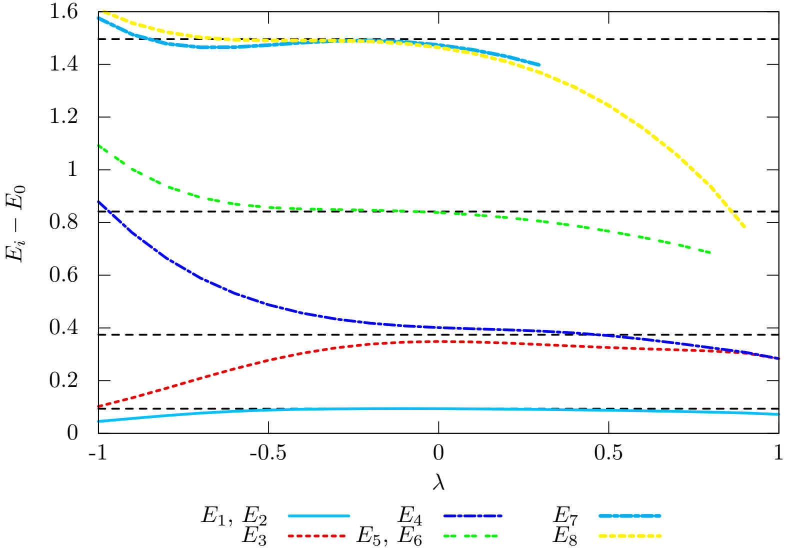
Probability distribution of order parameters: $L^2 = 24^2$



A soft mode arises near $\lambda \approx -0.2$ and extends all the way to the RK-point $\lambda = 1$.

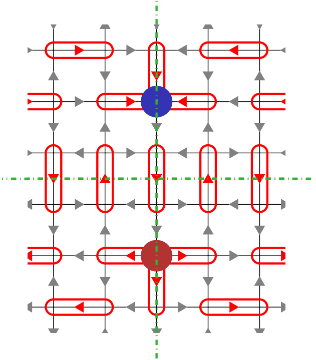
D. Banerjee, M. Bögli, C. P. Hofmann, F.-J. Jiang, P. Widmer, UJW, in preparation.

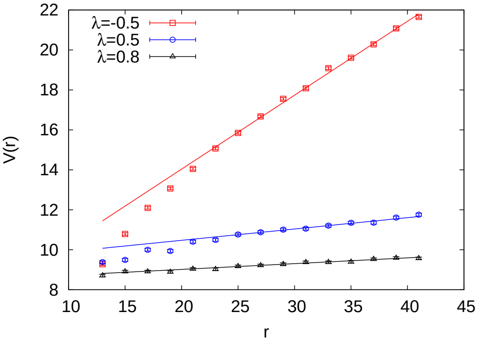
Soft mode leads to an almost perfect rotor spectrum: $L^2 = 8^2$



The small splitting $E_4 - E_3$ originates from an explicit breaking of the emergent $SO(2)$ symmetry.

External charges and confining potential: $L^2 = 120^2$, $\beta J = 100$





String tension:

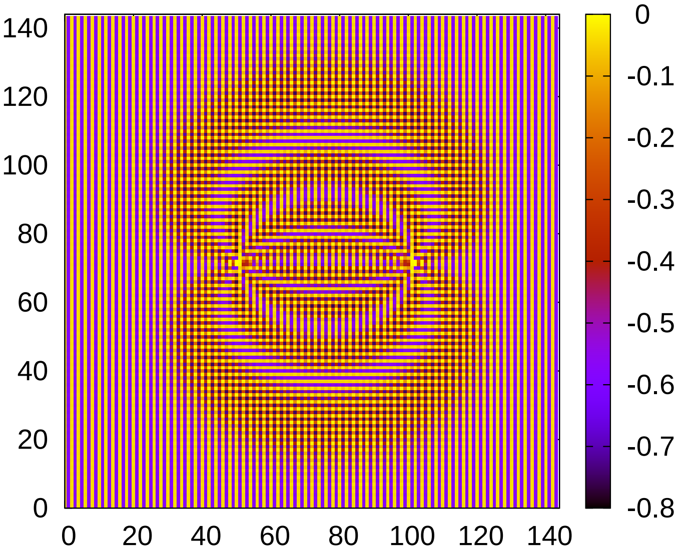
$\sigma(\lambda = -0.5) = 0.370(1) J/a,$

$\sigma(\lambda = 0.5) = 0.057(1) J/a,$

$\sigma(\lambda = 0.8) = 0.029(1) J/a$

The soft mode almost deconfines. Since for $\lambda \neq 1$ it is not exactly massless, it can not quite be identified as a dual photon.

Energy density of charge-anti-charge pair $Q = \pm 2$: $L^2 = 144^2$



The string separates into 8 strands of fractionalized flux $\frac{1}{4}$.

Outline

Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

Bosonic rishon representation of the quantum links

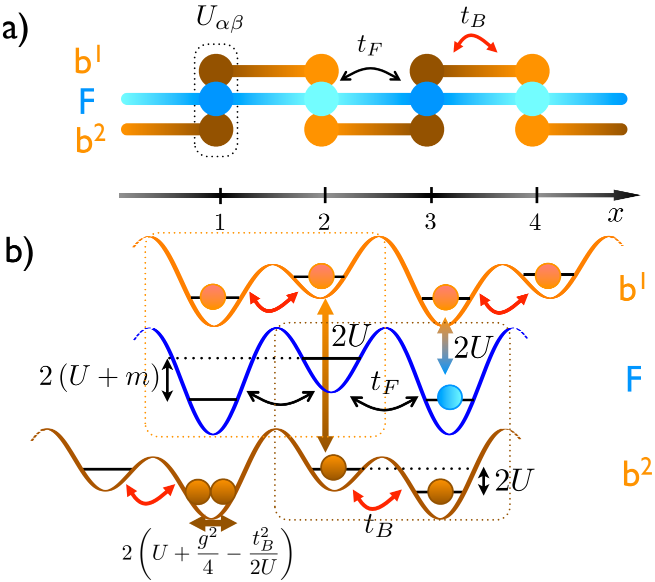
$$U_{x,x+1} = b_x b_{x+1}^\dagger, \;\; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

Microscopic Hubbard model Hamiltonian

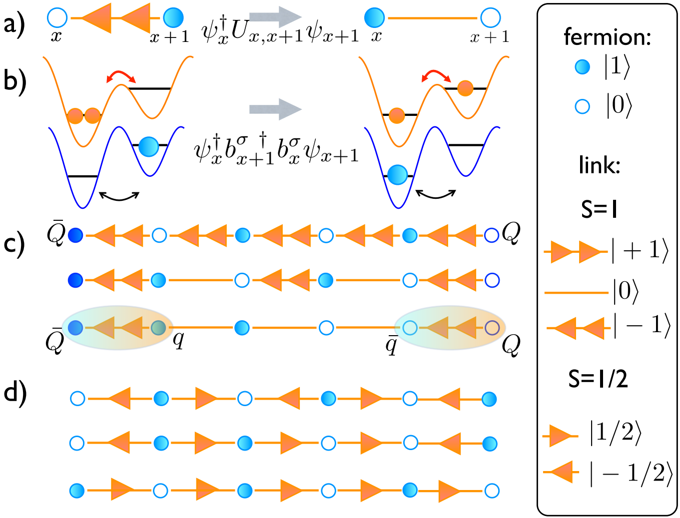
$$\begin{aligned}
\widetilde{H} &= \sum_x h^B_{x,x+1} + \sum_x h^F_{x,x+1} + m \sum_x (-1)^x n_x^F + U \sum_x \widetilde{G}_x^2 \\
&= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \mathrm{h.c.} \\
&+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha
\end{aligned}$$

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

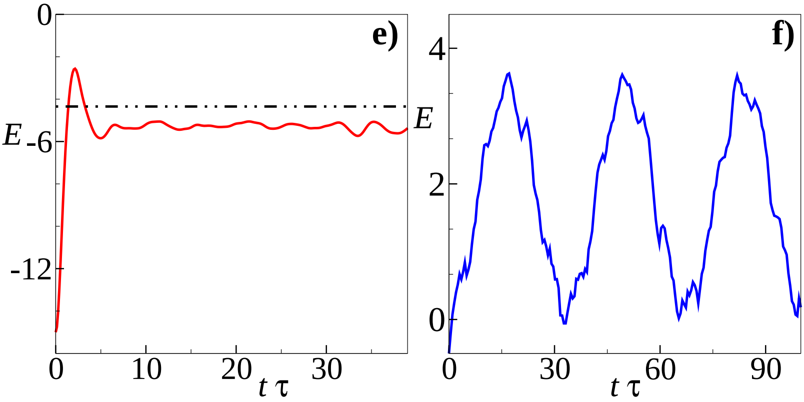
Optical lattice with Bose-Fermi mixture of ultra-cold atoms



From string breaking to false vacuum decay



Quantum simulation of the real-time evolution of string breaking and of coherent vacuum oscillations



Outline

$U(N)$ quantum link operators

$$U^{ij} = S_1^{ij} + iS_2^{ij}, \ U^{ij\dagger} = S_1^{ij} - iS_2^{ij}, \ i,j \in \{1,2,\dots,N\}, \ [U^{ij},(U^\dagger)^{kl}] \neq 0$$

$SU(N)_L \times SU(N)_R$ gauge transformations of a quantum link

$$[L^a, L^b] = if_{abc}L^c, \ [R^a, R^b] = if_{abc}R^c, \ a,b,c \in \{1,2,\dots,N^2-1\}$$

$$[L^a, R^b] = [L^a, E] = [R^a, E] = 0$$

Infinitesimal gauge transformations of a quantum link

$$[L^a, U] = -\lambda^a U, \ [R^a, U] = U\lambda^a, \ [E, U] = U$$

Algebraic structures of different quantum link models

$U(N)$: $U^{ij}, \ L^a, \ R^a, E, \ 2N^2 + 2(N^2-1) + 1 = 4N^2 - 1$ $SU(2N)$ generators

$SO(N)$: $O^{ij}, \ L^a, \ R^a, \ N^2 + 2\frac{N(N-1)}{2} = N(2N-1)$ $SO(2N)$ generators

$Sp(N)$: $U^{ij}, \ L^a, \ R^a, \ 4N^2 + 2N(2N+1) = 2N(4N+1)$ $Sp(2N)$ generators

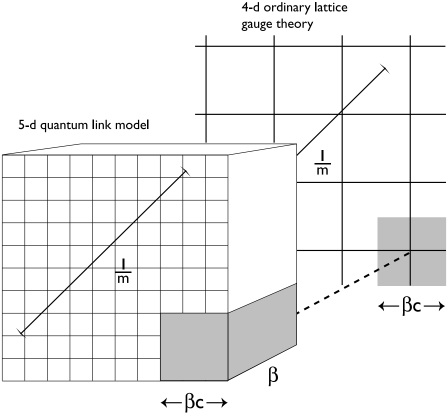
R. Brower, S. Chandrasekharan, UJW, Phys. Rev. D60 (1999) 094502

Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\text{Tr } G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \text{Tr } \partial_5 G_\mu \partial_5 G_\mu \right), \; G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

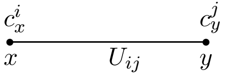
$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \text{Tr } G_{\mu\nu} G_{\mu\nu}, \; \frac{1}{g^2} = \frac{\beta}{e^2}, \; \frac{1}{m} \sim \exp\left(\frac{24\pi^2\beta}{11Ne^2}\right)$$



Fermionic rishons at the two ends of a link

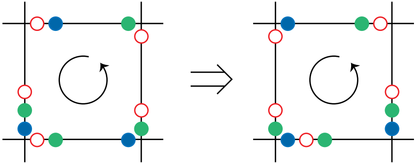
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

Rishon representation of link algebra

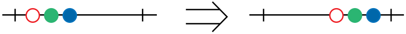


$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?

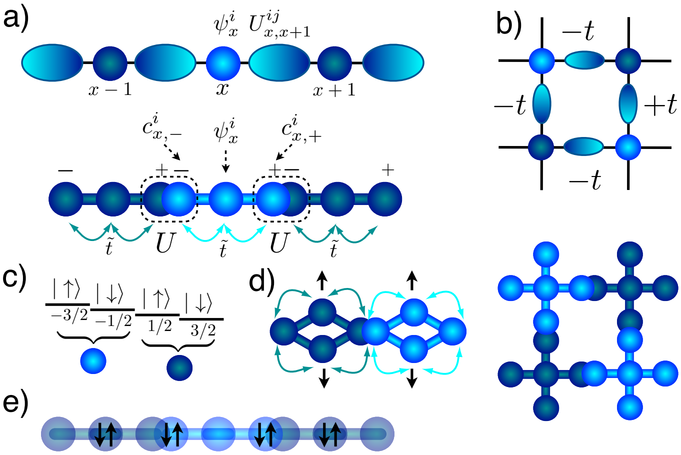


Tr Up



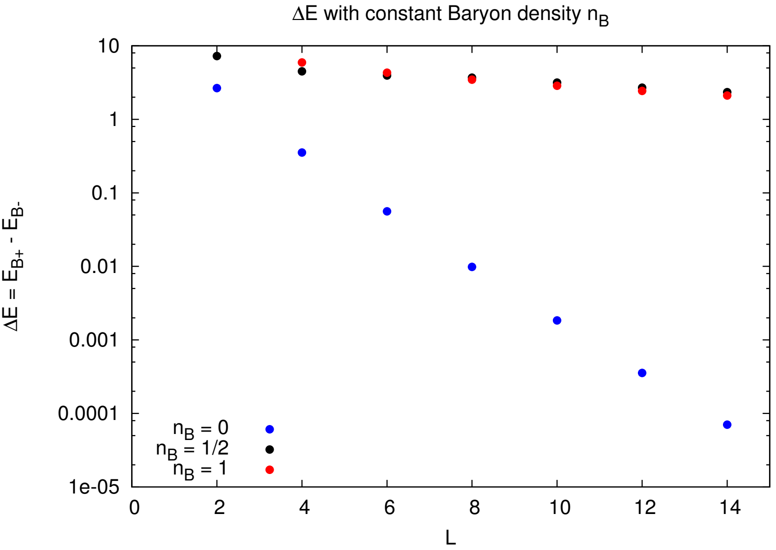
det $U_{x,\mu}$

Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Restoration of chiral symmetry at baryon density $n_B \geq \frac{1}{2}$



Other proposals for digital quantum simulators for Abelian and non-Abelian quantum link models

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.
L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.
L. Tagliacozzo, A. Celi, A. Zamora, M. Lewenstein, Ann. Phys. 330 (2013) 160.

Other proposals for analog quantum simulators for Abelian and non-Abelian gauge theories with and without matter

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.
E. Kapit, E. Mueller, Phys. Rev. A83 (2011) 033625.
E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.
E. Zohar, J. Cirac, B. Reznik, Phys. Rev. Lett. 109 (2012) 125302; Phys. Rev. Lett. 110 (2013) 055302; Phys. Rev. Lett. 110 (2013) 125304.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

Outline

Conclusions

• Quantum link models implemented with ultra-cold atoms serve as quantum simulators for Abelian and non-Abelian gauge theories, which can be validated in efficient classical cluster algorithm simulations.

• Such simulations have revealed a soft mode in the quantum dimer model. Can this play the role of "phonons" in high-T_c materials?

• Quantum simulator constructions have already been presented for the $U(1)$ quantum link model as well as for $U(N)$ and $SU(N)$ quantum link models with fermionic matter.

• This would allow the quantum simulation of the real-time evolution of string breaking as well as the quantum simulation of "nuclear" physics and dense "quark" matter, at least in qualitative toy models for QCD.

• Accessible effects may include chiral symmetry restoration, baryon superfluidity, or color superconductivity at high baryon density, as well as the quantum simulation of "nuclear" collisions.

• The path towards quantum simulation of QCD will be a long one. However, with a lot of interesting physics along the way.