Employer Learning, Firing Threat and the Peter Principle

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Abstract

The threat of firing is widely used as a discipline device in workplace. Despite its prevalence, theoretical foundations on the credibility of firing threats are not well studied. When firing is costly to the employer, it is not credible to carry out a firing threat unless a decrease in expected future return is associated with the employee's misbehavior. In this paper, we explore the role of learning in ensuring the credibility of firing threats and how a certain level of uncertainty is necessary to effectively induce compliance. Peter Principal arises as an outcome of the model, as workers who are known to be competent with almost certainty can no longer be disciplined and need to be promoted to more difficult tasks, even though they may be less productive at those tasks.

1 Introduction

When a boss asks his secretary to prepare a set of documents for an important meeting, the secretary does it, without bargaining for the price of doing so or receiving a piece rate for this action. This is a typical representation of an employment relationship: the employer has authority over the employee within
a set of tasks, and the employee executes what is commanded even though it may be costly for him to do so. But why does the employee follow his boss's order, when his employment contract is often too vague to bind him into that particular action and his contribution too subtle to be measured and compensated?

It has been recognized, as early as Shapiro and Stiglitz (1984), that firing threat acts as a discipline device when both action and output are hard to contract on. Firms offer higher than market clearing wages, and employees provide compliance and effort due to the fear of getting fired. Workers are disciplined by firing threat, because getting fired is costly to them.

Despite the well acceptance of firing threat as a discipline device, the credibility of it has not received as much attention in the literature. Shapiro and Stiglitz (1984) adopts a zero friction framework, in which there is no turnover cost to the employer, thus assumed away the credibility problem of the firing threat. In real life, however, turnover cost is an integral part of a firm’s cost of labor. Firms incur direct administrative firing cost and the cost of hiring, in addition to the indirect productivity loss during the time of unfilled vacancy. Under positive firing cost, the credibility of firing threat is questionable because it would not be optimal for the employer to carry out the firing threat ex-post.

Suppose the secretary failed to prepare the documents, and the boss knows him so well that, he did not question the secretary's ability or suitability to the job. Then, after the meeting, when the documents are no long needed and the damage is sunk, the boss would not want to fire the secretary, as doing so would incur an additional cost to himself. Expecting this, the secretary will not be disciplined by the firing threat, as he knows that the threat is empty.

In real life, however, a secretary who fails to do get his boss the documents is likely to get fired, or at least if he repeatedly fails to do so. The boss would willingly replace the secretary, even with the firing cost, because he would probably
believe the secretary is incompetent after a series of nonperformance. With uncertainty about the secretary's productivity, there is information revealed through nonperformance, and firing threat is made credible if ex-post believed productivity is reduced by more than the firing cost.

In this paper, I formalizes the idea that uncertainty and learning is essential for the credibility of firing threat and explore its implications on employment contract and promotion rules when action or output are not contractible.

The idea that uncertainty and learning provides incentive for workers has been explored in the literature since the foundational work of Holmstrom (1999) on career concerns. Holmstrom (1999) focuses on the impact of learning on the incentive of managers, when their output is observed by a competitive labor market and their compensation immediately adjusted to expected output once belief is updated. Managers would then exert high effort in trying to improve market's belief on their ability. Although the market is not fooled in equilibrium, managers are caught up in the effort race, because failing to provide the expected effort would negatively impact the market's belief.

While competitive market and immediately adjusted compensation may be a realistic assumption for managers, it is unlikely to be the case for workers lower down the hierarchy, as their contribution are more subtle and harder to be observed by outside market. A secretary would not expect a wage raise for preparing a set of documents correctly, but would more likely worry about being fired if he fails to do so. Although the secretary's effort also stems from learning, the way a secretary is disciplined by firing threat is very different from the way a manager is motivated by career concerns. A manager's compensation moves up and down with his perceived productivity, but a secretary's future payoff is only downward sensitive: he receives the same flat wage if stay employed, and incurs a cost of getting fired only if perceived productivity falls below the firing
Because the payoff of worker takes the special step function form, the effectiveness of firing threat is limited as it is only credible when the possibility of getting fired is not too remote. As learning progresses, when a worker is believed to be competent with almost certainty, it becomes hard to discipline him by firing threat, because the worker understand that the employer's opinion about him would not be impacted enough to warrant firing. At this stage, a well-performing worker may become complacent and reduce compliance or effort as he is no longer disciplined.

In section 2, I present a symmetric learning model with match-specific productivity and study the optimal employment contract when output is not directly contractible. Absent of other effective incentive device, firms offer higher wages than that indicated by efficient separation. Part of the separation is involuntary and implies a cost of getting fired for workers. Firms trade off efficient separation with effective discipline and choose the wage just enough to keep worker disciplined.

In section 3, I explore the limitations of discipline from firing threat, and show how firms could alleviate the complacency problem. In particular, promoting a complacent worker into more difficult jobs would re-introduce uncertainty about a worker's competency, thus reducing the cost of disciplining him.

2 Compliance and Credible Firing Threat

2.1 Model Setup

Let there be only one job position for each firm, and an employee for the job position can be either “competent” or “incompetent” for the job. There are two possible effort levels for employees, the “perfunctory effort” — the level of effort
that can be monitored and enforced, and the “consummate effort” – the level of effort that requires employee’s willingness to perform beyond the contracted terms.

An employee’s productivity on the job is affected by both competency and effort. Let \( y \) denote the output of an employee in each period and \( y^H, y^L, y^P, y^C \) be the expected output levels of competent and incompetent workers when they provide perfunctory and consummate effort respectively. We assume that \( y^H > y^L \) for both \( e = P \) and \( e = C \), and \( y^C > y^P \) for both \( a = H \) and \( a = L \).

Employees privately observe the cost of providing effort, and choose effort levels accordingly. As perfunctory effort can always be enforced, we normalize the cost of providing perfunctory effort to be 0, and let \( c \) denote the extra cost to the employee to provide consummate effort. Let \( c \sim G_H \) when the employee is competent and \( c \sim G_L \) when the employee is incompetent, where \( G_H \) and \( G_L \) are both distribution functions with support \((0, \infty)\). Assume that competent workers find it less costly to provide consummate effort in the first order stochastic dominant sense, i.e. \( G_H(c) \geq G_L(c) \forall c \in (0, \infty) \).

As we would like to model the environment similar to realistic workplaces, we adopt the harshest possible assumption on the contractability of individual employee’s output. We assume that, due to job complexity and team work, we only know the expected output levels of given competence and effort, while the actual realization of \( y \) cannot be accurately measured or observed. On the other hand, since supervisors usually closely observe employee’s actions and have a sense their working attitude, we assume the effort level of employee can be observed. The central issue in this framework is not private information on effort, but rather uncontractability of consummate effort and unobservability of actual output.
The Employment Contract

Since neither effort nor output is contractible, the employment contract in a 2-period setting is simply a pair of wages \((w_1, w_2)\), with the option to terminate by any party at the end of period 1. Under the employment contract, specified continuation wage must be paid conditional on the continuation of employment, unless renegotiated otherwise with mutual consent.

At the end of period 1, the only information available to the firm is the employee’s observed effort level, and it is the only thing that the continuation could be made contingent upon. For the concern over the continuation of current job to have any positive incentive effect, it must be that the continuation of current job provide the employee with higher payoff than termination and seeking another job. Let a firing threat be the firm’s conditional continuation policy that the original employment contract will be continued if consummate effort is observed, and terminated otherwise. Let \(T\) be the reduction in the worker’s payoff if the original contract is terminated, \(T\) then measures the effective incentive size from the firing threat.

Firm’s Learning and Updating of Beliefs

Neither the firm nor the employee knows whether he is competent at the beginning of the employment. Let \(\alpha\) be the prior probability of an employee is competent at a given job. Suppose that the incentive size from the firing threat is \(T\), an employee will choose to supply consummate effort whenever the private cost of doing so is \(c \leq T\).

Since competent and incompetent workers have different cost of supplying consummate effort, their choice of competence could be used to learn and update the belief about their competency.

Let \(\beta\) and \(\gamma\) be the updated probability of a worker being competent after
observing consummate effort. Since the probability of supplying consummate effort for the competent and incompetent employees are $G_H(T)$ and $G_L(T)$ respectively, according to Bayes Rule,

$$\beta(T) = \frac{\alpha G_H(T)}{\alpha G_H(T) + (1-\alpha)G_L(T)} = \frac{\alpha}{\alpha + (1-\alpha)G_H(T)/G_L(T)},$$

$$\gamma(T) = \frac{\alpha(1-G_H(T))}{\alpha(1-G_H(T)) + (1-\alpha)(1-G_L(T))} = \frac{\alpha}{\alpha + (1-\alpha)(1-G_H(T))/(1-G_L(T))}.$$ 

Since $G_H(c) \geq G_L(c) \forall c \in (0, \infty)$, it follows that $\gamma(T) \leq \alpha \leq \beta(T)$ for any $T$.

**Credibility of Firm’s Firing Threat**

For employees to be motivated through firing threat, the firm firing threat need to be credible. In other words, the conditional employment policy contingent upon observed efforts level need to be ex-post optimal. With the given contracted continuation wage and updated belief, ex-post optimality requires that the firm prefers keeping the employee than replacing him if he has provided consummate effort, and the opposite if perfunctory effort is observed.

According to the updated belief, the second period expected output of an employee is $\beta(T)(y_H^P - y_L^P) + y_L^P$ and $\gamma(T)(y_H^P - y_L^P) + y_L^P$ respectively if consummate or perfunctory effort is observed. Note that we know that perfunctory effort will be provided in the second period, as there is no future to incentivize the employee.

In a competitive labor market, the expected payoff of hiring a new employee is always 0. The firm’s decision on keeping or replacing a continuing employee thus solely depend on the contracted continuation wage and the expected second period output. Suppose there is no transaction cost of firing and firms keep the employee when payoff breaks even, the a firm’s firing threat is credible whenever

$$\gamma(T)(y_H^P - y_L^P) + y_L^P < w_2 \leq \beta(T)(y_H^P - y_L^P) + y_L^P \ (1).$$
2.2 Equilibrium with Positive Firing Threat Incentive

Throughout the previous analysis, we have taken the incentive size $T$ as given and have not examined whether it can be sustained as an equilibrium outcome. For simplicity, suppose there is no transaction cost of switching firms, and that a worker can always start a new career with the prior competency probability $\alpha$. In a competitive labor market, worker’s second period outside option is then $w_2^o = \alpha(y_H^P - y_L^P) + y_L^P$. With contracted continuation wage $w_2$, the value of staying at a worker’s current job is then

$$T = w_2 - w_2^o = w_2 - \left[\alpha(y_H^P - y_L^P) + y_L^P\right].$$

Note that, for the $T$ given in (2) to be sustained as an equilibrium, the firm’s firing threat must also be credible. In other words, condition (1) must be satisfied in conjunction to condition (2). Let $((w_1, w_2), T)$ be an employment contract with its associated incentive size, an equilibrium exist whenever $i)$ the firm’s expected profit is 0 and $ii)$ both condition (1) and (2) are satisfied.

The conditions for the existence of such equilibrium is stated in the following proposition.

**Proposition 1.** $i)$ If $G_H$ and $G_L$ are continuous and $\frac{\partial u(T)}{\partial T} \to 1$ as $T \to 0$, then there always exist $T > 0$ and corresponding wages such that $((w_1, w_2), T)$ is an equilibrium with positive incentive from firing threat. $ii)$ If $G_H$ and $G_L$ are continuous and that $\frac{\partial u(T)}{\partial T}$ is monotone decreasing in $T$, then there always exist a $T$ such that, for every $T \in (0, T]$, there exist wages such that $((w_1, w_2), T)$ is an equilibrium with positive incentive from firing threat.

Proposition 1 shows that, even when the frictional cost of labor turnover is 0, as long as an employee’s supply of consummate effort sends a positive signal about his competency, equilibrium with positive incentive can be always achieved, with mild regularity conditions. In Appendix I, I establish similar result and show that the regularity conditions could be further relaxed when
frictional cost of labor turnover is taken into account.

The significance of Proposition 1 is that, even under the situations when effort is not contractible and output is hard to measure, employees would be willing to provide more than perfunctory effort due to the fear of getting fired. Note that our model does not require the output to be measurable or observable by the outside market, nor do we require the outside market to observe an employee’s choice of effort. Thus, this model is a step forward from Holmstrom (1999) in applying the same idea to more general non-managerial employees, whose performances are not readily observed by the outside market. The two-period model also demonstrate that the working of firing threat incentives does not depend on very long employment horizon, thus compliments the relational contract literature which usually builds on the assumption of infinitely repeated interactions.

One interesting aspect of our model is the self-fulfilling nature of the firing threat incentive. Only when $T$ is believed to be positive, the firm is able to learn from the employee’s effort choice. The updating from the learning makes the firm’s conditional continuation policy credible, thus indeed create a positive $T$.

The importance of this self-fulfilling nature is that it enables the firm to select to learn only from actions that are both informative and productive. In the managerial career concerns model, managers work too hard early in their career, and are inclined to engage in self-image improving activities that are not necessarily good for the firm. In our non-managerial model, the firm is able to turn down unproductive activities that aimed only to demonstrate one’s ability by setting the expectations straight.
2.3 Limit on Learning and the Effectiveness of Firing Threat Incentive

In proposition 1, we showed that the firing threat can provide incentive for consummate effort as long as learning is involved, but have not discussed how effective it works. In our model, the effectiveness of the incentive is essentially measured by \( T \). The larger \( T \) is, the more likely that the effort cost \( e \) falls below it and consummate effort is provided.

As we have discussed before, learning and updating of beliefs is the key in our model. The credibility of the firm’s conditional continuation policy depends on the difference in expected output that resulted from learning and updating. At the same time, that learning process also sets a limit on the size of \( T \), in other words, the effectiveness of the firing threat incentive.

**Proposition 2.** Let \( T \) be the largest \( T \) such that there exist \( w_1 \) and \( w_2 \) that \(((w_1, w_2), T) \) is an firing threat equilibrium. Then \( T \) is bounded above by \((1 - \alpha)(y^H_P - y^L_P)\) in the two-period model.

We can see from Proposition 2 that the incentive size the firing threat provides can never exceed the difference learning makes on an employee’s expected future output. In a two-period model, the limit is simply the difference between a competent employee’s expected output and the average value. In a market where competent employees are expected to produce much higher output than average employees, it is possible for the firm to write an employment contract with strong incentive from the firing threat. On the other hand, if learning an employee is competent makes no much difference, the firm cannot credibly offer a continuation wage much higher than the outside option, and the firing threat incentive would be small as a result.

In the real world, when there are more than two periods involved, the direct effect of learning is the dissipation of uncertainty over time. As firms observe
their employees for more and more periods, they have better and better idea about their competency, and eventually reaching a point where there is little learning left. Even for outside firms who may not be able to observe and learn directly, competency can be inferred from the revealed preference of current firm's retention choice. As learning completes, the firing threat incentive in our model also lose its effectiveness. To retain effective incentives, firms may want to re-introduce uncertainty for competent employees. In the next section, we extend the model to incorporate more than one job positions, and study how the need to retain effective incentive affects firm's promotion policy.

3 Firing Threat and the Peter Principle

We have discussed in the previous section that the firing threat lose its effectiveness when learning completes. The loss of incentives could result in what we usually see as “complacency” – the unwillingness to put in extra effort when one is known to be competent for his job. In our model, “complacency” is more than just a psychological effect, it could be a rational choice of effort resulting from the loss of incentive to work hard and impress. In this section, we explore how the need to retain the effective incentives could affect a firm’s promotion policy.

When an employee is known to be competent for his current job with almost certainty, the firing threat incentive becomes ineffective as there is very little further learning left. If sustaining consummate effort is important enough for the firm’s output, firms may prefer to promote the employee into a more difficult job in order to reintroduce uncertainty, even when the promotion results in an immediate decrease in the expected base output of the worker. Thus, the need to sustain incentives provides a rational explanation for the Peter Principle – employees who are known to be competent for their current position
are promoted to the next level, even though they may be less productive at the higher level. In our model, the inefficiency of the Peter Principle is just an illusion because it does not take into account the value of extra consummate effort provided.

A. The Peter Principle without Incentives Consideration

Let there be two types of positions in each firm, a lower position that is relatively easy and an upper position that is more difficult. An employee can be either incompetent in both jobs, competent only for the lower position, or competent for both lower and upper position. Let $\alpha_0$ be the prior probability that an employee is competent for the lower position and $\eta_0$ be that for the upper position. Let $r = \frac{\eta_0}{\alpha_0}$ be the probability that an employee, who is competent at the lower position, is also competent for the upper position.

Let $y^H$ and $y^L$ be the expected output of competent and incompetent employees at the lower position, and $z^H, z^L$ be that at the upper position. Assume that $y^L + \alpha_0(y^H - y^L) \geq z^L + \eta_0(z^H - z^L)$, so that new employees are always admitted into the lower position first. Also, let $z^H > y^H$, so that competent employees produce more at the upper position.

Let the Peter Principle be the promotion rule that an employee is promoted to the upper position once he is learned to be competent at the lower position with probability $\alpha > \tilde{\alpha}$, where $\tilde{\alpha}$ is close to 1.

**Proposition 3.** Promoting an almost competent employee into the higher position always results in an immediate drop in expected output after promotion, if $r \leq \frac{y^H - z^L}{y^H - z^L}$.

Given that an employee is learned to be competent at the lower position with probability $\alpha$, the probability of him being competent at the upper position is $\eta = r\alpha$ according to Bayes Rule. Thus $r$ can be viewed as measuring how well
competency at the lower position predicts competency at upper position. When \( r \) is small, an employee's competency at the lower position does not provide much confidence in his competency at the upper position. Although increased \( \alpha \) leads to increased expected output at both the lower and upper position, the expected output is still higher at the lower position, even when \( \alpha \) gets close to 1. In such cases, the Peter Principle leads to inefficient allocation of job.

**B. The Peter Principle with Incentives Consideration**

Now, we would like to extend the model developed in section 2 and examine how the need to sustain effective incentives could provide rationale for promoting employees into positions with apparently lower expected output.

Let the settings for job positions and competency be the same as specified above, but now introduce the choice of consummate or perfunctory effort for employees. Same as in section 2, the cost of providing consummate effort is randomly distributed as \( c \sim G_H \) if the worker is competent for the job and \( c \sim G_L \) otherwise. Both \( G_H \) and \( G_L \) have support \((0, \infty)\) and \( G_H(c) \geq G_L(c) \) \( \forall c \in (0, \infty) \). Let the expected output be the same as specified above but add bonus value \( b \) of output whenever consummate effort is provided.

In the 2-period model in section 2, we discussed that concerns incentive is ineffective when an employees are known to be competent with almost certainty. Next, we would like to formally establish this result in a more general context, when multi-periods are allowed in an employee's career.

When there are more than 2 periods, the employment contract can no longer be specified by just \( w_1, w_2 \) and \( T \). Although the actual employment contract may take on very complex forms, we only need to focus on their continuation values. Let the \( \alpha_t \) be the starting prior, and \( \beta(\alpha_t, T) \), \( \gamma(\alpha_t, T) \) be the updated probability of competence with \( \alpha_t \) and expected incentive size \( T \). Denote the employee's continuation value by \( V_t(\alpha_{t+1}) \), then the actual incentive size would
be \( V_t(\beta(\alpha_t, T)) - V_t(\gamma(\alpha_t, T)) \). Incentives can be achieved when there exist \( T > 0 \) such that \( T = V_t(\beta(\alpha_t, T)) - V_t(\gamma(\alpha_t, T)) \).

**Lemma 4.** Let \( \alpha_t \) be the starting prior and \( T_t \) be the equilibrium firing threat incentive if no promotion is allowed. Then \( T_t \to 0 \) as \( \alpha_t \to 1 \). Let \( T'_t \) be the equilibrium firing threat incentive if the worker is promoted into the higher position. If the continuation value at the higher position \( V'_t \) is continuous and there exist \( \sim T > 0 \) such that \( \sim T = V'_t(\beta(r, \sim T)) - V'_t(\gamma(r, \sim T)) \), then \( T'_t \to \sim T \) as \( \alpha_t \to 1 \).

**Proposition 5.** Promoting an almost competent employee into the higher position results in higher incentive to provide consummate effort, if continuation value at the higher position is continuous and positive firing threat incentive can be achieved with starting prior \( r \).

When there is no chance of promotion, an employee who is competent at current job with almost certainty would lose the incentive to provide consummate effort. On the other hand, better incentive can be achieved with the reintroduced uncertainty at the higher position given appropriate conditions. Therefore, promoting an almost competent employee results in higher incentive for the employee to provide consummate effort. Depending on how large the value of consummate effort \( b \) is compared to the base output, the need to sustain incentives may justify promoting an employee into a more difficult position even when it results in an immediate drop in expected base output.

### 4 Conclusion

In this paper, we propose an employment model with employer learning to provide the foundation of credible firing threat in an employment relationship. With a two period model, we are able to show that credible firing can be achieved even
when the time horizon is extremely short. When the employer is not sure about an employee's type, the employee's actions acts as signals to the employer and changes the employer's belief about the employee's future productivity, which ensures the ex-post credibility of firing threats. As the ex-post credibility of firing threat is dependent on the learning and uncertainty over the employee, the use of firing threat as a discipline device becomes problematic when employer is almost sure about the employee's competence at the job. To sustain the employee's incentives, the employer may find it optimal to promote him into more difficult tasks, even though it results in an immediate decrease in expected base output. Our model thus provides an rational explanation for the Peter Principle: employees are promoted to levels that they are not undoubtedly competent in order to sustain their incentives to provide consummate effort.

References


