Evaluating Macroprudential Policy in a DSGE Framework with Financial Frictions

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Abstract

This paper studies the effectiveness of macroprudential policy in a New Keynesian DSGE model with financial frictions. The financial sector is modeled vis-à-vis Gertler and Karadi (2011) with the endogenous bank leverage ratio. The simulation results show that macroprudential policy can mitigate shocks and stabilize the economy. More specifically, two types of macroprudential instruments are examined. First, a countercyclical feedback rule to regulate the loan-to-value (LTV) ratio of the borrowing household is imposed. Furthermore, a lump-sum tax policy is implemented to reduce the leverage ratio of financial intermediaries during economic booms. Both policies aim to stabilize the credit cycle and interact with a standard monetary policy. The LTV ratio regulation significantly dampens economic fluctuations but shifts credit toward the business sector. Comparatively, the tax policy stabilizes the aggregate credit market more effectively by directly controlling the balance sheet of financial intermediaries. Additional welfare analysis selects the optimal simple rules and provides further insights to the search for an effective and implementable macroprudential policy.

Keywords: Macroprudential Policy, Financial Frictions, Loan-to-Value (LTV) Ratio, Optimal Macroprudential Policy

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1 Introduction

The 2008 financial crisis and the subsequent Global Recession have generated discussion among researchers and policymakers regarding the role of traditional monetary policy and the need for macroprudential policy to restore economic stability. Blanchard et al. (2010) discuss the failure of financial regulation to prevent or reduce systematic risk in financial markets. In addition, traditional monetary policy was insufficient to achieve a quick and effective recovery. The crucial role of the financial system in this crisis has prompted a search for macroprudential tools to promote financial and economic stability. These tools target broad financial markets instead of individual institution. The objective of this type of supervision and regulation is to mitigate the transmission of shocks to the general economy. However, arguments have been raised regarding the design of macroprudential instruments and their interaction with monetary policy.

Figure A.1 displays the real Housing Price Index of the U.S. from 1980 to 2013, and Figure A.2 illustrates the Case-Shiller U.S. National House Price Index from 1987 to 2013. Evidently from the graphs, U.S. housing price has increased dramatically during the first half of the 2000s and its collapse in 2007 marked the official start of the financial crisis. During the booming years, many economists and policymakers have overlooked the growing trend and maintained that the housing market could be explained by strong economic fundamentals. Jurgilas and Lansing (2012) document the 2005 media interview of Ben Bernanke, then Chairman of the President’s Council of Economic Advisory, who argued that the housing boom was the result of strong growth in jobs and income, along with low mortgage rates. He further commented that a substantial nationwide decline in housing prices was “a pretty unlikely possibility.” Similar comments were made by Alan Greenspan, then Chairman of U.S. Fed, that the lending industry has been dramatically improved by advances in information technology. He stated that lenders were now able to efficiently judge mortgage applications and issue loans to the “once more-marginal” applicants. McCarthy and Peach (2004) also comment on the unlikely existence of a home price bubble. Nonetheless, the over-optimistic view of economists and policymakers did not prevent the start of the Great Recession in late 2007.

In the booming years, the debt service ratio of U.S. households have grown dramatically. Figure A.3 displays the ratio of mortgage payment to household income during the period of 2000 to 2013. The graph shows that mortgage expenditures rose more than 20% since the beginning of the 2000s. In the peak of 2007, average house-
hold spent more than 11% of disposable income to service home debt in comparison to only 8.7% in 2000. Higher leverage imposes a significant distortion on consumer spending and bank lending, which further prevents a healthy recovery from the crisis. Controlling financial imbalances during economic booms is critical for the prevention of future crisis and it is important for policymakers to explore lean-against-the-wind policy measures that can be used to effectively control the credit market.

This paper contributes to the literature by addressing three key issues. First, it incorporates the housing sector to study the interaction between asset prices and business cycle fluctuations. This should shed some light on the recent financial crisis, which was initiated by a housing boom in the United States. A major portion of domestic borrowing is secured by real estate (Iacoviello 2005) and the housing market has been found to play an important role in driving business cycles (Case 2000, Higgins and Osler 1997). Second, this paper models financial frictions in the traditional New Keynesian DSGE framework. Incorporating the financial market captures a key element of the recent crisis and can be used to study the impact of financial regulation on the transmission of shocks. Furthermore, this paper examines two types of macroprudential instruments: a countercyclical loan-to-value (LTV) ratio rule that targets households’ borrowing constraints, and a lump-sum tax policy that directly affects the balance sheet of financial intermediaries. Using both monetary and macroprudential tools, I conduct a welfare analysis to determine the optimal and implementable rules that can effectively dampen economic fluctuations.

The economy consists of two types of households (as in Iacoviello 2005). Patient households own financial intermediaries and operate monopolistic competitive firms. They deposit savings into the banks each period, and serve as lenders of the economy. In contrast, impatient households make labor and consumption decisions and face borrowing constraints attached to the market value of their housing stock. The maximum allowable fraction of housing stock used as collateral is defined as the loan-to-value ratio, which is controlled by regulatory authorities.

The financial intermediaries are modeled similar to Gertler and Karadi (2011) with moral hazard and costly enforcement. They collect deposits from patient households each period and lend funds to impatient households and non-financial firms. The maximum amount of lending equals to the sum of household deposits and banks’ own net worth. The lending-to-net-worth ratio is defined as the banks’ leverage ratio and serves as a signal for the credit condition in the economy. In each period, managers of banks can choose to divert a fraction of available assets to their corresponding households.
The depositors can force diverted managers to resign but the transferred assets are non-recoverable under costly enforcement. Financial intermediaries face an incentive compatibility constraint, which implies that managers will only stay in operation when the continuing value of banks exceeds the value of diverted assets. Financial frictions create a wedge between saving and lending rates in the economy and introduce endogenous constraints on the intermediary leverage ratio, which is tied to the equity capital of the financial sector.

The model mechanism works as follows. Suppose there is a positive housing demand shock. Consumption and housing prices increase in response, expanding the borrowing constraint of the households. Higher consumer prices reduce the net value of debt and cause the lender’s balance sheet to deteriorate. Facing higher demand for loans, profit-maximizing banks optimally expand lending in the economy and raise their leverage ratios. This leads to a greater liquidity risk in the financial market and increases the systematic risk. Nonetheless, macroprudential policy can be used to mitigate the impact from the positive demand shock. More specifically, the regulatory authority lowers the target LTV ratio of the households, causing the collateral value to drop. This generates a contractionary effect on the borrowing constraint, which relieves the tension on financial intermediaries. Credit expansion is limited which alleviates the liquidity risk in the financial market. The opposite applies for negative shocks. The target LTV ratio is raised to stimulate lending in the economy, which promotes a fast and effective recovery. Nonetheless, when the household credit market is regulated, profit-maximizing banks seek to expand lending in the business sector. This creates a credit shift from restricted sector to unregulated sector. As a result, business lending and aggregate credit market can be more volatile in response to shocks. Policymakers have to consider the trade-off between different sectors when designing the appropriate macroprudential instrument.

The tax policy allows regulatory authorities to directly control the endogenous leverage ratio of banks. During economic booms, the government initiates a positive tax on banks’ lending and use the proceeds to subsidize intermediary equity capital. This policy targets the credit condition of financial intermediaries and is successful in creating direct control of banks’ leverage ratio. With a lower level of lending, the overheated credit market is calmed and the liquidity risk is reduced. Changes in the housing prices and household lending are significantly dampened, which could prevent the formation of asset bubbles. In comparison to the countercyclical LTV ratio rule, the tax policy demonstrates little evidence of credit shift towards the business sector.
as the policy targets the aggregate credit market. However, implementation may be
difficult and costly. The full functionality of this policy relies on the direct control of
banks’ balance sheet in a dynamic setting. The tax rate changes period by period and
the subsidies must be provided in a timely manner. The collection and distribution of
taxes and subsidies are the major obstacles for the implementation of this policy. In
addition, the cost of administering such complicated process may be significantly high.

The objective of this paper is to provide a preliminary theoretical analysis of po-
tential macroprudential instruments that can be effective in preventing asset bubbles
and possible financial crisis, which may be useful to policymakers in the search for an
appropriate and operative macroprudential policy. To provide practical suggestions,
a welfare analysis has been performed in this paper to identify the optimal policies.
Using second-order approximation, the welfare function is maximized with respect to
the coefficients entering various feedback rules. The numerical values indicate the de-
gree of intervention for policymakers to consider. Findings suggest that countercyclical
policy is desired and effective in mitigating economic fluctuations from unexpected
shocks, and the macroprudential policies suggested in this paper can be used jointly
with traditional monetary policy that targets inflation and output gap.

This paper is organized as follows. Section 2 reviews the existing literature on
housing market, financial frictions and macroprudential policies. Section 3 presents
the model. Section 4 reports the calibration of key parameters. Section 5 analyzes the
model implications. Section 6 conducts the welfare analysis and Section 7 concludes.

2 Literature Review

This paper is related to at least three major strands of literature. The first strand
of research studies the relationship between credit and housing. From the theoretical
perspective, Kiyotaki and Moore (1997) consider collateral constraints tied to the real
estate value of entrepreneurs. Iacoviello (2005) and Iacoviello and Neri (2010) discuss
the importance of housing in business cycle fluctuations. More recently, Liu, Wang and
Zha (2013) also report the crucial role of land prices in driving macroeconomic variables
arguing that land price is the dominant factor that affects housing price. Davis and
Heathcote (2007) find similar results using empirical analysis. Liu et al. (2013) recently
find that land price is capable of generating large volatility in unemployment. Research
by the IMF finds that credit and housing cycles are closely linked to business cycles
(IMF 2009). Fitzpatrick and McQuinn (2007) also find that the housing boom and
credit liberalization are mutually reinforcing in the long-run using data from Ireland. Similar results are reported for Norway in Anundsen and Jansen (2011). Favara and Imbs (2010) suggest bank deregulation in the United States has a significant impact on housing price.

This model is also closely related to the growing literature that incorporates financial frictions into DSGE models. Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) are among the pioneering works on this topic. The important role of financial friction has been stressed and carefully studied in a variety of contexts in the last decade\(^1\). Recent studies also focus on the important role of financial intermediation in driving business cycles. Christiano, Motto and Rostagno (2010) highlight the nominal friction (as lending contracts are denominated in nominal terms) that accounts for a significant portion of business cycle fluctuations. Gertler and Kiyotaki (2010) investigate how interruptions in the financial market could lead to a crisis and the effectiveness of several types of market intervention in mitigating the crisis. Gertler and Karadi (2011) further explore an effective but unconventional monetary policy in a simulated financial crisis.

Macroprudential policy refers to the financial regulation that directly targets financial stability and systematic risk. It is often mentioned as an alternative or complementary approach to traditional monetary policy. Common macroprudential tools target banks’ capital buffer and leverage ratio as well as households’ debt-to-income and LTV ratios. Gelati and Moessner (2013) provide an excellent survey of macroprudential policy research and comment on the current policy debate. Suh (2012) studies the interaction between monetary and macroprudential policy in a DSGE model with the financial accelerator. The findings suggest that macroprudential policy is effective in stabilizing the economy but creates a regulatory arbitrage that reallocates credit to a less regulated sector. Gelain et al. (2013) incorporate housing into a standard DSGE model and find that a permanent tightening of households’ collateral constraint is most effective in reducing excessive volatility. Lima et al. (2012) attempt to determine the optimal monetary and macroprudential policy in a DSGE model with financial frictions. The authors find that a tax-subsidy scheme aiming to reduce bank’s leverage ratio is effective in stabilizing the economy and is also welfare-improving. Nonetheless, common ground concerning the most effective and appropriate macroprudential policy has not been reached in the literature. Some policies are sector-specific while others

\(^1\)See, for example, Christiano, Trabandt and Walentin 2011, Smets and Wouters 2007, Gilchrist, Ortiz and Zakrajsek 2009, and Gertler, Gilchrist and Natalucci 2007.
can be difficult to implement. The objective of this paper is to find a practical, effective and welfare-improving macroprudential policy that can be used jointly with traditional monetary policy. In the next section, I present a model that is useful in addressing the recent financial crisis and can be utilized to study the issues mentioned above.

3 The Model

There exists two types of households differ by their discount factors. The impatient households (indexed $j = b$) are subject to a higher discount factor than the patient households (indexed $j = s$), and borrow funds to consume each period ($\beta_b < \beta_s$). Patient households optimally choose the amount of savings each period, earning the risk-free rate $R_t$. Using capital, household labor and housing input, the non-financial firms produce intermediate goods and sell to monopolistically competitive retailers. The capital good producers purchase used capital from intermediate-good producers, buy new capital, refurbish the old capital and sell to firms in the next period. The financial sector is modeled following Gertler and Karadi (2011). The existence of credit friction creates a wedge between the saving and lending rates. The central bank conducts monetary policy via a simple feedback rule that responds to the deviations of output and inflation. The model is then augmented to include macroprudential policies, with restrictions on the LTV ratio of borrowers and the leverage ratio of financial intermediaries. Figure A.4 displays the basic framework and key agents of the model.

Figure A.4: Basic Framework of Model
3.1 Households

There exists two types of infinitely lived, utility-maximizing households. Patient households deposit savings into the banking sector and receive the risk-free interest rate. Impatient households borrow from banks to consume and face a credit constraint limited by the value of their house, which serves as collateral. The difference in time preferences implies that in the equilibrium around the steady states, patient households will always save and impatient households will always borrow. I assume a continuum of households in each type with equal population. Both types of households obtain utility from housing goods and offer labor services in a competitive labor market.

3.1.1 Patient Households

Patient households, denoted by \( s \), are subject to a discount rate \( \beta_s \). In addition, they lease a fraction \( (1 - \kappa) \) of housing goods to non-financial firms and earn rent \( r^h_t \). Firms use this real estate for the production of intermediate goods. The representative household makes consumption, \( C_{t,s} \), housing, \( H_{t,s} \) and labor, \( L_{t,s} \) choices each period by maximizing the life-time expected utility, which is given by

\[
\max_{C_{s}, H_{s}, L_{s}, D} E_0 \left\{ \sum_{t=0}^{\infty} \beta_s^t \left[ v_c \log C_{t,s} + v_h \varepsilon^H_t \log(\kappa H_{t,s}) - v_l \frac{L_{t,s}^{1+\varphi}}{1+\varphi} \right] \right\}
\]

subject to budget constraint

\[
C_{t,s} + P^H_t (H_{t,s} - H_{t-1,s}) + D_t \leq R_t D_{t-1} + W_{t,s} L_{t,s} + r^h_t (1 - \kappa) P^H_t H_{t-1,s} + \Pi_t
\]

\( \varepsilon^H \) is a preference shock on housing goods, where a positive \( \varepsilon^H \) leads to a rise in housing demand. The parameter \( v_c \) controls the utility from consumption, \( v_h \) governs the utility from housing goods and \( v_l \) dictates the disutility of labor supply. \( P^H_t \) denotes the housing price in units of consumption. Patient households make real bank deposit \( D_t \), earning real return \( R_{t+1} \) in the following period. In addition, \( W_s \) is the real wage and \( \Pi_t \) denotes the net profits received from owning financial and non-financial firms. All variables are in real terms and the optimal choices are characterized by the following first-order conditions:

\[
v_l L_{t,s}^p = \frac{v_c}{C_{t,s}} W_{t,s}
\]

\[
\frac{1}{C_{t,s}} = \beta_s E_t \frac{1}{C_{t+1,s}} R_{t+1}
\]
\[
\frac{v_c}{C_{t,s}} P_t^H = \frac{v_h \xi_t^H}{H_{t,s}} + \beta_t E_t \frac{v_c}{c_{t+1,s}} P_{t+1}^H [1 + r_{t+1}^H (1 - \kappa)] \quad (5)
\]

Equation (3) is the labor supply decision and (4) is the standard Euler equation. (5) characterizes the housing demand for patient households, which shows that the shadow price of housing goods in period \( t \) is the sum of the period-\( t \) marginal utility from housing goods and the discounted value of the shadow price in \( t + 1 \).

### 3.1.2 Impatient Households

Impatient households make consumption, labor and housing decisions every period. The representative household, denoted by \( b \), is subject to a discount factor \( \beta_b \) and maximizes the expected utility given by

\[
\max_{C_{t,b},H_{t,b},L_{t,b}} E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^b [v_c \log C_{t,b} + v_h \xi_t^H \log H_{t,b} - v_l \frac{L_{t,b}^{1+\varphi_L}}{1+\varphi_L}] \right\} \quad (6)
\]

\[
C_{t,b} + P_t^H (H_{t,b} - H_{t-1,b}) + R_t^L B_{t-1,b} \leq W_{t,b} L_{t,b} + B_{t,b} \quad (7)
\]

\[
B_{t,b} \leq m_t E_t \frac{P_{t+1}^H}{P_{t+1}} H_{t,b} \quad (8)
\]

In equation (8), \( m_t \) represents the maximum allowable LTV ratio and \( E_t P_{t+1}^H H_{t,b} \) is the expected future value of the borrower’s real estate. Households borrow \( B_{t,b} \) in period \( t \) and repay \( R_{t+1}^L B_{t,b} \) in period \( t + 1 \). Constraint (8) implies that during period \( t \), the impatient households may only borrow up to a fraction \( m_t \) of the expected value of their housing stock in period \( t + 1 \), less the interest payment. In the equilibrium, this constraint binds under the assumption that borrowers are less patient than the savers.

Let \( \lambda_t \) be the Lagrange multiplier associated with the borrowing constraint; then, the first-order conditions that characterize the optimal choices are:

\[
v_t L_{t,b}^P = \frac{v_c}{C_{t,b}} W_{t,b} \quad (9)
\]

\[
\frac{v_c}{C_{t,b}} = \lambda_t E_t R_{t+1}^L + \beta_t E_t \frac{v_c}{C_{t+1,b}} R_{t+1}^L \quad (10)
\]

\[
\frac{v_c}{C_{t,b}} P_t^H = \frac{v_h \xi_t^H}{H_{t,b}} + \beta_t E_t \frac{v_c}{c_{t+1,b}} P_{t+1}^H + \lambda_t m_t E_t P_{t+1}^H \quad (11)
\]
Comparing the first-order conditions of the patient and impatient households, the importance of the collateral constraint emerges. From (10), a strictly positive $\lambda_t$ associated with the binding constraint implies that the traditional intertemporal optimal condition fails to hold with equality. In addition, the marginal utility of investment in housing increases with the magnitude of the Lagrange multiplier. Moreover, equation (11) shows that the borrower’s marginal return on housing goods depends on the LTV ratio. In a financially frictionless economy, the borrowing rate would be equivalent to the saving rate, $R^L_t = R_t$.

### 3.2 Financial Intermediaries

The banking sector modeled in Gertler and Karadi (2011) is introduced in this section to create a wedge between deposit and lending rates. Credit friction is essential to capture the role of bank capital in the transmission of shocks to the economy. Financial frictions are embedded in the funds available to banks, but I assume frictionless transfer of funds between financial intermediaries and borrowers. There exists a continuum of mass-one banks owned by the households with the timeline summarized as follows:

- In the beginning of period $t$, bank $j$ raises deposit $D^j_t$ from the patient household at deposit rate $R^L_{t+1}$ payable in period $t+1$.
- Bank $j$ issues one-period loans to the impatient households with real estate collateral, $B^j_{t,b} = m_t E_t \left[ \frac{P^H_{t+1}}{R^L_{t+1}} H^j_{t,b} \right]$.
- Bank $j$ also issues one-period loans to non-financial firms backed by equity capital, $B^j_{t,e} = Q_t K^j_{t+1}$.
- All loans are subject to interest rate $R^L_{t+1}$ payable in period $t+1$.

Here, $K_{t+1}$ is the capital holding of firms with unit price $Q_t$. This can be interpreted as the value of the financial claims that banks hold against non-financial firms. Intermediary $j$’s balance sheet consists of assets given by

$$B^j_t = B^j_{t,b} + B^j_{t,e} = m_t E_t \left[ \frac{P^H_{t+1}}{R^L_{t+1}} H^j_{t,b} \right] + Q_t K^j_{t+1} \quad (12)$$

In addition, with liabilities $D^j_t$ and residual net worth $N^j_t$, the following condition holds for every period:

$$B^j_t = D^j_t + N^j_t \quad (13)$$
The balance sheet of intermediary $j$ is as follows:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^j_t$</td>
<td>$D^j_t$</td>
</tr>
</tbody>
</table>

| Net Worth | $N^j_t$ |

The law of motion of intermediary $j$’s net worth is simply

$$N^j_{t+1} = R^L_{t+1}B^j_t - R_{t+1}D^j_t \quad (14)$$

The banker will only fund projects with an expected return of no less than the discounted cost of borrowing. Therefore, the following inequality must hold:

$$E_t\Lambda_{t,t+1+i}(R^L_{t+1+i} - R_{t+1+i}) \geq 0, i \geq 0 \quad (15)$$

where $\Lambda_{t,t+1+i}$ is the stochastic discount factor applied in period $t$ to earnings in $t+1+i$. The survival rate of the financial intermediary is $\theta$, a probability that is independent of job history. This implies that the average lifetime of a banker in any period is $\frac{1}{1-\theta}$.

Similar to the birth-and-death assumption of banks in Bernanke, Gertler and Gilchrist (1999), a positive exit probability prevents bankers from accumulating sufficient net worth to finance equity investments internally. In each period, $(1 - \theta)$ of bankers exit and become workers, while a similar number of workers take up jobs as financial intermediaries. Bankers who exit from the financial sector transfer their earnings back to their corresponding households, and the households provide some startup funds for new bankers.

Financial intermediary $j$ maximizes the expected discounted terminal net worth, $V^j_t$, by choosing the amount of assets to purchase:

$$V^j_t = \max_E E_t \sum_{i=0}^{\infty} (1 - \theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i}[(R^L_{t+1+i} - R_{t+1+i})B^j_t + R_{t+1+i}N^j_{t+i}] \quad (16)$$

Similar to Gertler and Karadi (2011), $V^j_t$ can be rewritten as follows:

$$V^j_t = v_t B^j_t + \eta_t N^j_t \quad (17)$$
with
\[ v_t = E_t[(1 - \theta)\beta_s A_{t,t+1}(B_{t+1}^j - R_{t+1}) + \beta_s A_{t,t+1}\theta \frac{B_{t+1}^j}{B_t^j}v_{t+1}] \] (18)
\[ \eta_t = E_t[(1 - \theta) + \beta_s A_{t,t+1}\theta \frac{N_{t+1}^j}{N_t^j}\eta_{t+1}] \] (19)

\( v_t \) is the expected discounted marginal benefit to banker \( j \) for a unit increase in asset holdings \( B_t^j \), keeping \( N_t^j \) constant. Analogously, \( \eta_t \) is the expected discounted marginal gain for a unit increase in net worth, holding total assets constant.

Following Gertler and Karadi (2011), I introduce a moral hazard with costly enforcement problem to limit the liability of financial intermediaries. This aims to prevent intermediaries from borrowing indefinitely from households given a positive expected risk premium, as shown in (15). At the beginning of any period, the banker can choose to divert a fraction \( \lambda \) of all available funds from the projects and transfer them back to the corresponding household. Upon this action, depositors can force the intermediary into bankruptcy and recapture the remaining fraction \( 1 - \lambda \) of total assets. Costly enforcement implies that it is too expensive for depositors to recover the diverted fraction \( \lambda \) of total assets. To ensure that depositors are willing to supply funds to bankers in each period, the following incentive constraint must be satisfied:

\[ V_{t}^j = v_t B_t^j + \eta_t N_t^j \geq \lambda B_t^j \] (20)

The left-hand side of (20) is the cost of diverting funds for banker \( j \), which is equivalent to the value of operating the intermediary. The right-hand side is the gain from diverting a fraction \( \lambda \) of available assets. The financial intermediary chooses not to divert only when the value of operating the intermediary is greater than or equal to the benefit from diverted assets.

Free of agency problems, the financial intermediary would continue to expand borrowing until \( R_{t+1}^j \) is adjusted to ensure \( v_t = 0 \). With the incentive compatibility constraint in place, the intermediary’s asset holdings are restricted by the equity capital. When the constraint binds, (20) can be rewritten as

\[ B_t^j = \frac{\eta_t}{\lambda - v_t} N_t^j \]
\[ = \phi_t N_t^j \] (21)

where \( \phi_t \) is the endogenous leverage ratio of the banks that depositors will tolerate.
Proposition 1  The incentive compatibility constraint binds if and only if $0 < \nu_t < \lambda$.

Proof. This is proven by contradiction. Suppose $\nu_t \geq \lambda$. Then given $\eta_t N^j_t > 0$, the left-hand side of (20) is always greater than the right-hand side. Therefore the constraint is not binding. This implies that the value of operating a bank is always greater than the benefit from diverting funds. Now, suppose $\nu_t \leq 0$. Then the marginal gain of increasing investment in financial assets is less than or equal to zero, implying that the financial intermediary will not take deposits from households to acquire assets $B^j_t$, which results in a slack constraint. It is important to emphasize that the credit condition tightens when $\lambda$ increases. □

When the constraint binds, the law of motion of intermediary $j$’s net worth becomes:

$$N^j_{t+1} = [(R^L_{t+1} - R_{t+1})\phi_t + R_{t+1}]N^j_t$$  \hspace{1cm} (22)

The leverage ratio does not depend on any bank-specific characteristics, therefore aggregate variables can be obtained simply by summing across all intermediaries. Let $N_t$ be the aggregate net worth of all banks, it can be written as the sum of the net worth of existing bankers, $N_{et}$, and the net worth of new bankers, $N_{nt}$.

$$B_t = \phi_t N_t = \phi_t (N_{et} + N_{nt})$$  \hspace{1cm} (23)

Given that only a fraction $\theta$ of bankers at $t - 1$ survive through period $t$, along with equation (22), $N_{et}$ can be expressed as

$$N_{et} = \theta N_t = \theta [(R^L_t - R_t)\phi_{t-1} + R_t]N_{t-1}$$  \hspace{1cm} (24)

New bankers receive startup funds from their respective households. It is assumed that these funds equal a fraction of the value of assets that exiting bankers intermediated in their final operating period, which is given by $(1 - \theta)B_t$. Households are assumed to transfer $\frac{\omega}{1-\theta}$ of the total final-period assets of exiting bankers to newly entering financial intermediaries. $N_{nt}$ can then be written as

$$N_{nt} = \omega B_t$$  \hspace{1cm} (25)

Therefore, we can rewrite the law of motion for $N_{t+1}$ as follows

$$N_t = \theta [(R^L_t - R_t)\phi_{t-1} + R_t]N_{t-1} + \omega B_t$$  \hspace{1cm} (26)
3.3 Entrepreneurs

There is a continuum of infinitely lived entrepreneurs of measure one, who produce a homogeneous good utilizing household labor, capital and housing stock. Entrepreneurs use a standard Cobb–Douglas production function with constant return-to-scale. Note that there is imperfect substitution between impatient and patient household labor, which may be explained by assuming the patient households to be managers of firms and impatient households to be workers. The representative entrepreneur produces an intermediate good $Y_t$, using capital goods $K_t$, housing stock leased from patient households, $(1 - \kappa)H_{t,s}$, and the labor input of the patient and impatient households according to the production function:

$$ Y_t = A_t(U_t\xi_t K_t)^\eta[(1 - \kappa)H_{t,s}]^\nu L_{t,s}^{(1-\eta-v)} L_{t,b}^{(1-\alpha)(1-\eta-v)} $$

(27)

$A_t$ is the technology shock, $\xi_t$ is an exogenous shock to the quality of capital and $U_t$ is the utilization rate of capital. $\xi_t K_t$ is the effective quantity of capital at time $t$. Firms raise funds from financial intermediaries by issuing claims against working capital $K_{t+1}$ at price $Q_t$. More specifically,

$$ B_{t,e} = Q_t K_{t+1} $$

(28)

The endogenous borrowing constraints are only applicable to banks since non-financial firms face no credit friction. Banks have perfect information about firms and there is perfect enforcement. The financial constraints discussed in Section 3.2, however, directly affect the supply of funds to the firms.

Intermediate-good producers choose a capital utilization rate $U_t$, capital $K_t$, housing goods $H_{t,s}$ and labor rate $(L_{t,s}, L_{t,b})$ to produce $Y_t$, and sells this product at price $P_{mt}$. The firm solves

$$ \max_{U_t, L_{t,s}, L_{t,b}, K_t} P_{mt} Y_t + Q_t \xi_t K_t - \delta(U_t) \xi_t K_t - W_{t,s} L_{t,s} - W_{t,b} L_{t,b} - R_t L_{t-1} K_t - \tau^h (1 - \kappa) P_t H_{t-1,s} $$

(29)

where $\delta(\cdot)$ is the depreciation rate on capital. The first-order conditions are given by

$$ L_{t,s} : W_{t,s} = P_{mt} (1 - \eta - v) \alpha \frac{Y_t}{L_{t,s}} $$

(30)

$$ L_{t,b} : W_{t,b} = P_{mt} (1 - \eta - v) (1 - \alpha) \frac{Y_t}{L_{t,b}} $$

(31)
\[ U_t : \delta'(U_t) \xi_t K_t = P_{mt} Y_t \]

The representative firm pays out the ex-post return to capital to the banks and earns zero profit. The ex-post return to capital is given by

\[ R^L_{t+1} = \frac{[P_{mt+1} Y_{t+1} + Q_{t+1} - \delta(U_{t+1})] \xi_{t+1}}{Q_t} \]

The intuition is that the ex-post gross rate of return on capital equals the sum of the marginal productivity of labor and the capital gain from changes in capital prices. Note that the valuation shock \( \xi_{t+1} \) provides additional variation to the return on capital. The value of capital stock at the end of period \( t+1 \) is \( \xi_{t+1} U_{t+1} [Q_{t+1} - \delta(U_{t+1})] \).

Similarly, the non-financial firm pays ex-post return to housing stock as rents to the patient households. Taking the derivative with respect to \( H_{t,s} \), the ex-post rent is given by

\[ r^h_{t+1} = -\frac{P_{mt+1} Y_{t+1}}{(1 - \kappa) P^H_{t+1} H_{t,s}} \]

### 3.4 Capital Producers

Capital producers are essential to introduce capital adjustment costs in a tractable way. Following the literature on financial accelerators, capital adjustment costs induce additional variations in the price of capital in response to changes in capital stock. At the end of period \( t \), competitive capital-producing firms purchase used capital from intermediate-good producers, refurbish the old and produce new capital. They sell both repaired and new capital. Assuming the cost of replacing depreciated capital is one, the price of a unit of new or repaired capital is denoted by \( Q_t \). Let \( I_t \) be the gross capital created, \( I_{nt} \) the net capital created and \( \bar{I} \) the steady state value of \( I_t \), then the capital-good producer solves

\[ \max_{I_{nt}} E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_0 t [Q_t - 1] I_{nt} - \Phi(\frac{I_{nt + \bar{I}}}{I_{nt - 1 + \bar{I}}})(I_{nt} + \bar{I}) \]

s.t. \( I_{nt} = I_t - \delta(U_t) \xi_t K_t \)
\( F(\cdot) \) is the capital adjustment cost function, with \( F(1) = F'(1) = 0 \) and \( F''(1) > 0 \). The first-order condition that characterizes the net investment \( Q \) relation is given by

\[
Q_t = 1 + \Phi\left( \frac{I_{nt} + \bar{I}}{I_{nt-1} + \bar{I}} \right) + \Phi\left( \frac{I_{nt} + \bar{I}}{I_{nt-1} + \bar{I}} \right) (I_{nt} + \bar{I}) - E_t \beta_A N_{t,t+1} \Phi'\left( \frac{I_{nt+1} + \bar{I}}{I_{nt} + \bar{I}} \right) (I_{nt+1} + \bar{I})^2
\]

(37)

Note that given no idiosyncratic shock among capital-good producers, all firms choose the same net investment rate. In this setup, the price of capital increases when total investment expenditure expands.

The depreciation rate and adjustment cost function are assumed to take the following functional forms:

\[
\delta(U_t) = \bar{\delta} - \frac{\delta}{1 + \omega} + \frac{\delta}{1 + \omega} U_t^{1+\omega}
\]

(38)

\[
\Phi\left( \frac{I_{nt} + \bar{I}}{I_{nt-1} + \bar{I}} \right) = \frac{\phi_I}{2} \left( \frac{I_{nt} + \bar{I}}{I_{nt-1} + \bar{I}} - 1 \right)^2
\]

(39)

where \( \bar{\delta} \) is determined by the steady state and \( \omega \) and \( \phi_I \) are parameters.

### 3.5 Retail Firms

Retail firms are present in the model to introduce sticky prices. There is a continuum of monopolistic competitive retailers who purchase intermediate output from intermediate-good producers and produce the final output, \( Y_t \). The CES composite of final goods is given by

\[
Y_t = \int_0^1 Y_{st}^{\frac{\varepsilon - 1}{\varepsilon - 1}} ds
\]

(40)

\( Y_{st} \) is the output by retailer \( s \),

\[
Y_{st} = \left( \frac{P_{st}}{P_t} \right)^{-\varepsilon} Y_t
\]

(41)

\[
P_t = \int_0^1 P_{st}^{1-\varepsilon} ds
\]

(42)

One unit of intermediate goods can be used to produce one unit of final goods. The marginal cost is simply the price of intermediate output, \( P_{nt} \). To introduce nominal rigidities, only a fraction \( 1 - \gamma \) of retailers can reset the price freely in any period (Calvo 1983). Retailers that do not re-optimize prices will index their prices with respect to inflation and parameter \( \gamma_P \). Specifically, the retailer chooses the optimal reset price
\[ P_t^* \text{ to solve} \]

\[
\max_{P_t^*} E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i}[P_t^* \frac{\prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p}}{P_{t+i}} - P_{mt+i}]Y_{st+i}
\]  

(43)

\( \pi_t \) is the rate of inflation from \( t-1 \) to \( t \). The first-order condition is

\[
E_{t} \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i}[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (1 + \pi_{t+k-1})^{\gamma_p}]^{1-\varepsilon} - \frac{1}{1 - 1/\varepsilon} P_{mt+i}]Y_{st+i} = 0
\]  

(44)

Using the law of large numbers, the law of motion for the price level can be derived:

\[
P_{t}^{1-\varepsilon} = (1 - \gamma)(P_{t}^*)^{1-\varepsilon} + \gamma((1 + \pi_{t-1})^{\gamma_p} P_{t-1})^{1-\varepsilon}
\]  

(45)

Let \( \dot{P}_t^* = P_t^*/P_t \), then the law of motion for the relative price level is given by

\[
1 = (1 - \gamma)(\dot{P}_t^*)^{1-\varepsilon} + \gamma[\frac{(1 + \pi_{t-1})^{\gamma_p}}{1 + \pi_t}]^{1-\varepsilon}
\]  

(46)

### 3.6 Central Bank and Monetary Policy

The central bank in this economy administers a log-linearized Taylor rule of the following form:

\[
\dot{i}_t = \rho_r(\dot{i}_{t-1}) + (1 - \rho_r) [\gamma_\pi \hat{\pi}_t + \gamma_\gamma \hat{Y}_t] + \varepsilon_t^R
\]  

(47)

where \( i_t \) is the nominal interest rate. The central bank adjusts interest rates according to the log deviation of inflation and output from their steady-state levels. \( \gamma_\pi > 0 \) and \( \gamma_\gamma > 0 \) are the parameters chosen by the central bank to conduct the monetary policy. When \( \rho_r > 0 \), this rule also includes a first-order autoregressive component that captures the interest rate inertia à la Woodford (2000). The relationship between real and nominal interest rates is characterized by the Fisher equation

\[
1 + \dot{i}_t = R_{t+1} \frac{E_t P_{t+1}}{P_t} = R_{t+1} E_t (1 + \pi_{t+1})
\]  

(48)

\( \varepsilon_t^R \) is a random monetary shock with zero mean and variance \( \sigma_R^2 \).
3.7 Macroprudential Policy

3.7.1 Dynamic Loan-to-Value (LTV) Ratio

Impatient households can only borrow up to a fraction of the expected value of their housing stock, which is characterized by equation (8). The regulatory authority controls for the loan-to-value ratio in order to moderate credit growth in the economy. Under the assumption that the impatient households are subject to a lower discount factor than the patient households, the borrowing constraint will always bind. Therefore, a high LTV ratio releases the tension of the collateral constraint and induces borrowing. A lower LTV ratio implies a tighter constraint that restricts the amount of lending in the real economy. The first experiment is to allow a fixed LTV ratio for the impatient households. More specifically, I set $m_t = m$ in the baseline model, where $m$ is the steady-state value of $m_t$. With the constraint binding, changes in the maximum LTV ratio directly signal credit liberalization or tightening policies. This assists the study of the impact of credit conditions on the real economy.

The 2008 financial crisis grew out of great economic conditions associated with a period of housing price boom and credit liberalization. A fixed LTV ratio is unable to capture the dynamic and corrective measures policymakers may want to implement. A countercyclical LTV ratio is thus desired to limit lending during booms and stimulate the economy during downturns. In an alternative experiment, I adopt a dynamic simple feedback rule of the form:

$$\hat{m}_t = \rho_m \hat{m}_{t-1} - \varphi_Y \hat{Y}_t - \varphi_P \hat{P}_t^H$$

where $\varphi_Y > 0$ and $\varphi_P > 0$ are the parameters chosen by the regulatory authority. In this setup, policymakers adjust the LTV ratio corresponding to the log deviations of output and housing price from their steady-state values. In the literature, Gerlach and Peng (2005) also documents the regulation of household credit as a stabilizing tool for housing prices in Hong Kong and South Korea during the 1997 East Asian crisis.

3.7.2 Pigouvian Lump-Sum Tax

The financial intermediaries face an endogenous capital-to-loan ratio of $\frac{N_t}{B_t} = \frac{1}{\sigma_t}$, which is the reciprocal of their leverage ratio. Similar to the discussion above, the regulatory authority wants to raise banks’ capital and restrict credit growth during economic upturns. Here, I introduce a Pigouvian lump-sum tax scheme adapted from
Lima et al. (2012). In this setup, the government collects tax on banks’ loans and uses the proceeds to subsidize the banks’ net worth. This tax-subsidy policy directly affects the leverage ratio and allows policymakers to stabilize the credit market via the balance sheets of financial intermediaries.

Let \( \tau_t^s \) be the subsidy rate on net worth and \( t \) be the tax rate on loans, then the balance sheet of the bank (equation 13) can be rewritten as follows:

\[
(1 - \tau_t)B_t = D_t + (1 - \tau_t^s)N_t
\]  
(50)

The new law of motion for the banks’ net worth is given by

\[
N_{t+1} = (R_{t+1}^L - R_{t+1})(1 - \tau_t)B_t + (1 - \tau_t^s)R_{t+1}N_t + \tau_t^sN_t
\]  
(51)

The last term on the right-hand side of equation (51) is the lump-sum subsidy. Intermediary \( j \) now solves a new problem

\[
V_t^j = \max E_t \left[ \sum_{i=0}^{\infty} (1 - \theta)^i \beta^{i+1} \Lambda_{t,t+1+i} [(R_{t+1+i}^L - R_{t+1+i})(1 - \tau_{t+i})B_{t+i}^j + ((1 - \tau_{t+i}^s)R_{t+1+i} + \tau_{t+i}^s)N_{t+i}^j] \right]
\]

\[
= v_t^m B_t^j + \eta_t^m N_t^j
\]  
(52)

with

\[
v_t^m = E_t[(1 - \theta)\beta_s \Lambda_{t,t+1}(R_{t+1}^L - R_{t+1})(1 - \tau_t) + \beta_s \Lambda_{t,t+1+1} \frac{B_{t+1}^j}{B_t^j} v_{t+1}]
\]

\[
\eta_t^m = (1 - \theta)(1 - \tau_t^s) + E_t[(1 - \theta)\beta_s \Lambda_{t,t+1} \tau_t^s + \beta_s \Lambda_{t,t+1} \theta \frac{N_{t+1}^j}{N_t^j} \eta_{t+1}]
\]  
(53)

Equation (26) can also be rewritten as

\[
N_t = \theta[(R_t^L - R_t)(1 - \tau_{t-1})\phi_{t-1} + (1 - \tau_{t-1}^s)R_t + \tau_{t-1}^s]N_{t-1} + \omega(1 - \tau_t)B_t
\]  
(55)

The aggregation across banks is identical to section 3.2 with a newly added balanced budget condition

\[
\tau_tB_t = \tau_t^sN_t
\]  
(56)

In addition, there exists a cost of administering this tax scheme, which is quadratic
in the tax rate, $\psi\tau^2B_t$. The tax rate is administered through a feedback rule that corresponds to changes in output. The log-linearized rule is given by

$$\hat{\tau}_t = \rho_r\hat{\tau}_{t-1} + \theta_Y\hat{Y}_t$$

where $\theta_Y > 0$ is a parameter selected by the regulatory authority. When output increases above its steady-state level, the positive tax rate imposed on bank’s lending asserts a negative pressure on the leverage ratio. Subsequent subsidies of banks’ net worth lead to a further decrease in leverage, which increases the capital-to-loan ratio. During economic recessions, the tax and subsidy reverse their positions, and a tax will be imposed on banks’ capital holdings to stimulate borrowing. As a result, the leverage ratio falls, and aggregate lending is expanded to enhance economic recovery. Unlike in Lima et al. (2012), the steady-state tax rate in this model is zero instead of some small positive value. This implies that the economy bears no cost of raising taxes in the equilibrium and only faces such a burden in response to a random shock. This allows better comparison across models because the steady-state values remain unchanged.

The central bank, regulatory authority, and government play different roles in shaping the economy. First, the central bank determines the nominal interest rate through monetary policy, which affects the real interest rate via the Fisher equation. This influences the funding cost of financial intermediaries and their lending rates. Second, regulations on the LTV ratio have a direct impact on household borrowing, which is then passed on to banks’ balance sheet. However, the borrowing condition of entrepreneurs is only influenced indirectly through lending rates. The Pigouvian tax scheme targets the leverage ratio of financial intermediaries directly, which has an impact on both business and household borrowing conditions. With the understanding that each policy affects different agents via different channels, the interaction between monetary policy and macroprudential tools is further examined in Section 6.

### 3.8 Competitive Equilibrium

The shocks to productivity, housing preference, monetary policy and the quality of capital follow the AR(1) process in a log-linearized form:

$$\dot{a}_t = \rho_Aa_{t-1} + \epsilon_t^A$$

$$\dot{z}_t^H = \rho_Hz_{t-1}^H + \epsilon_t^H$$

where $\rho_A, \rho_H < 1$ are the autoregressive coefficients, and $\epsilon_t^A, \epsilon_t^H$ are zero-mean, stationary disturbances.
\[
\hat{\xi}_t = \rho \xi_{t-1} + \varepsilon_t^{\xi} \\
\hat{\xi}_R = \rho R\hat{\xi}_{Rt-1} + \varepsilon_t^{R} \tag{58.2}
\]

A competitive equilibrium is a sequence of allocations

\[
\{H_{t,s}, H_{t,b}, L_{t,s}, L_{t,b}, C_{t,s}, C_{t,b}, D_t, B_{t,b}, B_{t,e}, B_{t}, K_{t+1}, Y_t, N_t, I_t, I_{nt}, v_t, \eta_t, \phi_t, \tau_t, \tau^s_t, m_t\}_{t=0}^\infty \tag{59}
\]

together with a sequence of prices

\[
\{W_{t,s}, W_{t,b}, A_t, A_{t,t+1}, U_t, P^H_t, P^L_t, P^*_t, Q_t, \lambda_t, R_{t+1}, R^L_{t+1}, i_t, r^h_{t+1}, \pi_t\}_{t=0}^\infty \tag{60}
\]

and exogenous processes

\[
\{A_t, \xi_t^R, \xi_t^H, \xi_t\}_{t=0}^\infty
\]

such that i) the allocations solve each household’s, bank’s, entrepreneur’s, capital-good producer’s, and retailer’s maximization problem at equilibrium prices given predetermined variables and ii) all markets clear. The aggregate clearing conditions are given by

\[
H_{t,s} + H_{t,b} = \bar{H} \tag{61}
\]
\[
L_{t,s} + L_{t,b} = 1 \tag{62}
\]
\[
Y_t = C_{t,s} + C_{t,b} + I_t + \Phi(\frac{I_{nt} + \bar{I}}{I_{nt-1} + \bar{I}})(I_{nt} + \bar{I}) + \psi \tau^2_{t}B_t \tag{63}
\]
\[
\tau_t B_t = \tau^s_t N_t \tag{64}
\]

Equation (64) and the term $\psi \tau^2_{t}B_t$ in (63) only appear when the Pigovian tax scheme is imposed. The full log-linearized model is presented in Appendix A.2.

4 Calibration

Table 1 in the appendix provides a summary of the parameters and their calibrated values. The time period in the model is one quarter. Parameter values are mostly taken from Gertler and Karadi (2011) and Iacoviello (2005). The discount factors for saving and borrowing households are chosen to be 0.985 and 0.95, respectively. This implies that the annual net equity return for the patient households is 6.1%. The weights of consumption goods and housing goods in the utility function $(v_c, v_h)$ are calibrated as 0.9 and 0.1, respectively. The weight of labor in the utility function $(v_l)$ is set to 2. The
inverse of the Frisch elasticity of labor supply ($\varphi_L$) is 0.276. According to the Bureau of Economic Analysis, investments in commercial real estates as a percentage of GDP are, on average, twice that of investments in residential property. In this model, I assume that patient households lease a fraction $1 - \kappa = 0.67$ of housing stock to firms. This is also consistent with the estimation results from Yepez (2012), where the posterior mean of $1 - \kappa$ from the Bayesian estimation is around 0.6.

The financial sector consists of three suggestive parameters ($\theta, \lambda, \varpi$), which are the survival rate of bankers, the fraction of wealth a banker could divert and the proportional wealth transfer to the entering banks, respectively. The parameters are chosen to meet two goals. First, the steady-state interest rate spread is 1%. Moreover, the steady state leverage ratio of banks is four. In Gertler and Karadi (2011), the fraction of assets a banker can divert ($\lambda$) is extraordinarily high, more than 30%. This is set to achieve a life expectancy of ten years for bankers. In this model, I set the survival rate of bankers ($\theta$) to be 0.948, which implies an average career horizon of five years. This lowers the fraction of assets a banker can divert to 17%. Entering bankers get a wealth transfer $\varpi \frac{\varpi}{1 - \theta}$ with $\varpi = 0.002$.

In the production sector, the share of capital ($\eta$) and housing stock ($v$) in the Cobb-Douglas production function are set to be 0.33 and 0.03, respectively. Patient household’s labor share ($\alpha$) is set to 0.64. The steady-state utilization rate is normalized to 1 with rate of depreciation $\delta(U) = 0.025$. This implies that capital takes an average of ten years to fully depreciate. The capital adjustment cost parameter $\phi_I$ is 4 and the elasticity of marginal depreciation with respect to the utilization rate ($w$) is 7.2. Retail firm’s elasticity of substitution is 4.167, implying a steady-state real markup of 1.316. The probability of keeping prices fixed ($\gamma$) is 0.779 and the measure of inflation indexation ($\gamma_P$) is 0.241.

The baseline calibration for the degree of intervention in monetary policy is set following Gertler and Karadi (2011) where $\gamma_\pi = 1.5$ and $\gamma_Y = 0.125$. The degree of inertia ($\rho_\pi$) is 0.8. The autoregressive coefficients in the exogenous processes are $\rho_A = 0.85, \rho_H = 0.95, \rho_R = 0.8$ and $\rho_\xi = 0$.

The steady-state consumption ratio between impatient and patient households ($C_b/C_s$) is 0.89 and the ratio of labor supply ($L_b/L_s$) is 1.55. The annualized household debt-to-GDP ratio ($B_b/Y$) is 0.26 and the business-to-household credit ratio ($B_e/B_b$) is 3.9. The aggregate consumption-to-GDP ratio ($C/Y$) is 0.82, the investment-to-GDP ratio ($I/Y$) is 0.12 and the capital-to-GDP ratio ($K/Y$) is 5.02. The steady-state LTV ratio ($m$) and leverage ratio ($\phi$) are 0.7 and 4, respectively. The tax and subsidy rates
are zero in equilibrium. The real interest rate ($R$) is 1.015 and the lending rate ($R^L$) is 1.025.

## 5 Model Analysis

This section reports the simulation results from three alternative specifications with four shocks\(^2\). First, the baseline model ("BLM") considers a fixed LTV ratio with $m = 0.7$, which is the steady-state value for $m_t$. This implies that the impatient households can only borrow up to 70\% of the expected value of their housing stock. In fact, this number is compatible with the average LTV ratio for U.S. residential mortgages (76\%) before the recession in 2008 (IMF 2011). The central bank conducts monetary policy according to equation (47) with no other active regulatory policy. The second model adopts a dynamic LTV ratio rule ("LTVM"), which is characterized by (49). The regulatory authority lowers the LTV ratio during economic booms to limit credit expansion and increases the ratio during recessions. The degree of inertia ($\rho_m$) is 0.85 and the degrees of intervention are $\varphi_P = 1.5$ and $\varphi_Y = 0.15$. In the last specification, the baseline model is augmented with the Pigovian tax scheme ("PTM"), which is characterized by equation (57). The government sets positive tax and subsidy rates during economic booms to increase the capital-to-loan ratio of financial intermediaries. The degree of inertia ($\rho_r$) is 0.75 and the degree of intervention ($\theta_Y$) is 0.57. These values are taken from Lima et al. (2012). The monetary policy is effective in all three models with no change in the parameter values.

The impulse response functions from positive productivity, housing demand, capital quality shocks and expansionary monetary policy are displayed in Figures 1 to 4 in the appendix. The standard deviation for each shock is $\sigma_A = \sigma_\xi = 0.01$, $\sigma_R = 0.01$ (annualized) and $\sigma_H = 0.0021$. The magnitude of the housing demand shock is taken from Suh (2012), who matches the historical volatility of U.S. housing prices. Moreover, 1\% productivity and capital quality shocks are standard as in Gertler and Karadi (2011). The macroprudential policies, if effective, should stabilize the economy relative to the baseline model. Furthermore, fluctuations of household lending and housing prices should be significantly dampened.

\(^2\)The results from a model with both macroprudential policies are not reported here because the impulse response functions with both policies are indistinguishable from those of the tax policy alone. Since the tax policy targets the aggregate credit market and directly controls the balance sheet of financial intermediaries, the LTV ratio rule becomes uneffective as it targets only the housing market.
5.1 Technology Shock

In the baseline model, output, consumption, and investment increase, while inflation decreases in response to a 1% technology shock (Figure 1 solid lines). An increase in consumption drives up housing demand, causing housing prices to rise under a fixed housing supply. Consequently, household lending expands resulting in a higher leverage ratio for financial intermediaries. It should be emphasized that the leverage ratio has increased more than 150% in comparison to the steady-state level, creating excessive liquidity risk in the credit market. Consistent with findings in Gertler and Karadi (2011), the credit spread falls in response to positive productivity shock that causes the banks’ net worth to decrease. With a dynamic LTV ratio rule, the impulse response functions are moderately dampened for consumption, investment, output, inflation and housing price (Figure 1 dashed lines). The LTV ratio decreases by around 6% in response to higher output and housing price. The contractionary LTV ratio further leads to a decrease in household lending since impatient households now face a lower collateral value. However, decreasing net worth and an increasing leverage ratio cause the aggregate lending to increase in response to the shock (as is evident from equation 21). This suggests that financial intermediaries seek to expand business lending when the household credit market is regulated. This creates a credit shift from the impatient households to entrepreneurs, which is consistent with findings in Suh (2012). The regulatory authorities should consider this possibility when choosing the appropriate macroprudential policy.

The tax policy, PTM, is comparably more effective (Figure 1 dash-dotted lines). The leverage ratio remains stable as the government imposes a 1% tax on aggregate lending and simultaneously subsidizes the banks’ net worth. Furthermore, the percentage change in all other variables is significantly smaller than that of the BLM and LTVM. Household lending and housing prices increase slightly in response to the technology shock but revert back to the steady state levels quickly. Comparatively, small changes in the leverage ratio and banks’ net worth provide minimal evidence of credit shift. Both macroprudential policies are effective in stabilizing the economy, but the tax policy may be more desirable in this particular setup.

5.2 Housing Demand Shock

Figure 2 displays the impulse response functions from a housing demand shock of 21 bps, calibrated following Suh (2012). A positive shock implies that both patient and
impatient households now obtain more utility from housing stock. Given an exogenous and fixed housing supply, the price is completely driven by demand. In the BLM, consumption, output and inflation increase while investment falls. As more people demand housing goods, prices increase and lending expands. Financial intermediaries also become more leveraged. In the LTVM, impatient households face a 1% drop in the target LTV ratio, which limits the maximum loan size. This reduces household lending and dampens the growth of housing prices. In addition, the increase in the leverage ratio is reduced by more than 50% in comparison to the BLM. This significantly limits the liquidity risk of financial intermediaries. Similar to the technology shock, a small decrease in banks’ net worth and a relatively large increase in the leverage ratio imply a rise in aggregate lending. Growing business lending is the result of banks’ profit-maximizing behavior. In this model, the entrepreneurs face no credit constraint and the amount of lending depends solely on the price and quantity of capital. This setup allows the banks to seek alternative markets to expand asset holdings. It should be emphasized that the steady-state business-to-household lending ratio is 3.9, which implies that a more volatile credit market in the business sector increases the volatility of the overall financial market, despite of the dampened effect in household lending.

In contrast, the PTM is also effective in response to a housing demand shock. All variables other than housing price are associated with small fluctuations, and the tax rate is positive at around 30 bps. Housing price increases slightly less than the LTVM, but reverts to the steady-state level quickly. Figures 1 and 2 suggest that the dynamic LTV ratio rule is effective in controlling for household lending and housing price, but creates additional volatility in the business sector. The tax policy is effective under both shocks but achieves a better stabilizing role in response to technology disturbance.

5.3 Capital Quality Shock

In Figure 3, a 1% capital quality shock is imposed to examine the effects on the housing market. With an autoregressive coefficient of zero, the disturbance is considered temporary. The overall effect has two stages. First, a positive shock to quality increases the effective quantity of capital, which enhances the balance sheet of financial intermediaries. Consequently, increasing demand for capital drives up the price, $Q_t$, and banks’ leverage ratios. The effects of the shock are amplified by the presence of financial frictions. Demand for housing drops initially as resources are directed toward the production sector, causing the housing price to fall. In the second stage, positive income growth naturally induces greater housing demand and puts additional pressure
on the credit market. In the BLM, the housing prices first decreases by 4%, and then quickly rises to 2% above the steady-state level. Consumption and the leverage ratio are hump-shaped given the two-stage adjustment process and the banks’ net worth grows by 30%.

The introduction of the dynamic LTV ratio effectively dampens the effects from the shock and restricts credit expansion. Because the housing price falls immediately after the shock, the target LTV ratio first increases above its steady-state level, then drops significantly. The borrowing limit for impatient households increases first, which leads to greater household lending. As output and housing prices both increase, the LTV ratio falls below the long-run level, which leads to a gradual decrease in household loans. This macroprudential policy actively adjusts the borrowing constraints to reduce fluctuations in the credit market. As a result, disturbances in the economy are effectively mitigated. In contrast with technology and housing demand shocks, business lending expands naturally in response to the higher price and quantity of capital with little evidence of credit reallocation.

The tax policy is more effective in stabilizing the credit market but not the aggregate economy. A positive tax on total lending reduces the growth of household borrowing and alleviates the increase in housing price. Changes in the net worth of financial intermediaries have two parts. The first part comes from increases in the quantity and price of capital, as shown in the BLM. However, the positive subsidy further enhances the banks’ net worth. As a result, the net present value of financial intermediaries increases by nearly 40% in response to the capital quality shock. Consequently, the credit market is more restricted with a smaller increase in the leverage ratio. Since households own the financial intermediaries, consumption and output growths are higher in comparison to the LTVM, but are still noticeably smaller than those from the BLM.

5.4 Monetary Shock

Figure 4 displays the impulse response functions from an unexpected 1% (annualized) decline in the nominal interest rate. Consumption, investment, output and inflation rise in response to the shock. This expansionary shock lowers the borrowing expense of banks and induces more lending. The leverage ratio, housing price and household lending increase significantly in the BLM. The greater risk premium also increases banks’ net worth. With monetary policy solely in effect, the economy experiences large fluctuations and faces higher financial risk. In contrast, the countercyclical loan-to-value ratio policy is effective in stabilizing the economy. In the LTVM, the percentage de-
viations are clearly smaller. The LTV ratio decreases by around 2% before gradually returning to the steady-state level. The housing price and household lending increase by about 50% less than in the baseline case. However, business lending significantly expands, creating the aforementioned credit shift issue. As the cost of borrowing decreases, financial intermediaries have a larger incentive to expand lending in all sectors. Regulatory control in the housing sector promotes active lending in the business sector.

The tax scheme is effective in mitigating the economic response to disturbances, as demonstrated in previous sections. The tax rate increases by 1% in response to a 2% rise in output. Although the initial deviation of output is greater than in the LTVM, the recovery is also more rapid. The tax policy effectively reduces the changes to banks’ balance sheets, resulting in small fluctuations of the housing price and household lending. In this case, business lending also expands more than household lending because of the lower borrowing cost for banks, suggesting some evidence for a credit shift. However, the magnitude is smaller in comparison to the LTVM.

5.5 Volatility of Economic Variables

Macropurudential policy aims to stabilize the economy and dampen fluctuations caused by unexpected shocks. The impulse response functions presented above provide direct measures of the percentage change of major variables in response to shocks. However, evaluation of the relative volatility is essential for a complete assessment. More specifically, a successful macroprudential policy should not only reduce the relative change of an economic variable, but also lessen the overall variation. Table 2 summarizes the unconditional standard deviation of consumption, output, investments, inflation, housing price, banks’ net worth, household lending and business lending. The percentage differences between the baseline model and other models are listed in brackets.

Both the dynamic LTV ratio rule and Pigovian tax policy significantly reduce the volatility of the housing price. In addition, household lending is around 10% less volatile in models with macroprudential policy. However, the results are not uniform among the other variables. In the LTVM, the standard deviations of investment and business lending are higher than the baseline model. This suggests that banks reallocate towards the less regulated production sector when a restriction is applied to the housing market. Financial intermediaries expand business lending to maximize profit, which leads to a more volatile credit market. Since the ratio of business-to-household lending is 3.9 in the steady state, aggregate lending in the economy fluctuate even more than in the
case with a fixed LTV ratio.

The tax policy is more effective in stabilizing the economy. All variables are less volatile in comparison to the BLM and LTVM. Furthermore, the standard deviation of business lending and investments are significantly lessened. In summary, the regulatory authority needs to consider the possible trade-off between the business and household sectors when choosing the appropriate policy. In the following section, I search for the optimal degree of intervention for each model by maximizing the social welfare of the economy. This provides a more practical suggestion for policymakers seeking an effective and implementable macroprudential policy.

Table 2: Unconditional Standard Deviation of Major Economic Variables

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Y</th>
<th>I</th>
<th>π</th>
<th>PH</th>
<th>N</th>
<th>B₀</th>
<th>Bₑ</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLM</td>
<td>0.01356</td>
<td>0.01621</td>
<td>0.01296</td>
<td>0.0079</td>
<td>0.0197</td>
<td>0.0780</td>
<td>0.1036</td>
<td>0.0908</td>
</tr>
<tr>
<td></td>
<td>(-3.9%)</td>
<td>(-1.1%)</td>
<td>(6.7%)</td>
<td>(-5.1%)</td>
<td>(-16.8%)</td>
<td>(-15.5%)</td>
<td>(-9.4%)</td>
<td>(33.9%)</td>
</tr>
<tr>
<td>LTVM</td>
<td>0.01302</td>
<td>0.01603</td>
<td>0.01383</td>
<td>0.0075</td>
<td>0.0164</td>
<td>0.0659</td>
<td>0.0939</td>
<td>0.1216</td>
</tr>
<tr>
<td></td>
<td>(-9.4%)</td>
<td>(-2.5%)</td>
<td>(-3.5%)</td>
<td>(-7.6%)</td>
<td>(-28.4%)</td>
<td>(-12.4%)</td>
<td>(-10.2%)</td>
<td>(-19.1%)</td>
</tr>
<tr>
<td>PTM</td>
<td>0.01229</td>
<td>0.01581</td>
<td>0.01250</td>
<td>0.0073</td>
<td>0.0141</td>
<td>0.0683</td>
<td>0.0930</td>
<td>0.0735</td>
</tr>
</tbody>
</table>

6 Optimal Policy and Welfare Analysis

In this section, I conduct welfare analysis to search for the optimal monetary and macroprudential policies outlined in Section 3. The policy problem faced by the central bank and regulatory authority is essentially the choice of the optimal coefficients entering the feedback rules. More specifically, I look for the optimal monetary, LTV ratio and tax rules described in equations (47), (49), and (57). In most DSGE models, a practical and implementable rule is optimal if it yields the highest value for an appropriate welfare criterion. The traditional linear-quadratic approach introduced by Rotemberg and Woodford (1997) has been criticized for producing inaccurate results, as it neglects the higher moments necessary for the evaluation of risk and welfare. Kim and Kim (2003) show that in a two-agent economy, welfare evaluation based on linear approximation to the policy function may yield biased result that an autarky economy generates a higher welfare in comparison with an economy under risk sharing. In this paper, I follow Schmitt-Grohe and Uribe (2004) and use the second-order approximation method for welfare comparison.
In this model, both monetary and macroprudential policy are based on observable variables to allow better implementation. In addition, the parameter values are restricted in a pre-set range to ensure a unique solution. Welfare is measured using the unconditional expectation of the aggregate household utility at period zero. Equation (65) characterizes the social welfare function.

\[
W_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \nu_c \log C_{t,s} + v_h \varepsilon^H_t \log(\kappa H_{t,s}) - v_l \frac{L^{1+\varphi_L}_{t,s}}{1 + \varphi_L} \right] \right\} + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \nu_c \log C_{t,b} + v_h \varepsilon^H_t \log(H_{t,b}) - v_l \frac{L^{1+\varphi_L}_{t,b}}{1 + \varphi_L} \right] \right\} (65)
\]

The sum of household utility arises from the assumption that the population of patient and impatient household are equalized. The objective is to calculate the welfare measure using different combination of policy parameters to select the optimal rules.

For monetary policy, the values of \( \gamma_\pi \) and \( \gamma_Y \) are set to be within the ranges of \([1, 2]\) and \([0, 1]\), respectively. This is consistent with the findings of Schmitt-Grohe and Uribe (2004). For macroprudential policy parameters, the range of \( \varphi_Y \) (the response of the LTV ratio rule to output), \( \varphi_P \) (the response of the LTV ratio rule to housing price), and \( \theta_Y \) (the response of the tax rate to output) are restricted to be within \([0, 2]\). In the numerical search for optimal parameters, the grid size is chosen to be 0.05 and \( W_0 \) is calculated for each combination. The optimal combination of monetary and macroprudential policy is identified at the maximum welfare measure. The results are presented in Table 3.

Table 3: Optimal Policy and Welfare Comparison

<table>
<thead>
<tr>
<th>Model</th>
<th>( \gamma_\pi )</th>
<th>( \gamma_Y )</th>
<th>( \varphi_Y )</th>
<th>( \varphi_P )</th>
<th>( \theta_Y )</th>
<th>Welfare</th>
<th>% Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>1.5</td>
<td>0.125</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>-107.11</td>
<td>–</td>
</tr>
<tr>
<td>Monetary Policy only</td>
<td>1.65</td>
<td>0.105</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>-106.09</td>
<td>0.95%</td>
</tr>
<tr>
<td>Monetary + LTV Ratio</td>
<td>1.9</td>
<td>0.15</td>
<td>0.13</td>
<td>1.45</td>
<td>–</td>
<td>-105.96</td>
<td>1.07%</td>
</tr>
<tr>
<td>Monetary + Tax Scheme</td>
<td>1.85</td>
<td>0.1</td>
<td>–</td>
<td>–</td>
<td>0.35</td>
<td>-105.82</td>
<td>1.20%</td>
</tr>
</tbody>
</table>

Four different models are considered here. In the baseline model, the central bank administers only monetary policy which is calibrated according to Gertler and Karadi (2011). The "Monetary Policy only" panel presents the optimal parameters for monetary policy, where \( \gamma_\pi = 1.65 \) and \( \gamma_Y = 0.105 \). These two models utilize a fixed
LTV ratio and consider no macroprudential tools. The optimal monetary policy found implies that the central bank responds more than one-to-one to changes in inflation, which is consistent with the Taylor principle. The second last column reports the welfare measure characterized by equation (65). The percentage gains in the last column are the percentage increase in welfare compared to the baseline model. For the optimal monetary policy, the net welfare gain is 0.95%.

In the "Monetary + LTV Ratio" panel, the central bank administers monetary policy along with the dynamic LTV ratio rule presented in Section 3.7.1. The regulatory authority adopts a countercyclical target LTV ratio rule to reduce credit expansion during economic booms. The optimal monetary policy responds more aggressively to inflation but less to output. The optimal parameters for the LTV ratio rule are \( \varphi_Y = 0.13 \) and \( \varphi_P = 1.45 \). This implies that the regulatory authority lowers the target LTV ratio more than 1% for a 1% increase in housing price. The welfare gain compared to the baseline case is 1.07%. In the last model, monetary policy and the Pigovian tax-subsidy scheme operate jointly to stabilize the economy. The optimal parameters for monetary policy remain similar to the model with a dynamic LTV ratio rule. For the tax policy, the optimal parameter value for \( \theta_Y \) is 0.35. This implies that the policymakers should increase the tax rate by 0.35% per 1% increase in output. The welfare gain amounts to 1.20% in this model, the highest among all models. This result reinforces the previous findings that the tax scheme is more effective in stabilizing the economy and also improves social welfare. The optimal simple rules provide policymakers with practical and implementable policies to effectively mitigate fluctuations from shocks to the economy.

7 Conclusion

This paper models a New Keynesian DSGE economy with the housing sector and financial frictions to examine the implication of macroprudential policies. Findings suggest that the countercyclical loan-to-value ratio rule responding to output and housing price changes is effective in stabilizing the economy but causes a credit shift to the business sector. Profit-maximizing banks expand lending in the business sector when restrictions are applied in the housing market. The policy that subsidizes the net worth of banks financed by a tax on lending appears to be more efficient. A welfare analysis shows that macroprudential policies are welfare-improving and that credit stabilization should be an objective for policymakers in the design of regulatory instruments.
In addition, optimal macroprudential policies are countercyclical aiming to create a buffering effect for the economy under shock. They help to limit credit growth and price boom in good economic conditions while facilitating more lending in downturns.

The main contribution of this paper is the examination of an economy that is closely related to the recent financial crisis, which began with a housing price boom and coupled with the failure of financial regulation. This model explicitly addresses the financial sector and housing market. Macroprudential instruments target the financial market directly, and can help to reduce systematic risk. The results suggest that if these policies had been preventively adopted before the housing boom in 2007, the housing price and credit growth may have been significantly limited to prevent the later crash. However, the implementation of macroprudential policies may pose a challenge to policymakers because of the great administration and monitoring cost. Nonetheless, this paper offers preliminary insights into the implications of macroprudential tools in a New Keynesian DSGE model with housing and financial frictions. An alternative consideration of financial frictions such as the financial accelerators (Suh 2012) can also be utilized to address the issues studied here. Moreover, investment in housing is not incorporated in this paper and can be extended further to study the real economy more extensively. In addition, explicit credit constraints on non-financial firms are not modeled. The credit shift phenomenon can be further examined when the firms are regulated jointly with the housing sector.

Further research is needed to study the issues related to macroprudential policy. Appropriate and effective financial regulation poses a challenge to many countries with active financial markets. The literature has not yet reached a consensual view toward the design and implementation of regulatory instruments, and future research is strongly desired to promote financial stability.
References


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Appendix

A.1 Tables and Figures

Table 1: Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_s )</td>
<td>0.985</td>
<td>saver’s discount factor</td>
</tr>
<tr>
<td>( \beta_b )</td>
<td>0.95</td>
<td>borrower’s discount factor</td>
</tr>
<tr>
<td>( v_C )</td>
<td>0.9</td>
<td>weight of consumption in utility</td>
</tr>
<tr>
<td>( v_h )</td>
<td>0.1</td>
<td>weight of housing in utility</td>
</tr>
<tr>
<td>( \psi )</td>
<td>2</td>
<td>weight of labor in utility</td>
</tr>
<tr>
<td>( \varphi_L )</td>
<td>0.276</td>
<td>inverse Frisch’s elasticity of labor supply</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.33</td>
<td>fraction of leased housing stock</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.948</td>
<td>survival rate of banker</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.17</td>
<td>fraction of asset that can be diverted</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.002</td>
<td>proportional transfer to entering bankers</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.64</td>
<td>patient household’s labor share</td>
</tr>
<tr>
<td>( U )</td>
<td>1</td>
<td>steady state capital utilization rate</td>
</tr>
<tr>
<td>( \delta(U) )</td>
<td>0.025</td>
<td>steady state rate of depreciation</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.03</td>
<td>share of housing in production function</td>
</tr>
<tr>
<td>( \phi_I )</td>
<td>4</td>
<td>capital adjustment cost parameter</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.779</td>
<td>probability of keeping price fixed</td>
</tr>
<tr>
<td>( w )</td>
<td>7.2</td>
<td>elasticity of marginal depreciation</td>
</tr>
<tr>
<td>( \gamma_P )</td>
<td>0.241</td>
<td>measure of inflation indexation</td>
</tr>
<tr>
<td>( \gamma_Y )</td>
<td>0.125</td>
<td>degree of intervention for output</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>0.8</td>
<td>degree of inertia, nominal interest rate</td>
</tr>
<tr>
<td>( \rho_A )</td>
<td>0.85</td>
<td>autocorrelation, technology shock</td>
</tr>
<tr>
<td>( \rho_H )</td>
<td>0.95</td>
<td>autocorrelation, housing shock</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>0.8</td>
<td>autocorrelation, monetary shock</td>
</tr>
<tr>
<td>( \rho_\xi )</td>
<td>0</td>
<td>autocorrelation, capital shock</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.33</td>
<td>share of capital in production function</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.33</td>
<td>share of housing in production function</td>
</tr>
</tbody>
</table>

Figure A.1: U.S. real housing price index from 1980 to 2013, with 1980=100. Data is taken from the St. Louis Fed.
Figure A.2: Case-Shiller U.S. national house price index, with 2000=100. Data is taken from the St. Louis Fed.

Figure A.3: U.S. mortgage payment to income ratio from 2000 to 2013. Data is taken from the Federal Reserve Board.
Figure 1: Impulse response functions given a productivity shock
Figure 2: Impulse response functions given a housing demand shock
Figure 3: Impulse response functions given a capital quality shock
Figure 4: Impulse response functions given a monetary shock
A.2 Log-Linearized Model

Let \( \hat{x} \) be the log-deviation of variable \( X \) from steady state value \( \bar{X} \).

Patient households:

\[
\varphi_L \hat{t}_{t,s} = \hat{w}_{t,s} - \hat{c}_{t,s} \tag{A1}
\]

\[
0 = \hat{c}_{t,s} - E_t(\hat{c}_{t+1,s} - \hat{r}_{t+1}) \tag{A2}
\]

\[
\frac{v_c}{C_s} (\hat{p}^H_t - \hat{c}_{t,s}) = \frac{v_h}{H_s} (\hat{c}^H_t - \hat{h}_{t,s}) + \beta_s \frac{v_c}{C_s} E_t \{ [1 + \hat{r}^h(1 - \kappa)] (\hat{p}^H_{t+1} - \hat{c}_{t+1,s}) + (1 - \kappa) \hat{r}^h \} \tag{A3}
\]

\[
\hat{\Lambda}_{t,t+1} = \hat{c}_{t,s} - E_t \hat{c}_{t+1,s} \tag{A4}
\]

Impatient households:

\[
\varphi_L \hat{t}_{t,b} = \hat{w}_{t,b} - \hat{c}_{t,b} \tag{A5}
\]

\[
- \frac{v_c}{C_b} \hat{c}_{t,b} = \bar{\lambda} \bar{R}_L E_t (\Lambda_t + \hat{r}_{t+1}) + \beta_b \frac{v_c}{C_b} \bar{R}_L E_t (\hat{r}_{t+1} - \hat{c}_{t+1,b}) \tag{A6}
\]

\[
(\hat{p}^H_t - \hat{c}_{t,b}) \frac{v_c}{C_b} = \frac{v_h}{H_b} (\hat{c}^H_t - \hat{h}_{t,b}) + \beta_b \frac{v_c}{C_b} E_t (\hat{p}^H_{t+1} - \hat{c}_{t+1,b}) + \bar{\lambda} \bar{m} E_t (\hat{p}^H_{t+1} + \Lambda_t + \hat{m}_t) \tag{A7}
\]

Financial Intermediaries:

\[
\bar{B}_b \hat{b}_{t,b} = \bar{m} \frac{\bar{H}_b}{\bar{R}_L} E_t [\hat{m}_t + \hat{p}^H_{t+1} - \hat{r}_{t+1}^L + \hat{h}_{t,b}] \tag{A8}
\]

\[
\hat{b}_{t,e} = \hat{q}_t + \hat{k}_{t+1} \tag{A9}
\]

\[
\bar{B}_e \hat{b}_t = \bar{B}_b \hat{b}_{t,b} + \bar{B}_e \hat{b}_{t,e} \tag{A10}
\]

\[
\bar{B} \hat{b}_t = \bar{D} \hat{d}_t + \bar{N} \hat{n}_t \tag{A11}
\]

\[
\bar{\nu} \hat{v}_t = E_t [((1 - \theta) \beta_s (\bar{R}^L \hat{r}_{t+1}^L - \bar{R} \hat{r}_{t+1} + \Lambda_{t,t+1} (\bar{R}^L - \bar{R}))) + \theta \beta_s \bar{\nu} (\hat{b}_{t+1} - \hat{b}_t + \hat{v}_{t+1})] \tag{A12}
\]

\[
\eta_t = \theta \beta_s E_t [\hat{\eta}_{t+1} + \hat{\Lambda}_{t,t+1} + \hat{n}_{t+1} - \hat{n}_t] \tag{A13}
\]

\[
\hat{b}_t = \hat{\phi}_t + \hat{n}_t \tag{A14}
\]

\[
\hat{\phi}_t = \hat{\eta}_t + \frac{\bar{\nu}}{\bar{\lambda} - \bar{\nu}} \hat{v}_t \tag{A15}
\]

\[
\hat{n}_t = \theta \hat{\phi}_t [\bar{R}^L \hat{r}_t^L - \bar{R} \hat{r}_t] + (\bar{R}^L - \bar{R}) (\hat{\phi}_{t-1} + \hat{n}_{t-1}) + \theta \bar{R} (\hat{r}_t + \hat{n}_{t-1}) + \bar{\omega} \hat{\phi}_t \tag{A16}
\]
Entrepreneurs:

\[
\dot{y}_t = \dot{a}_t + \eta(\dot{u}_t + \dot{\xi}_t + \dot{k}_t) + v\hat{h}_{t,s} + (1 - \eta - v)(a\hat{l}_{t,s} + (1 - \alpha)\hat{l}_{t,b}) \tag{A17}
\]

\[
\dot{w}_{t,s} = \dot{p}_{mt} + \dot{y}_t + \hat{l}_{t,s} \tag{A18}
\]

\[
\dot{w}_{t,b} = \dot{p}_{mt} + \dot{y}_t + \hat{l}_{t,b} \tag{A19}
\]

\[
(1 + \omega)u_t + \dot{\xi}_t + \dot{k}_t = \dot{p}_{mt} + \dot{y}_t \tag{A20}
\]

\[
\bar{R}^L(r_{t+1}^L + \dot{q}_t) = \bar{P}_m\bar{Y}\frac{\bar{Y}}{K}(\hat{p}_{mt+1} + \hat{y}_{t+1} - \hat{k}_{t+1}) + \hat{q}_{t+1} + (1 - \bar{\delta})\hat{\xi}_{t+1} - \bar{\delta}\hat{u}_{t+1} \tag{A21}
\]

\[
\ddot{r}_{t+1} = \ddot{p}_{mt+1} + \ddot{y}_{t+1} - \ddot{h}_{t,s} \tag{A22}
\]

\[
\ddot{\xi}_{nt} = \ddot{I}_{nt}^{invest} - \bar{\delta}\bar{K} (\dot{u}_t + \ddot{\xi}_t + \ddot{k}_t) \tag{A23}
\]

Capital producers:

\[
\dot{K}\dot{k}_{t+1} = \bar{K}\dot{k}_t + \ddot{\xi}_t + \hat{\xi}_{nt} \tag{A24}
\]

\[
q_t = \frac{\phi_t}{T}(\hat{i}_{nt} - \hat{i}_{nt-1}) - \beta_s E_t[\frac{\phi_t}{T}(\hat{i}_{nt+1} - \hat{i}_{nt})] \tag{A25}
\]

Retailers:

\[
\dot{a}_t = \dot{a}_t \tag{A26}
\]

\[
\ddot{a}_t = \ddot{Y}(\ddot{p}_t^* + \ddot{y}_t) + \gamma\beta_s\ddot{a}_t E_t[\ddot{p}_t^* - \bar{p}_{t+1} + \bar{a}_{t+1} + (1 - \varepsilon)(\gamma_p\bar{\pi}_t - \bar{\pi}_{t+1})] \tag{A27}
\]

\[
\dddot{a}_t = \dddot{Y}(\dddot{p}_t + \dddot{y}_t) + \gamma\beta_s\dddot{a}_t E_t[\dddot{p}_t - \dddot{p}_{t+1} + \dddot{a}_{t+2} - \varepsilon(\gamma_p\ddot{\pi}_t - \ddot{\pi}_{t+1})] \tag{A28}
\]

\[
0 = (1 - \gamma)(1 - \varepsilon)p_t^* + \gamma(1 - \varepsilon)(\gamma_p\pi_{t-1} - \pi_t) \tag{A29}
\]

Monetary and macroprudential policy:

\[
\dot{\iota}_t = \rho_r(\hat{\iota}_{t-1}) + (1 - \rho_r)[\gamma_{\pi}\ddot{\pi}_t + \gamma_{\gamma}\ddot{\gamma}_t] + \varepsilon_t^R \tag{A30}
\]

\[
\dot{\iota}_t = \ddot{r}_{t+1} + E_t\ddot{\pi}_{t+1} \tag{A31}
\]

\[
\dot{m}_t = \rho_m\hat{m}_{t-1} - \varphi_Y\ddot{Y}_t - \varphi_P\hat{P}_t^H \tag{A32}
\]

Market clearing conditions:

\[
\bar{H}_s\dot{h}_{t,s} + \bar{H}_b\dot{h}_{t,b} = 0 \tag{A33}
\]

\[
\bar{L}_s\dot{\iota}_{t,s} + \bar{L}_b\dot{\iota}_{t,b} = 0 \tag{A34}
\]
\[
\tilde{Y}_t \tilde{y}_t = \bar{C}_S \tilde{c}_{t,s} + \bar{C}_b \tilde{c}_{t,b} + \tilde{I}_t^{\text{Invest}} 
\]  \hspace{1cm} (A35)

Shock processes:

\[
\dot{a}_t = \rho_A \dot{a}_{t-1} + \epsilon_t^A 
\]  \hspace{1cm} (A36)

\[
\dot{z}_t^H = \rho_H \dot{z}_{t-1}^H + \epsilon_t^H 
\]  \hspace{1cm} (A37)

\[
\dot{z}_t^R = \rho_R \dot{z}_{t-1}^R + \epsilon_t^R 
\]  \hspace{1cm} (A38)

\[
\dot{\xi}_t = \rho_Q \dot{\xi}_{t-1} + \epsilon_t^\xi 
\]  \hspace{1cm} (A39)