Chiral fermions and precision lattice field theory

PPCM – Boston University

May 9, 2014

Norman H. Christ

Columbia University

RBC and UKQCD Collaborations

Outline

- Lattice fermions
- State-of-the-art lattice QCD
- Apply to two processes:
  - $K \rightarrow \pi\,\pi$ decay
  - $K_L - K_S$ mass difference

RBC Collaboration

- BNL
  - Chulwoo Jung
  - Chris Kelly
  - Christoph Lehner
  - Amarjit Soni
- RBRC
  - Tomomi Ishikawa
  - Taku Izubuchi (BNL)
  - Shigemi Ohta (KEK)
  - Eigo Shintani
- Connecticut
  - Tom Blum
- Columbia
  - Ziyuan Bai
  - Norman Christ
  - Luchang Jin
  - Jasper Lin
  - Robert Mawhinney
  - Greg McGlynn
  - David Murphy
  - Jianglei Yu
  - Daiqian Zhang

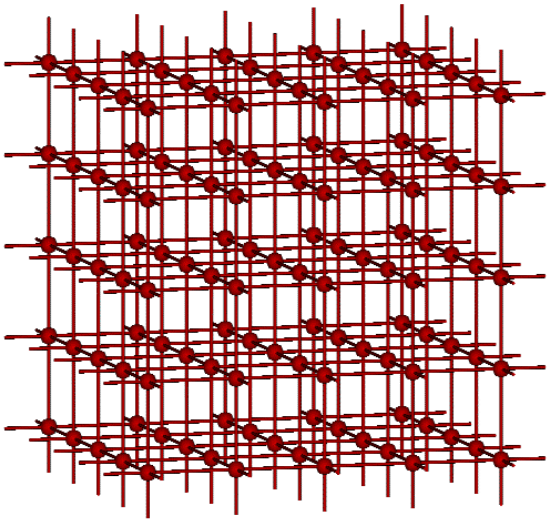
UKQCD Collaboration

- Edinburgh
  - Peter Boyle
  - Julien Frison
  - Nicolas Garron (Trinity)
  - Jamie Hudspith
  - Karthee Sivalingam
- Southampton
  - Shane Drury
  - Tadeusz Janowski
  - Andreas Juttner
  - Andrew Lytle (Mumbai)
  - Marina Marinkovic
  - Antonin Portelli
  - Chris Sachrajda

Lattice Fermions

Lattice QCD

- First-principles treatment of low-energy, non-perturbative QCD.
- All approximations understood and controlled:
  - Non-zero lattice spacing: $a \rightarrow 0$.
  - Finite volume: $L \rightarrow \infty$
  - Typically neglect E&M and $m_u \neq m_d$, $\alpha_{\text{EM}} << 1$



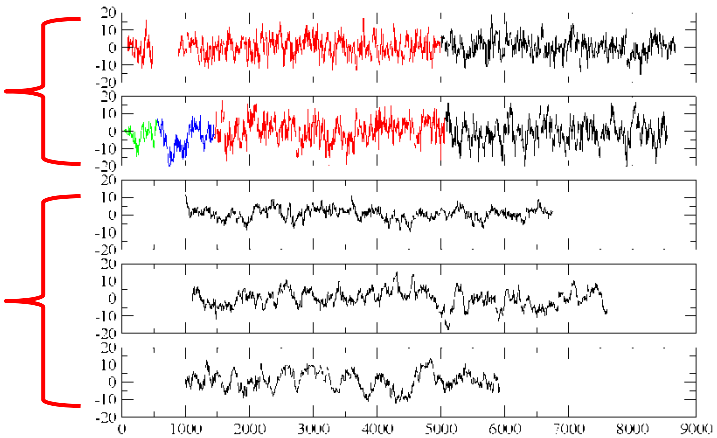
- Supports not only rough phenomenology but also accurate theoretical physics (where it can be applied).

Lattice QCD – gauge action

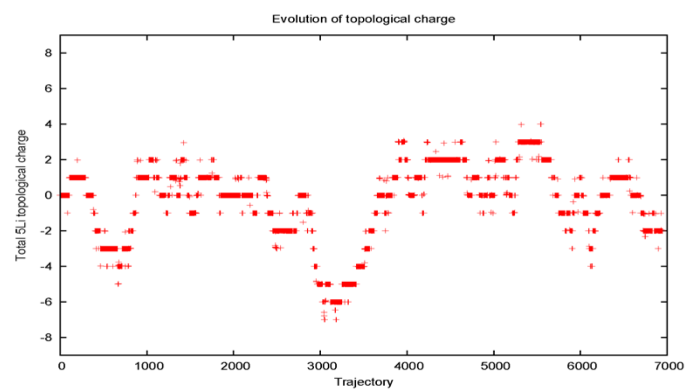
- Theories which differ at the lattice scale can describe the same the long distance physics.
- Lattice gauge action chosen to achieve:
  - Reduced lattice artifacts: *Symanzik improved*, *Iwasaki* (improvement effects are expected but not verified)
  - Alter the short-distance behavior to vary the rate of topological tunneling. (effects are visible and important)

1/a = 1.73 GeV

1/a = 2.28 GeV



1/a = 3.00 GeV



Lattice QCD – fermion action

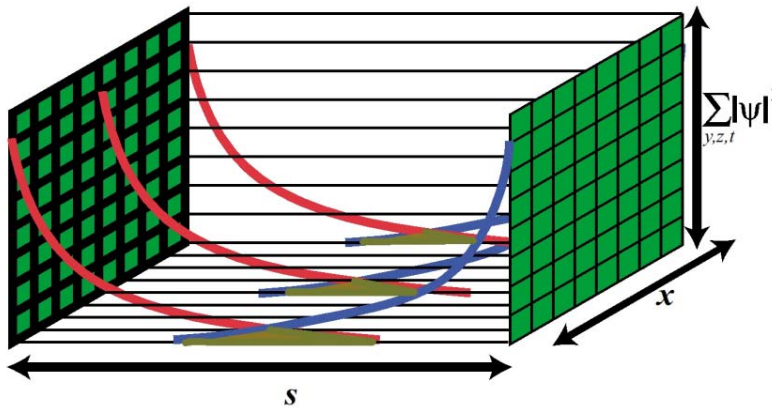
- Great variety of fermion actions in use:
  - *Wilson* (*clover*)
  - *staggered* (*naïve, p4, ASQTAD, HISQ*)
  - *domain wall* (*Shamir, Mobius, optimal*)
  - *overlap*
- Fermion action chosen to achieve:
  - Reduced lattice artifacts: *clover*, *p4, ASQTAD, HISQ*
  - Accurate chiral symmetry (*DWF*)
  - Exact chiral symmetry (*overlap*)

Lattice QCD – fermion action

- The choice of fermion action does have important consequences!
- Wilson: large breaking of chiral symmetry
- Staggered:
  - Large lattice artifacts without improvement.
  - Four extra species (*tastes*) partially removed by *rooting*
  - Chiral symmetry broken, mass protected by *taste* symmetry.
- Domain wall: 12 – 48x more costly than Wilson
- Overlap:
  - ~100x more costly than Wilson
  - Monte Carlo simulation requires fixed topology
    → $1/L$ corrections and dangerous non-ergodicity

Domain wall fermions

- With adequate computer power, DWF become an interesting choice!
- Use 5-dimensional Wilson fermion action with Dirichelet BC on $s = 0$ and $s = L_s{-}1$ slice.

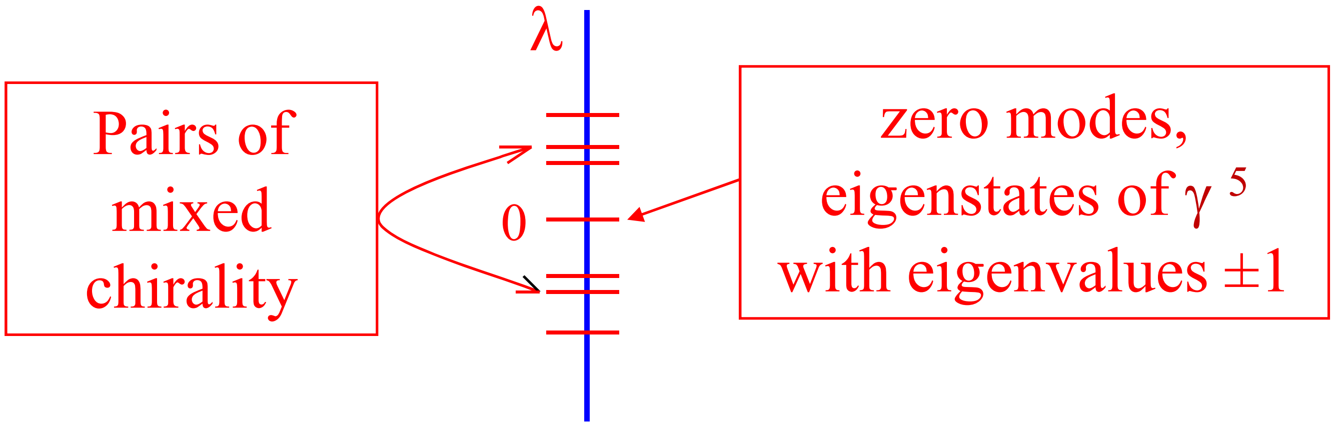


- 4-D chiral bound states form on the s = 0 and $L_s{-}1$ walls.
- 5-D propagating states are large-action, lattice artifacts.
- 4-D states disappear as $p \rightarrow \pi/a$, solving the doubling problem.
- Accurate chiral symmetry at all energies, broken by left-right mixing: residual chiral symmetry breaking.

Chiral symmetry and topology

- Continuum Dirac operator $D = D_\mu \gamma^\mu$ obeys

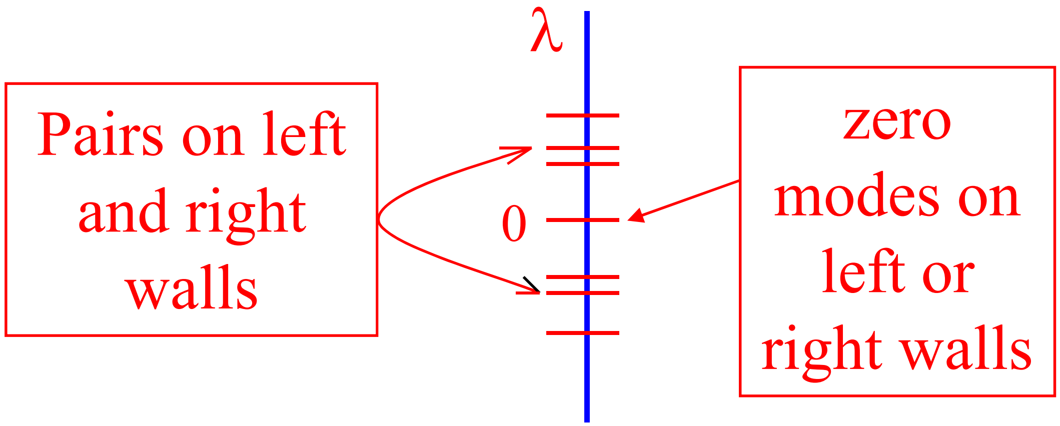
$\{D, \gamma^5\} = 0 \rightarrow$ if $D\psi_n = \lambda_n \psi_n$ then $D\gamma^5\psi_n = -\lambda_n \gamma^5 \psi_n$



- Topological invariance of $N_R - N_L$ easy to see!
- Atiyah-Singer theorem relates topological charge of gauge field to $N_R - N_L$ of fermions.

Connection with Topology

- For the domain wall operator, low eigenvalues show this chiral pattern

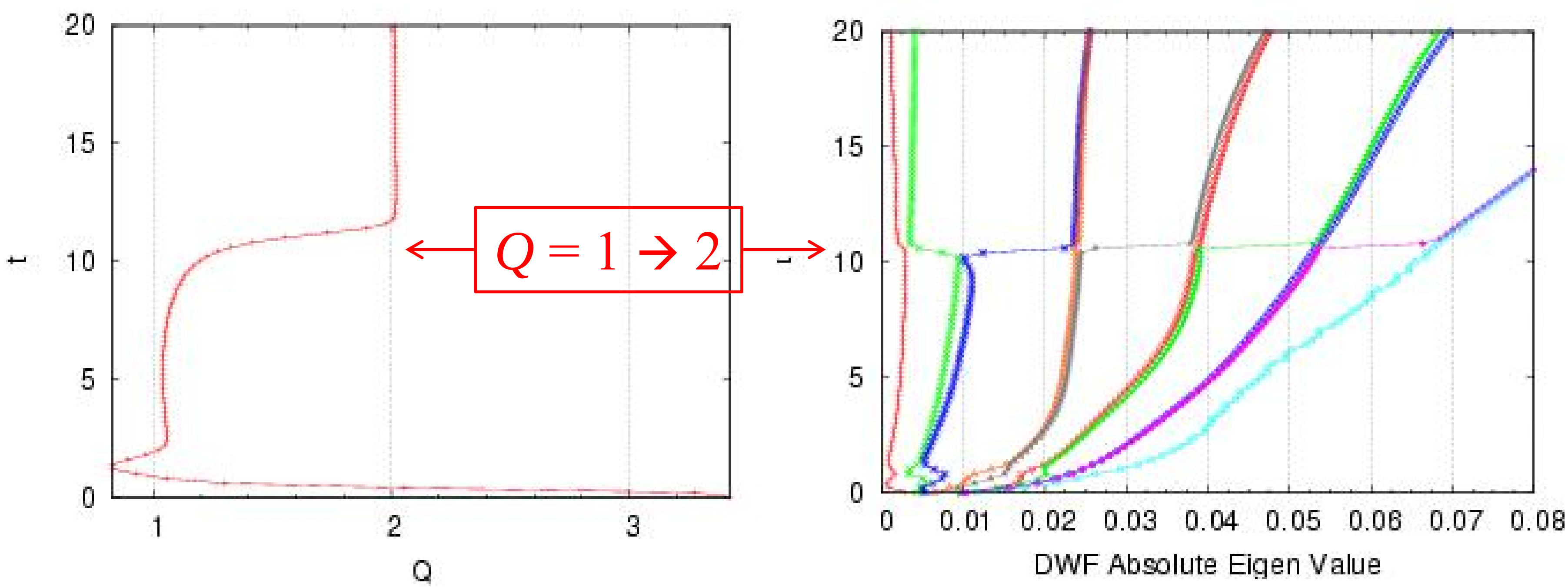


- If the topological changes, all modes must mix between left and right walls.
- Tearing the gauge field must violate chirality!

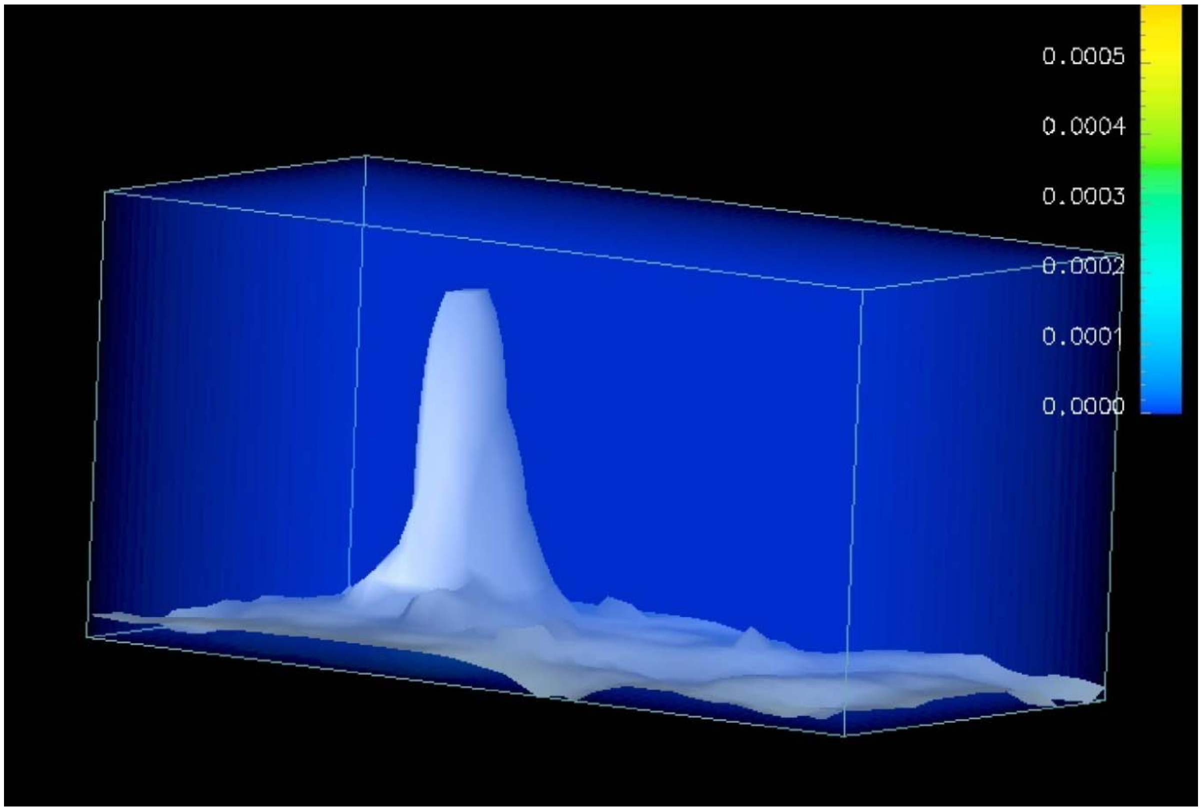
Wilson Flow

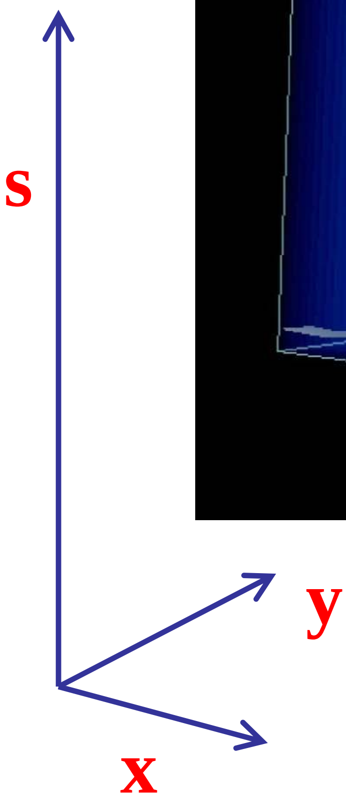
(Qi Liu)

- Instructive to examine topology and Dirac spectrum as the gauge fields are smoothed:



Local chirality violation

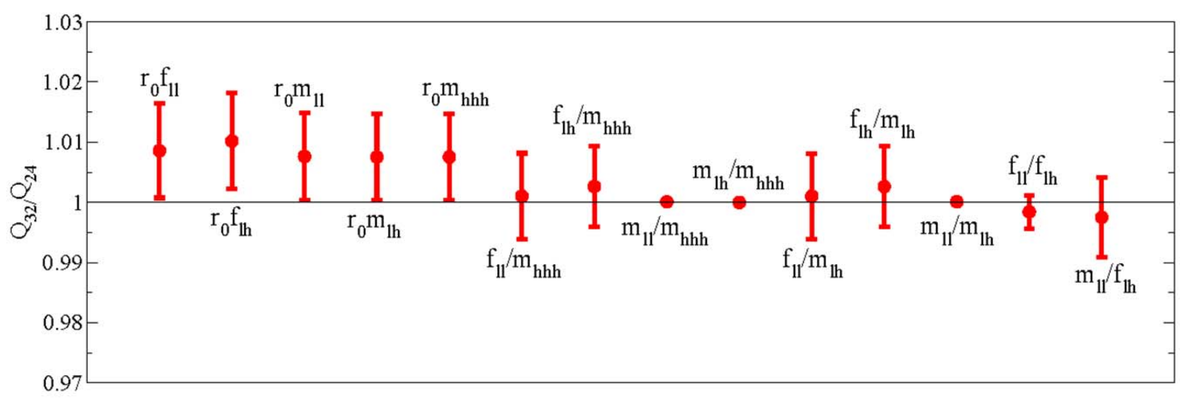




Peter Boyle

DWF at low energy

- At low energy $E << 1/a$, 5-D DWF theory looks like a chiral 4-D theory (QCD) with small chiral asymmetry:
  - Leading, dim-3 operator: $m_{res}\,\overline{q}\,q$ (mass term)
  - Next leading dim-5 operator: $m_{res}\,\overline{q}\,\sigma^{\upsilon\nu}F^{\upsilon\nu}q$ (clover term)
- Very small discretization errors:



Ratios of dimensionless combinations of physical quantities computed using $1/a$ = 1.73 and 2.28 GeV.

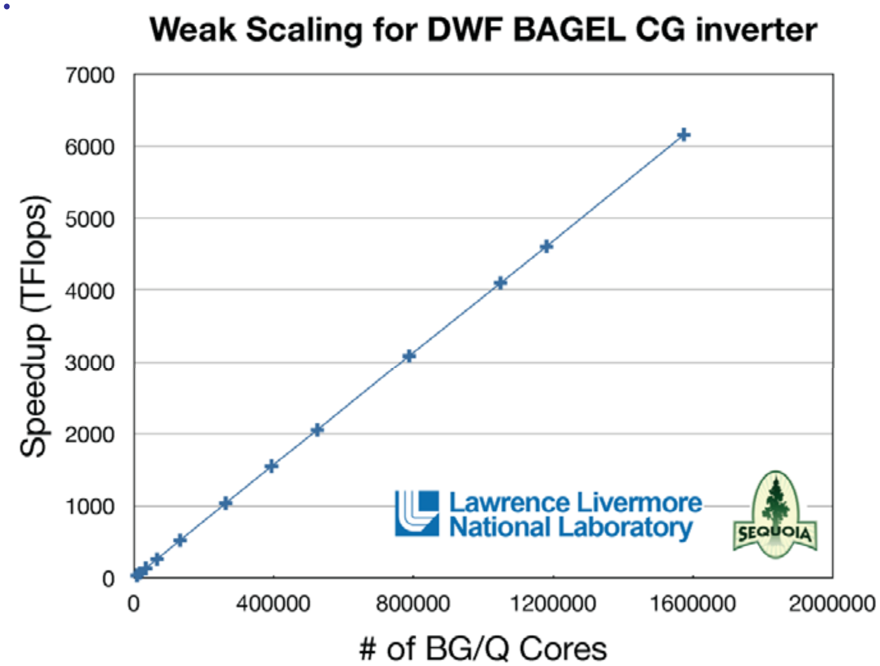
State-of-the-art Lattice QCD

Current state-of-the-art

- Physical m_π=135 MeV and L = 4 – 6 fm now possible.
- Generate 48^3 x 96 and 64^3 x 128 ensembles of gauge field configurations.
- Complete set of measurements takes 5.3 hours on a 32-rack BG/Q machine (sustains 1 Pflops)
- Large collaboration essential:
  - Highly optimized code (64 threads, SPI comms., wide-vector FP)
  - Sophisticated algorithms (deflation, FG $(\Delta t)^3$ integrator)
  - Complex measurement strategies (NPR, G-parity BC, 4-pt functions)

Efficient code is essential

- High-performance, Blue Gene/Q-optimized software underlies these results:
  - Peter Boyle's BG/Q inverter
  - Chulwoo Jung and Hantao Yin's threaded evolution and measurement code based on this inverter.
- Figure shows weak scaling on 96 BG/Q (Sequoia) racks.
- Sustained speed of 6 Pflops!

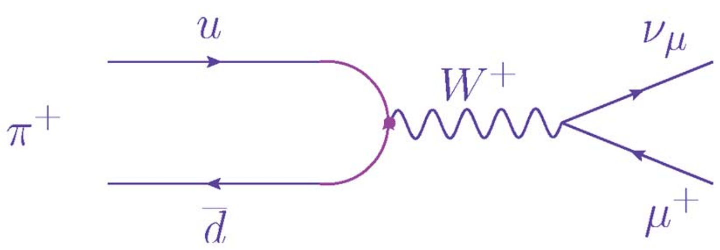


Code developed by Peter Boyle at the STFC funded DiRAC facility at Edinburgh

Measurement Techniques

- Use eigenvectors to solve $\not{D}\, G_n = h_n$ for multiple right-hand sides (deflation).
- Amortize eigCG or Lanczos set up time by using sources (h_n) on each time slice (128) and measuring all quantities in the same job.
- Use All-Mode-Averaging technique (Blum, Izubuchi & Shintani, arXiv:1208.4349 [hep-lat])
  - Loosen CG stopping condition $10^{-8} \rightarrow 10^{-4}$.
  - Obtain accurate result for 8 out of 128 time slices
  - $O(10^{-8})$ accurate result $= \langle G_{10^{-4}} \rangle_{124} + \langle G_{10^{-8}} - G_{10^{-4}} \rangle_{8}$
  - Achieve 5-20 x speed-up.

Simple state-of-the-art example: f_π



$$\langle 0|\overline{d}\gamma^5\gamma^\mu u|\pi^+(\vec{p})\rangle = f_\pi \frac{p^\mu}{\sqrt{4E_\pi(\vec{p})}}$$

$$f_\pi = N \sum_{\vec{r}} \frac{\langle A^0(\vec{r},t)O_\pi(t=0)\rangle}{\langle O_\pi^\dagger(t)O_\pi(t=0)\rangle^{\frac{1}{2}}} e^{m_\pi t/2}$$

- 2012 (elaborate chiral fit): $f_\pi = 127(3)_{stat}(3)_{sys}$ MeV
- 2013 (m_π=135 MeV): $f_\pi = 130.0(0.3)_{stat}$ MeV (40 cnfgs.)
- Experiment: $f_\pi = 130.4(0.04)(0.2)$ MeV

New Opportunities

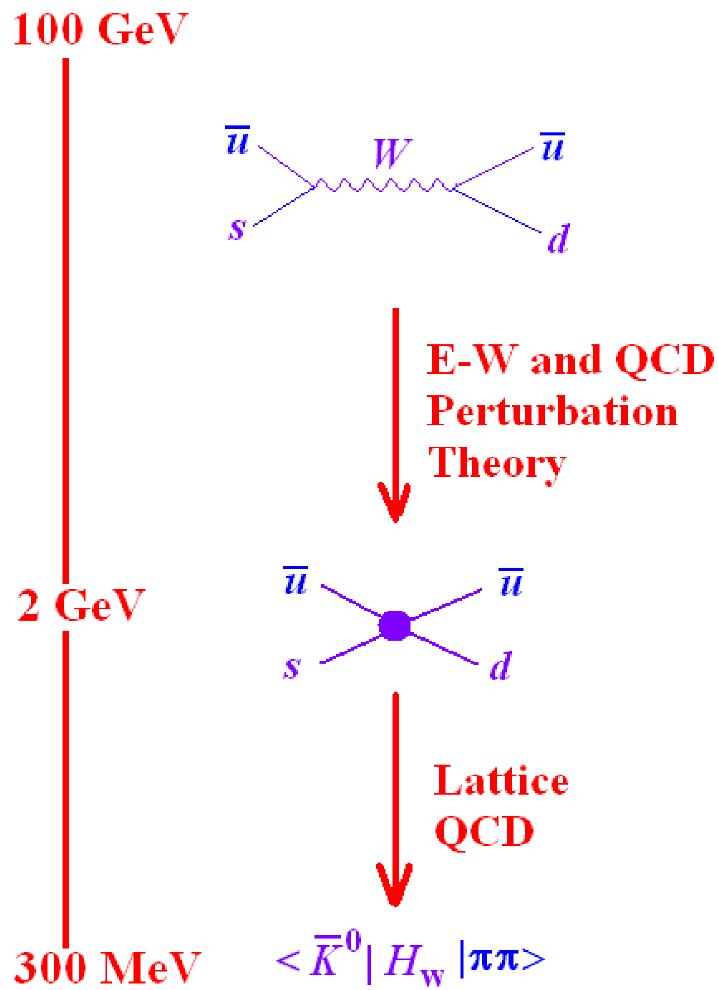
- With physical pion masses, large volumes and no need for chiral extrapolation we can tackle more complex and important quantities.
- Work with up, down and strange quarks (including some charm quark loops).
- $K \rightarrow \pi\pi$ decay
  - Δ I = 1/2 rule
  - Direct CP violation and ε'
- $K_L - K_S$ mass difference

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ud}} \frac{V_{ts}^*}{V_{us}^*} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

$K \rightarrow \pi\,\pi$ decay

$K \to \pi\pi$ phenomenology

- Final $\pi\pi$ states can have $I = 0$ or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$
$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}\right)$$

Direct CP violation

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi - \pi$ state.
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{\begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix}\right\}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and K^0

$$|K_S\rangle = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$|K_L\rangle = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$\overline{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\, \Gamma_{0\bar{0}}}{\mathrm{Re}\, M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\, \Gamma_{0\bar{0}}}\right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^-|H_w|K_L\rangle}{\langle \pi^+\pi^-|H_w|K_S\rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|H_w|K_L\rangle}{\langle \pi^0\pi^0|H_w|K_S\rangle} = \epsilon - 2\epsilon'$$

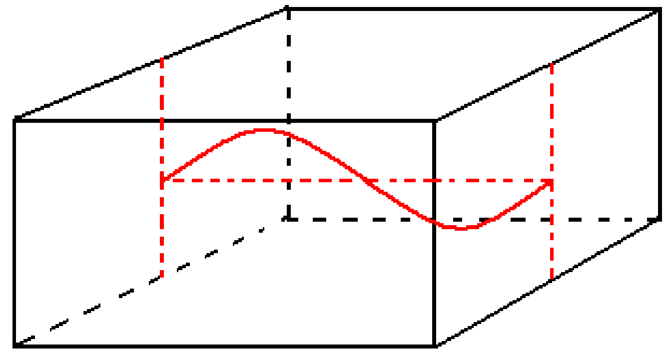
- Where: $\epsilon = \overline{\epsilon} + i\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.65 \pm 0.26) \times 10^{-3}$

Lattice Aspects

Physical $\pi\,\pi$ states – Lellouch-Luscher



$p = 2\pi/L$

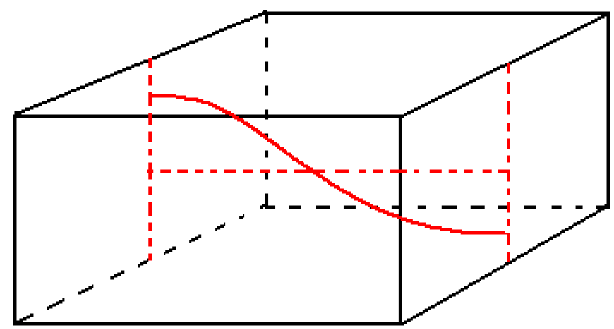
- Euclidean e^{-Ht} projects onto $|\pi\pi\,(\vec{p}=0)>$
- Use finite-volume quantization.
- Adjust volume so 1st or 2nd excited state has correct p.
- Correctly include p - p interactions, including normalization.
- Requires extracting signal from non-leading large ***t*** behavior:

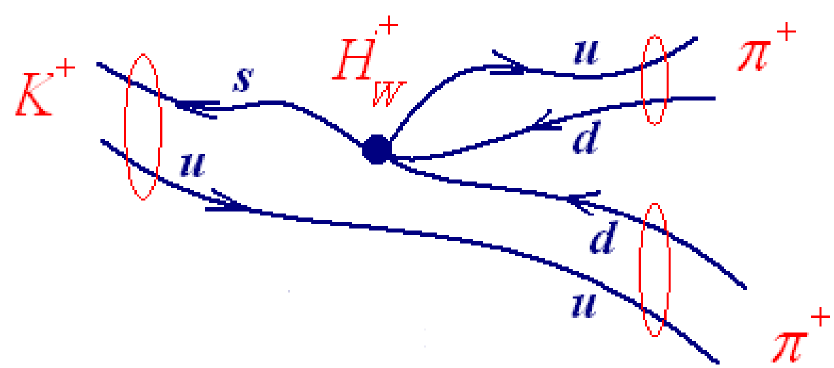
$$G(t) \sim c_0 e^{-E_0 t} + c_1 e^{-E_1 t}$$

$$\Delta \mathbf{I} = \mathbf{3/2}$$

Δ I = 3/2 $K \to \pi\pi$

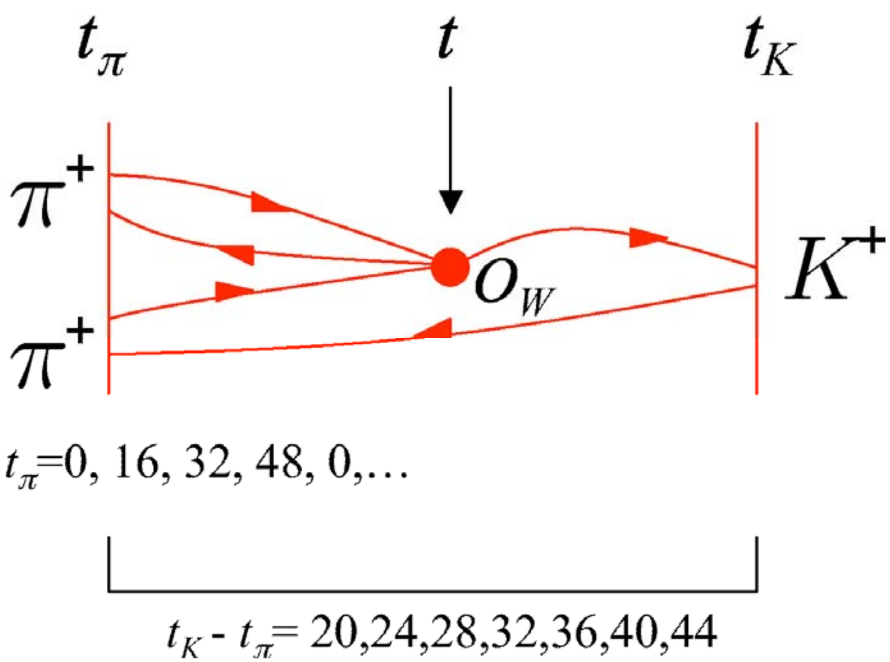
- Three operators contribute $O^{(27,1)}$, $O^{(8,8)}$ and $O^{(8,8)m}$.
- Use isospin to relate to $K^+ \to \pi^+\pi^+$.
- Use anti-periodic boundary conditions for *d* quark. (Changhoan Kim, hep-lat/0210003).
- **Achieve essentially physical kinematics!** (63 → 146 configurations)
  - m_π = 142.9(1.1) MeV
  - m_K = 511.3(3.9) MeV
  - $E_{\pi\pi}$ = 492(5.5) MeV



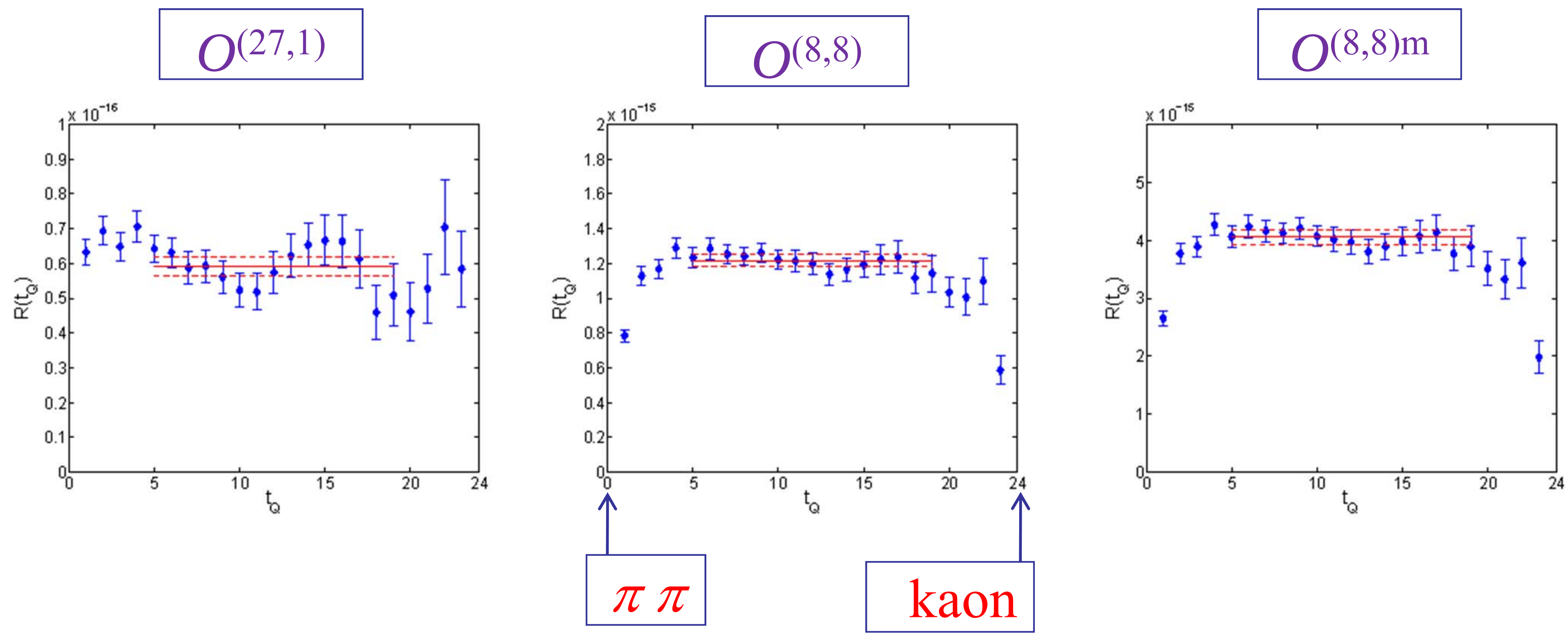


Computational Set-up
(Lightman and Goode)

- Use anti-periodic boundary conditions for *d* quark in two directions (average over three choices).
- Fix $\pi - \pi$ source at $t = 0$, vary location of O_W and kaon source.



$< \pi\, \pi \,|\, O \,|\, K >$ from 146 configurations



Plot ratio of correlators:

$$\frac{C^{i}_{K\pi\pi}(t)}{C_K(t_K - t)\,C_{\pi\pi}(t)} = \frac{\mathcal{M}_i}{Z_K Z_{\pi\pi}}$$

Determine physical A_2

- $\mathrm{Re}(A_2) = (1.436 \pm 0.063_{\mathrm{stat}} \pm 0.258_{\mathrm{sys}})\ 10^{-8}$ GeV

Experiment: $1.479(4)\ 10^{-8}$ GeV

- $\mathrm{Im}(A_2) = -(6.29 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{sys}})\ 10^{-13}$ GeV

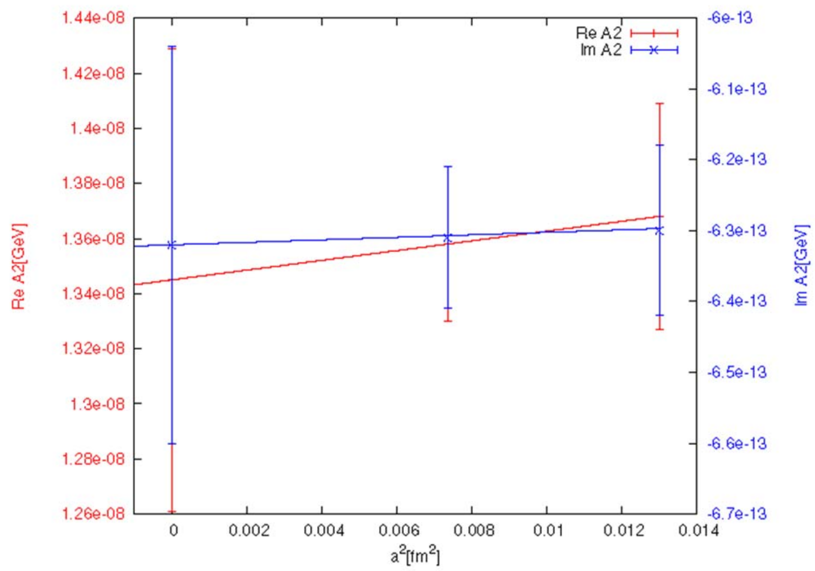
- Error estimates:

	$\mathrm{Re}A_2$	$\mathrm{Im}A_2$
lattice artefacts	15%	15%
finite-volume corrections	6.2%	6.8%
partial quenching	3.5%	1.7%
renormalization	1.8%	5.6%
unphysical kinematics	0.4%	0.8%
derivative of the phase shift	0.97%	0.97%
Wilson coefficients	6.6%	6.6%
Total	18%	19%

Δ I = 3/2: Next results

(Tadeusz Janowski and Daiqian Zhang)

- Use two new large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV

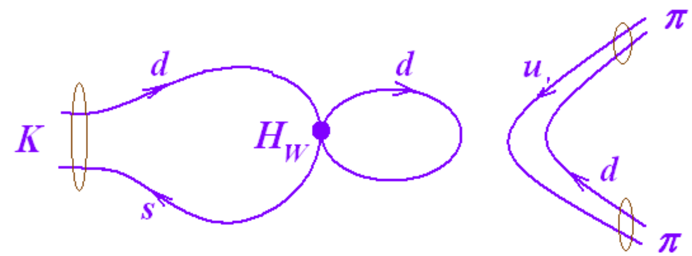


- First continuum results, (preliminary):
  - $\mathrm{Re}(A_2) = (1.345 \pm 0.084_{stat})\times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -\,(6.32 \pm 28_{stat})\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV

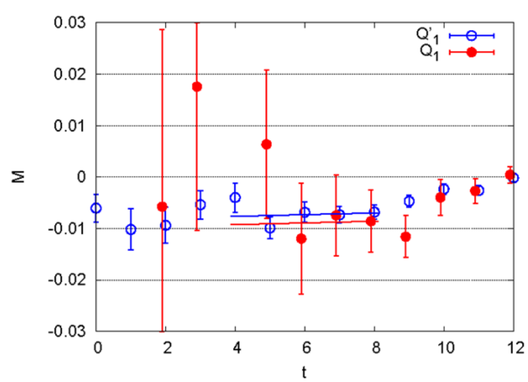
$$\Delta \mathbf{I} = \mathbf{1/2}$$

$\Delta I = 1/2 \ K \rightarrow \pi\pi$

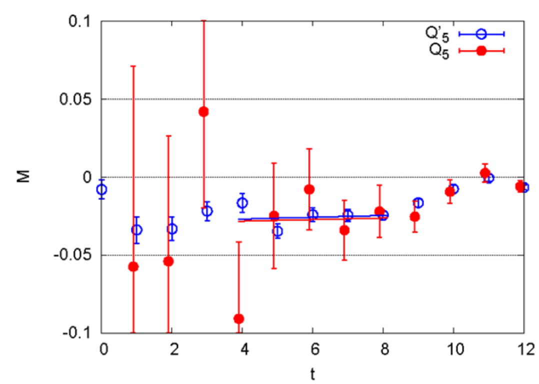
- Made much more difficult by disconnected diagrams:



- Many more diagrams (48) than $\Delta I = 3/2$.
- Initial threshold decay calculation successful (Qi Liu)
  - Re (A_0): 25% statistical errors
  - Im (A_0): 50% statistical errors



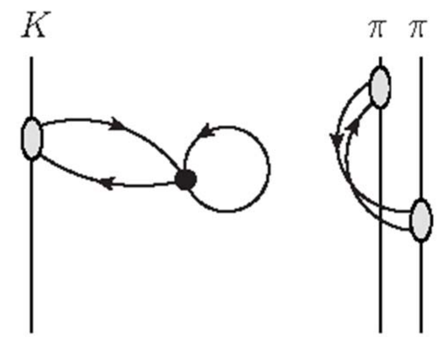
Q2 - largest part of Re(A_0)



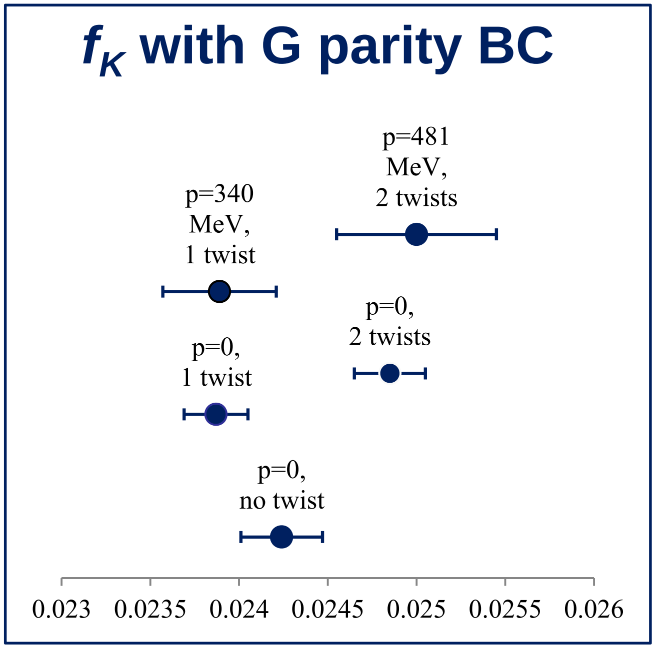
Q6 - largest part of Im(A_0)

$\Delta I = ½\ K \rightarrow \pi\pi$ – Next steps
(Chris Kelly & Daiqian Zhang)

- Use **all-2-all** propagators (Trinity/KEK)
  - Sum over localized sources – further suppress vacuum coupling
  - See 5x improvement in statistics for $I = 0$, π–π scattering



- Use **G-parity** BC to obtain $p_\pi = 205$ MeV
  - $G = C\, e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$
  - Extra $I = 1/2$, s' quark adds $e^{-m_K L}$ error.
  - Tests: f_K and B_K correct within errors.



$\Delta I = 1/2$ $K \rightarrow \pi\pi$: Physical kinematics

- Goal is a 20% calculation of ε'/ε with all errors controlled
- Repeat $\Delta I = 3/2$ kinematics
  - Use 32^3 x 64 volume with $1/a = 1.37$ GeV
  - Achieve $p = 205$ MeV from **G-parity** boundary conditions in 3 directions
- BG/Q gives 20 x speedup
- Configuration generation at 500 time units
- Complete measurements performed on 10 configurations!
- Result expected in 1 year

$K_L - K_S$ mass difference

$K^0 - \overline{K^0}$ Mixing

- Time evolution of $K^0 - \overline{K^0}$ system given by familiar Wigner-Weisskopf formula:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where:

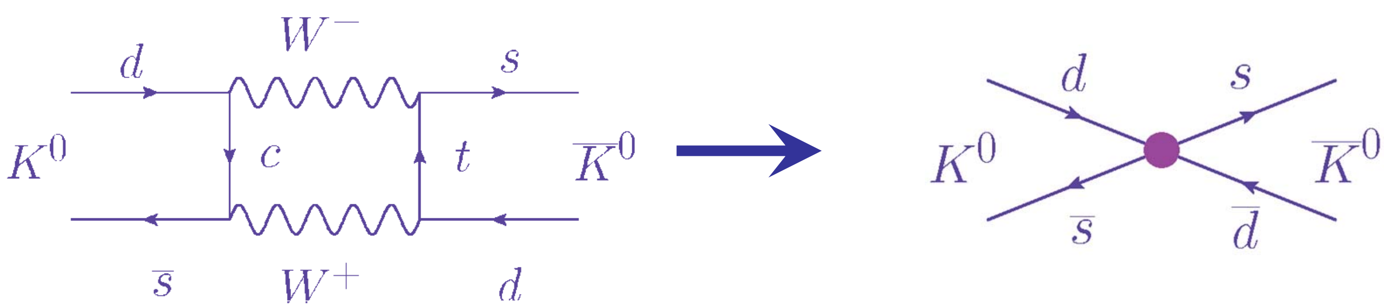
$$\Gamma_{ij} = 2\pi \sum_{\alpha} \int_{2m_\pi}^{\infty} dE \langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle \delta(E - m_K)$$

$$M_{ij} = \sum_{\alpha} \mathcal{P} \int_{m_\pi}^{\infty} dE \frac{\langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle}{m_K - E}$$

$K^0 - \overline{K^0}$ Mixing

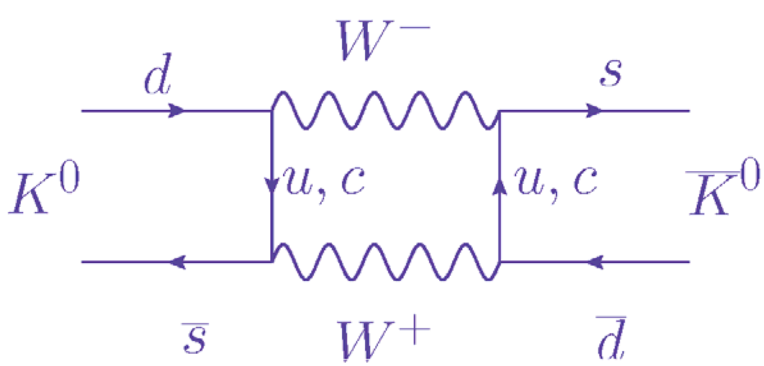
- CP violating: $p \sim m_t$

$$\bar{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\}$$



- CP conserving: $p \leq m_c$

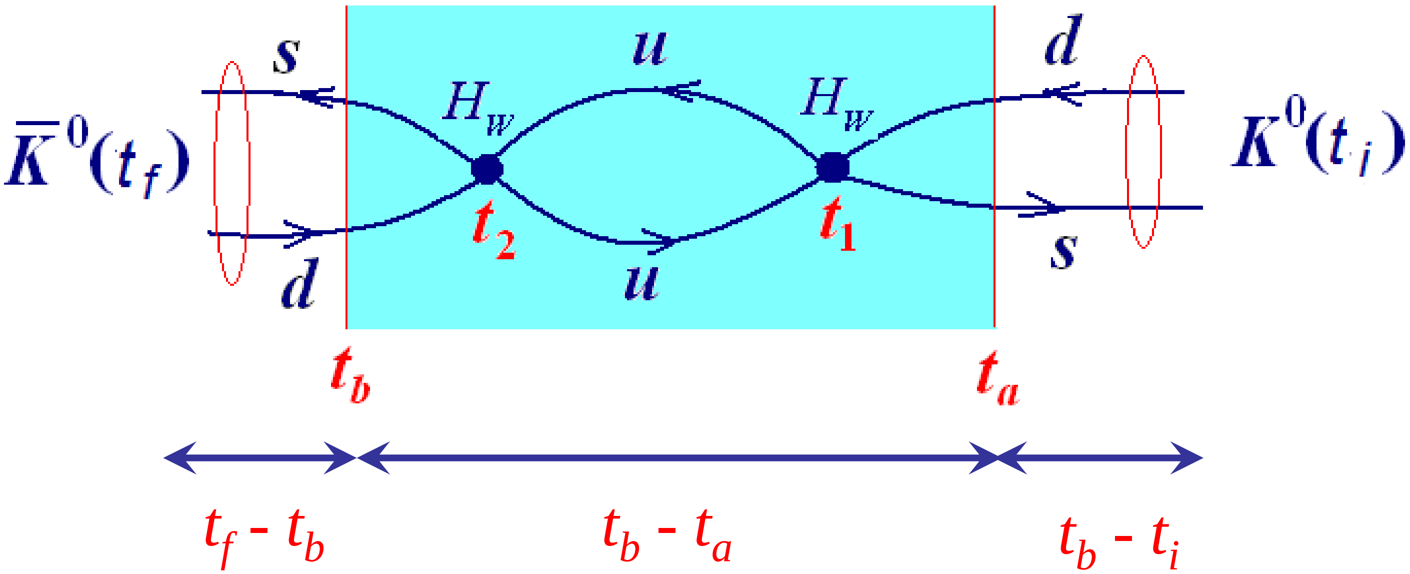
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



Lattice Version
(Jianglei Yu)

- Evaluate standard, Euclidean, 2nd order $K^0 - \overline{K^0}$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\dagger}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K - E_n)(t_b - t_a)}}{M_K - E_n}}_{3.} \right)$$

1. $\Delta m_K{}^{\mathrm{FV}}$
2. Uninteresting constant
3. Growing or decreasing exponential:
   $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_{n_0}} - \frac{E_{n_0}^2}{2 k_n M_K} \frac{d(\phi + \delta_0)}{dk} \bigg|_{m_K} V \, |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0) \big|_{M_K}$$

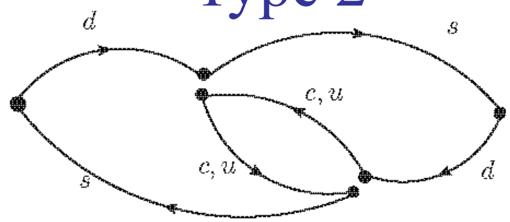
N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda

Lattice setup
(Jianglei Yu)

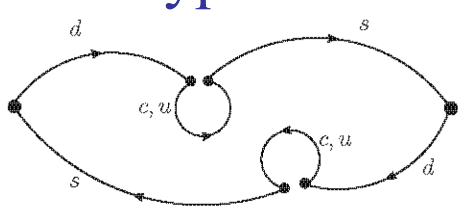
- Must include charm quark (GIM *u–d* cancellation)
- Two calculations performed
  - $16^3 \times 32$, m_p = 420 MeV, types 1 & 2 (arXiv:1212.5931)
  - $24^3 \times 64$, m_p = 330 MeV, all graphs included

Type 1

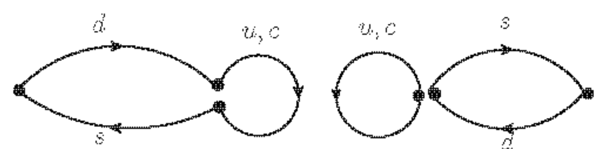
Type 2



Type 3

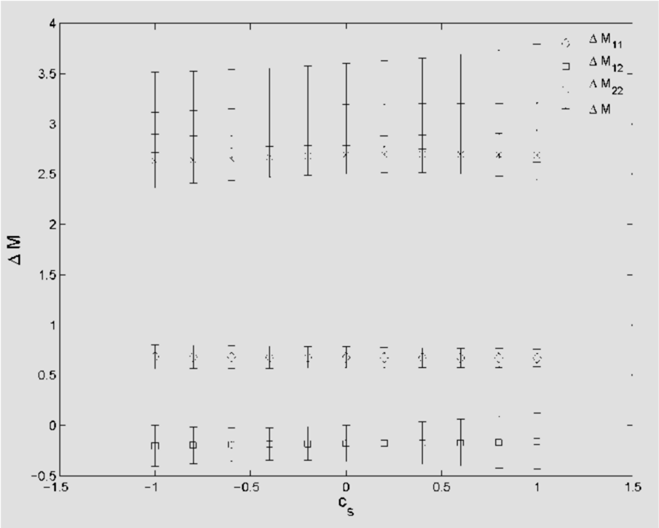


Type 4

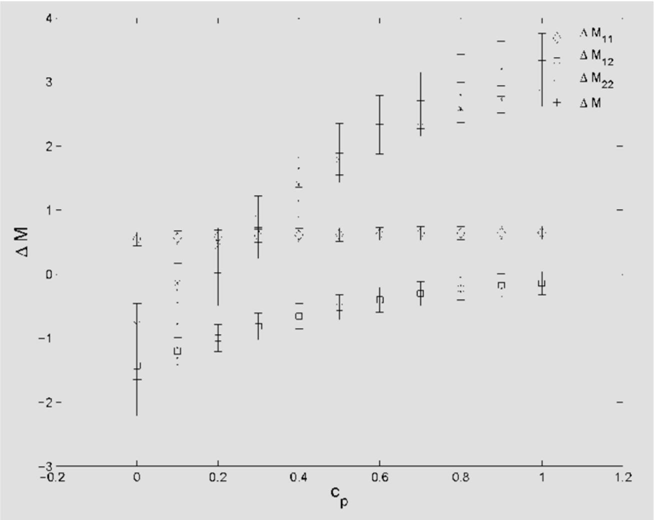


Exponentially growing terms

- The vacuum, π^0 and η require special treatment:
  - Calculate $\langle\, X \mid H_W \mid K^0 \,\rangle$ directly and subtract, $X = |0\rangle$, π^0, η
  - Fit the exponential time dependence in the 4-point function
  - Adjust $c_s\, \bar{s}\, d$ or $c_p\, \bar{s}\, \gamma^5 d$ terms to completely remove an unwanted state.



vary c_s



vary c_p

Remove extra η contribution

- Calculate $\langle\, \eta \,|\, H_W \,|\, K^0 \,\rangle$ directly and remove
- Has an ~10% effect on the result

PRL **105**, 241601 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

η and η' Mesons from Lattice QCD

N. H. Christ,[1] C. Dawson,[2] T. Izubuchi,[3,4] C. Jung,[3] Q. Liu,[1] R. D. Mawhinney,[1] C. T. Sachrajda,[5] A. Soni,[3] and R. Zhou[6]

(RBC and UKQCD Collaborations)

[1]*Physics Department, Columbia University, New York, New York 10027, USA*
[2]*Department of Physics, University of Virginia, 382 McCormick Road, Charlottesville, Virginia 22904-4714, USA*
[3]*Brookhaven National Laboratory, Upton, New York 11973, USA*
[4]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[5]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[6]*Physics Department, University of Connecticut, Storrs, Connecticut 06269-3046, USA*
(Received 24 February 2010; published 8 December 2010)

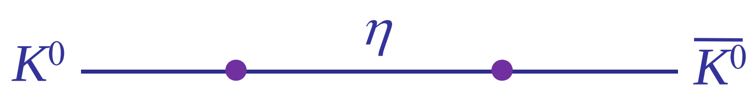
The large mass of the ninth pseudoscalar meson, the η', is believed to arise from the combined effects of the axial anomaly and the gauge field topology present in QCD. We report a realistic, 2 + 1-flavor, lattice QCD calculation of the η and η' masses and mixing which confirms this picture. The physical eigenstates show small octet-singlet mixing with a mixing angle of $\theta = -14.1(2.8)^\circ$. Extrapolation to the physical light quark mass gives, with statistical errors only, $m_\eta = 573(6)$ MeV and $m_{\eta'} = 947(142)$ MeV, consistent with the experimental values of 548 and 958 MeV.

DOI: 10.1103/PhysRevLett.105.241601 PACS numbers: 12.38.Gc, 11.15.Ha, 11.30.Rd, 14.40.Be

The relatively large mass of the ninth pseudoscalar meson, the η', provides a significant challenge for quantum chromodynamics (QCD), the component of the standard model which describes the interactions of quarks and gluons. On a naive classical level, there are nine conserved axial-vector currents. Given the vacuum breaking of the symmetries which these currents generate, this should diagrams to decrease exponentially with increasing time separation. For mesons this falloff roughly matches the exponential time dependence of the massive, Euclidean-space meson propagator, and good numerical signals can be seen over a large range of times. For terms in which the source and sink of the meson propagator are not joined by quark propagators, the needed exponential decrease comes

$$\eta' = \frac{1}{\sqrt{3}}\left(\overline{u}u + \overline{d}d + \overline{s}s\right)$$

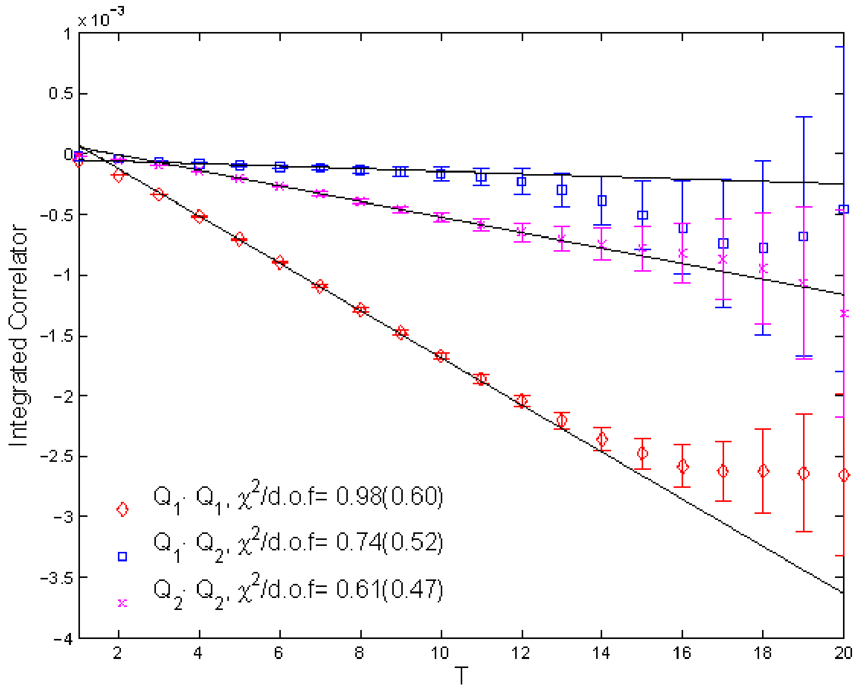
$$\eta = \frac{1}{\sqrt{6}}\left(\overline{u}u + \overline{d}d - 2\overline{s}s\right)$$



$$\frac{\left|\langle\eta|H_W|K^0\rangle\right|^2}{M_K - M_\eta}\left(-(t_b - t_a) - \frac{1}{M_K - M_\eta} + + \frac{e^{(M_K - M_\eta)(t_b - t_a)}}{M_K - M_\eta}\right)$$

Latest results
(Jianglei Yu)

- N_f=2+1, 24^3 x 32, m_π = 330 MeV, $m_c^{\overline{MS}}$(2 GeV) = 949 MeV
- Incorporate GIM cancellation



- Large statistics (800 configurations, 64 measurements each).

Results

Δ_K	T_{min}	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
	6	0.754(42)	-0.16(15)	2.70(18)	3.30(34)
7	7	0.755(42)	-0.18(15)	2.66(18)	3.23(34)
	8	0.751(42)	-0.18(15)	2.62(19)	3.18(35)

x 10^{-12} MeV

Diagrams	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
Type 1,2	1.485(80)	1.567(38)	3.678(56)	6.730(96)
All	0.754(42)	-0.16(15)	2.70(18)	3.30(34)

x 10^{-12} MeV

- Unphysical, $m_\pi = 330$ MeV
- Active charm but $m_c\, a = 0.55$
- Result:
  $\Delta M_K \quad = \quad 3.30(34)\ 10^{-12}$ MeV
- $\Delta M_K^{expt} = 3.483(6)\ 10^{-12}$ MeV

- Agreement fortuitous!
- 32^3 x 64, $1/a$=1.37 GeV, $m_\pi = 330$ MeV started (Z. Bai)
- 80^2 x 96 x 192, $1/a$=3.0 GeV calculations planned!

Outlook

- DWF with physical quark masses reproduce QCD at the ≤ 2% level on a 64^3 x 128 lattice.
- NPR give continuum-like control of operator normalization and mixing.
- Theoretical advances allow rescattering effects to be correctly computed in Euclidean space (so far only for low energy π–π states).
- Many critical quantities can now be computed:
  - $K \rightarrow \pi\pi$, ΔI=3/2 and 1/2, ε'/ε
  - $M_{K_L} - M_{K_S}$
  - $K \rightarrow \pi l \bar{l}$, $K \rightarrow \pi \nu\bar{\nu}$
  - Quark effects on g_μ - 2 at $O(\alpha^3)$ (L. Jin's poster)