The Origins of Aggregate Fluctuations in a Credit Network Economy*

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December 20, 2015

Abstract

This paper shows that inter-firm lending plays an important role in business cycle fluctuations. I first build a network model of the economy in which trade in intermediate goods is financed by supplier credit. In the model, a financial shock to one firm affects its ability to make payments to its suppliers. The credit linkages between firms then transmit financial shocks across firms, amplifying their effects on aggregate output. To calibrate the model, I construct a proxy of inter-industry credit flows from firm- and industry-level data. I find that the credit network of the US generates 22 percent of the fall in GDP in response to an aggregate financial shock. Finally, I use a structural factor approach to estimate the shocks which hit US industrial production (IP) industries from 1997-2013. I find that most aggregate volatility in IP was driven by aggregate financial shocks and idiosyncratic productivity shocks, and that the credit network of IP industries generated 17 percent of observed aggregate volatility. During the Great Recession, three-quarters of the drop in aggregate IP was due to an aggregate financial shock.

*I am very greatful to my advisors Stefania Garetto, Simon Gilchrist, and Adam Guren for their guidance. I also thank Jean-Noel Barrot, Giacomo Candian, Mirko Fillbrunn, Illenin Kondo, Fabio Schiantarelli, Stephen Terry, and participants of the BU macro workshop, Federal Reserve Board seminars, and BU-BC Green Line Macro Meeting for comments which substantially improved this paper. All errors are my own.

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Introduction

The recent financial crisis and ensuing recession have underscored the importance of external finance for the real economy. Generally, firms obtain most of their short-term external financing from their suppliers, in the form of delayed payment terms for their purchases. In spite of its importance, the aggregate implications of these lending relationships remain poorly understood.

In this paper, I show that inter-firm lending plays an important role in business cycle fluctuations. To this end, I introduce supplier credit into a network model of the economy and show, analytically and quantitatively, that the credit network of an economy amplifies the effects of financial shocks. I then use my framework to empirically shed light on the origins of observed business cycle fluctuations in the US.

My approach involves three steps. First, I provide intuition with a stylized model in which trade in intermediate goods is financed by supplier credit. In this model, a shock to one firm's liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects. Second, I calibrate the model to assess the quantitative importance of this propagation mechanism. For this, I construct a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data. I find that the credit network of the US economy accounts for 22 percent of the fall in GDP following an aggregate financial shock. Finally, I investigate which shocks drive the US business cycle when we account for the credit linkages between industries. To do so, I use a structural factor approach to estimate the contribution of productivity and financial shocks to observed aggregate fluctuations in US industrial production (IP) from 1997-2013, and find that these fluctuations were driven primarily by aggregate financial shocks and idiosyncratic productivity shocks. During the Great Recession, productivity shocks played a minimal role; rather, most of the drop in aggregate IP was driven by an aggregate financial shock.

The credit linkages that I model take the form of trade credit, or delayed payment terms, that suppliers of intermediate goods often extend to their customers. Trade credit is the single most important source of short-term external finance for firms, and facilitates most inter-firm trade. In the US, trade credit was three times as large as bank loans and fifteen times as large as commercial paper outstanding on the aggregate balance sheet of non-financial corporations in 2012.[1] In most OECD countries, trade credit accounts for more than half of firms' short-term liabilities and more than one-third of their total liabilities.[2] All of these facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the financial distress of their customers. A notable example of this is the US automotive industry in 2008, when the Big Three automakers (Chrysler, Ford, and General Motors) faced an acute shortage of liquidity. While Ford did not require a bailout from the US government itself, it requested one on behalf of its competitors, fearing that a bankruptcy by Chrysler or General Motors would transfer the liquidity shortage to their

[1] During the Great Recession, the dry-up of trade credit was comparable to that of bank lending, with a peak-to-trough decline of about 25 percent. See the Federal Reserve Board's Flow of Funds.

[2] A large empirical literature documents the pervasiveness of trade credit. Generally, trade credit contracts are last for as short as 15 days to as long as several months. See Petersen and Rajan (1997) for more information.

common suppliers, as the money owed to them could not be paid until they exited bankruptcy. This episode suggests that when firms play a dual role of supplier and creditor, shocks may not only affect trade directly, but may also affect the availability of liquidity to finance trade.

There is growing evidence to suggest that this intuition is empirically relevant. A number of studies - including Boissay and Gropp (2012), Jacobson and von Schedvin (2015), and Raddatz (2010) - have found that firm- and industry-level trade credit linkages propagate financial shocks from firms to their suppliers. In spite of this evidence, the macroeconomic implications of trade credit have been largely overlooked in the literature. I therefore develop a framework for understanding how inter-firm trade and credit interact in response to credit conditions.

I consider an economy similar to that of Bigio and La'O (2015), in which firms are organized in a production network and trade intermediate goods with one another. Limited enforcement problems require firms to make cash-in-advance payments to their suppliers before production takes place. However, firms can delay part of these payments by borrowing from their suppliers. I assume that, to obtain this credit, a firm can credibly pledge some fraction of its revenue to repay its suppliers. Importantly, this implies that the cash-in-advance payments collected by each firm are endogenous to the model, and depend on the prices of its customers' goods. As it turns out, endogenous changes in firms' cash-in-advance constraints are crucial for how the economy behaves in response to shocks.

When one firm is hit with an adverse shock to its cash on hand, there are two channels by which other firms in the economy are affected. First is the standard input-output channel, which has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2015): the shocked firm cuts back on production, reducing the supply of its good to its customers. Second is a new credit linkage channel in which the shock disrupts the cash flow from firms to their suppliers. That is, when the shocked firm cuts back on production, the price of its good rises. This increases the collateral value of its receivables, allowing the firm to reduce the cash-in-advance payments it makes to its suppliers. With less cash on hand, these suppliers may themselves be forced to cut back on their own production. Thus, the credit linkages propagate the shock up the production chain, amplifying the fall in aggregate output.

Next, I evaluate the quantitative relevance of the mechanism. I first generalize the model to capture more features of the economy. In order to calibrate the model, I then construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis. With this, I produce a map of the credit network of the US economy at the three-digit NAICS level of detail. I calibrate the model to match this proxy and the input-output matrices of the US. I also allow for substitutability between cash-in-advance payments and bank credit, so that firms can partially offset a loss in customer payments with increased bank borrowing. I set this parameter to match firm-level evidence from Omiccioli (2005) on how much Italian firms collateralize their trade credit for bank borrowing.

In response to a one percent financial shock to every industry in the US, I find that GDP falls by 2.9 percent. The credit linkage channel accounts for nearly a quarter of this drop, reflecting the quantitative importance of the propagation mechanism. Compared to a model without inter-firm lending, the credit network of the economy amplifies the fall in GDP by 28 percent. I then explore which features of the US economy contribute to this amplification. In particular, I find that industries which are important suppliers of intermediate goods to the rest of the economy are also more vulnerable to nonpayment by their customers - a feature which exacerbates the impact of financial shocks.

Having shown that the credit network of the economy *can* generate quantitatively significant aggregate fluctuations, I then ask whether it does in the data. In the empirical part of the paper, I evaluate how much of observed fluctuations in output can be attributed to the credit network of the economy. I also investigate which shocks drive the US business cycle when we account for the effects of credit linkages in propagating financial shocks across industries.

To this end, I use quarterly output and employment data on US industrial production industries over 1997-2013, from the Federal Reserve Board's Industrial Production Indexes and the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, respectively. With this data, I use a structural factor approach similar to that of Foerster et al. (2011) to estimate shocks which hit these industries over my sample period. This approach involves two stages.

In the first stage, I use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. The model is able to separately identify these shocks because each type of shock has differential effects on industry output and employment. Namely, productivity shocks affect the ratio of output to employment through Cobb-Douglas production functions. On the other hand, financial shocks do not affect production functions. Rather, they work through cash-in-advance constraints and thereby affect industries' first-order conditions. In the second stage, I use standard factor methods to decompose each of these quarterly productivity and financial shocks into an aggregate component and an idiosyncratic component. In all, I thus estimate four types of shocks: aggregate and idiosyncratic productivity shocks, and aggregate and idiosyncratic financial shocks.

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks estimated using the model with a measure of the risk-bearing capacity of the US financial sector - namely, the excess bond premium of Gilchrist and Zakrajsek (2012). There turns out to be a fairly strong negative correlation between the two time series (approximately negative one-half), suggesting that the aggregate financial shocks picked up by the model are indeed reflective of changes in the supply of credit from the financial sector to the IP industries.

With the estimated financial and productivity shocks at hand, I perform a variance decomposition of aggregate IP to estimate how much of aggregate volatility was driven by each of these shocks. I find that, before the Great Recession, aggregate fluctuations were driven primarily by idiosyncratic productivity shocks and aggregate financial shocks. Moreover, the credit network of US IP played a quantitatively important role in propagating these financial shocks, generating at least 17 percent of observed aggregate volatility over this period.

During the Great Recession, however, productivity shocks seemed to have played little role. Rather, three-quarters of the peak-to-trough drop in aggregate IP can be attributed to an aggregate financial shock to these industries. In addition, I show that idiosyncratic financial shocks to the three most systemically important IP industries - oil and coal manufacturing, chemical manufacturing, and auto manufacturing - accounts for between 9 and 27 percent of the drop in aggregate IP during the recession. Furthermore, the credit and input-output linkages between industries played a significant role in propagating these industry-level shocks across the economy.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for such shocks. This paper, however, finds a minimal role for aggregate productivity shocks in the US business cycle, but a vital one for aggregate financial shocks. Hence, my results suggest that when one accounts for the effects of credit linkages between industries, aggregate financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle. This finding is in line both with growing evidence on the importance of the financial sector for real activity, and with standard interpretations of the causes of the Great Recession. Thus, by explicitly accounting for the propagation generated by credit linkages, this paper captures the importance of financial shocks for aggregate fluctuations in the real economy.

The rest of the paper is organized as follows. The next section reviews some of the literature to which this paper is related. In Section 1, I introduce the stylized model and derive the analytical results. In Section 2, I generalize the production network structure, discuss the construction of my proxy for credit flows and calibration, and summarize the quantitative results. In Section 3, I perform the structural factor analysis and discuss the empirical results.

Related Literature

This paper contributes to several strands of the literature. A growing literature examines how network effects across firms can generate aggregate fluctuatinos from idiosyncratic shocks. Much of this literature builds on the multi-sector real business cycle model of Long and Plosser (1983). Most notably, these include Acemoglu et al. (2012), Shea (2002), Dupor (1999), Horvath (1998), Horvath (2000), and Acemoglu et al. (2015). These studies all focus on the role of input-output linkages between firms. Input-specificity in the production of intermediate goods prevents firms from easily switching suppliers or customers in response to productivity shocks. However, most of this literature do not model how trade in intermediate goods is financed. Indeed, most abstract away from financial frictions, leaving no role for financial factors in aggregate fluctuations.

A notable work to which this paper is closely related is that of Bigio and La'O (2015), who examine the role of financial frictions in the context of an input-output network. They find that the input-output structure of an economy is an important determinant of the aggregate impact of firm-level financial shocks. However, they do not explicitly model any credit relationships between firms; the cash-in-advance constraints faced by firms are therefore fixed exogenously. In contrast, I explicitly model these credit relationships, in turn endogenizing these constraints. Moreover, I show that the structure of the credit network of an economy is also an important determinant of the aggregate impact of financial shocks.

There is a growing literature on origins of trade credit and its implications for the transmission of shocks. A question which was once the focus of this literature is why trade credit exists when there are financial intermediaries who specialize in lending? Burkart and Ellingsen (2004) attribute this to a monitoring advantage that suppliers have over banks which allows them to lend to a customer when a bank will not. They find that, in a setting in with a moral hazard problem, the optimal trade credit contract between supplier and customer is one in which the credit limit is proportional to the customer's revenue. In addition, they assert that firms will grant credit to their customers, even when they themselves are liquidity constrained, because trade credit can be collateralized for bank borrowing. Their results are line with evidence of trade credit use presented in Petersen and

Rajan (1997). A number of studies have looked at how trade credit relationships transmit financial distress across trading firms. For example, Boissay and Gropp (2013) find evidence that firms pass over a fifth of their liquidty shocks to their firms via their trade credit linkages: an increase in the default probability by one firm increases its supplier's chance of defaulting by 0.2%. Jacobson and von Schedvin (2015) both use firm-level data to show that firms pass a significant fraction of their financial shocks to their suppliers via trade credit lending. Barrot (2015) examines data on trucking firms in France and finds that delayed payment terms are associated with greater financial distress. Raddatz (2010) shows that, even controlling for input-output linkages, greater intensity of trade credit use linking two industries increases their correlation in output growth. In my appendix, I follow a strategy similar to that of Raddatz (2010) to test my model's implications for the comovement of sectoral output growth, using industry- and firm-level data. My empirical findings are in line with the literature: even when accounting for the (direct and indirect) input-output linkages between two industries, a model-based measure of the credit linkages between industry-pairs increases the cross-sectional correlation of their output growth. Furthermore, I utilize a theoretical framework to assess the aggregate implications these findings.

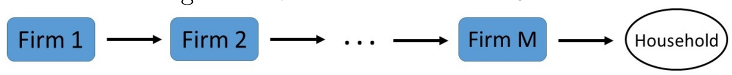
A growing empirical literature tries to evaluate the origins of aggregate fluctuations by measuring the contribution of idiosyncratic versus aggregate shocks. In this context, the seminal work of Gabaix (2011) examined the role of granularity in the firm size distribution. Taking cue from the network literature, a few studies have incorporated input-output linkages as a mechanism by which idiosyncratic shocks may account for larger portion of fluctuations. Broadly speaking, there are two approaches: a more structural approach (e.g. Horvath (2000)) and a more statistical approach. Foerster et al. (2011) and Stella (2014) bridge these approaches using structural and factor approaches together; they account for the effects of input-output linkages in propagating idiosyncratic shocks. My empirical approach follows the same methodology, but uses additional data on industry employment growth to decompose the data into an additional source of fluctuations - financial shocks. The presence of credit linkages between firms implies a greater role for financial shocks in driving the business cycle. I show that failing to account for the credit linkages created by trade credit underestimates the importance of idiosyncratic shocks, and over-attributes aggregate volatility to aggregate productivity shocks. I also explicitly estimate the contribution of the production and credit networks US industrial production in generating aggregate volatility.

Jermann and Quadrini (2012) evaluate the importance of financial shocks by explicitly modelling the tradeoff between debt and equity financing. They find these shocks explain about half of observed aggregate volatility in the US. My paper, which excludes equity financing, produces empirical results in line with theirs by accounting for the importace of trade credit in financing firms' working capital.

1 Stylized Model: Vertical Production Structure

In this section introduce and analyze the stylized model to build intuition. The simplicity of the production structure of the economy and preferences of the household permits closed-form expressions for equilibrium variables. I will later generalize both the production structure and preferences.

Figure 1: Vertical Production Chain



There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are M goods, each produced by a continuum of competitive firms with constant returns-to-scale in production. We can therefore consider each good as being produced by a representative, price-taking firm. Each good can be consumed by the household or used in the production of other goods.

1.1 Representative Household

The representative household supplies labor competitively to firms and consumes a final consumption good. . It has preferences over consumption C and labor N given by $U(C, N)$.

$$U(C, N) = log\, C - N$$

Later I will generalize the preferences. Let w denote the competitive wage earned from working, and π_i the profit earned by firm i. The household chooses how much to work and how much of each good to consume to maximize its utility subject to the following budget constraint.

$$C = wN + \sum_{i=1}^{M} \pi_i \tag{1}$$

The household's optimality condition is given by

$$\frac{V'(N)}{U'(C)} = w \tag{2}$$

This equates the competitive wage with the marginal rate of substitution between labor and consumption.

1.2 Firms

There are M firms who each produce a different good. Suppose for now that firms are arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with $i = M$ denoting the producer of the final good. The flow of goods is depicted in Figure 1.

Firms are price-takers. The production technology of firm i is Cobb-Douglas over labor and intermediate goods.

$$x_i = \begin{cases} z_i n_i^{\eta_i} & for\ i = 1 \\ z_i n_i x_{i-1}^{(1-\eta_i)} & for\ i > 1 \end{cases}$$

Here, x_i denotes firm i's output, n_i its labor use, and x_{i-1} its use of good $i-1$. Parameter z_i denotes firm i's total factor productivity, η_i the share of labor in its production, and $\omega_{i,i-1}$ the use of good $i-1$ in firm i's production. Let p_s denote the price of good s. The value of the sales from firm s to firm c is then $p_s x_{cs}$.

Limited enforcement problems create a need for *ex ante* liquidity. The household cannot force any debt repayment. Therefore, firm i must pay the full value of wage bill, wn_i, up front to the household before production takes place. In addition, each firm i must pay for its intermediate goods purchases, $p_{i-1}x_{i-1}$ up front to its supplier. Thus, firms are required to have some funds at the beginning of the period before any revenue is realized.

Firm i can delay payment to its supplier by borrowing some amount τ_{i-1} from its supplier. This represents the trade credit loan given from $i-1$ to i. In addition, each firm can obtain a cash loan b_i from the bank. The net payment that firm $i-1$ receives from its customer at the beginning of the period is therefore $p_{i-1}x_{i-1} - \tau_{i-1}$. Firm i's cash-in-advance constraint takes the form

$$\underbrace{wn_i}_{wage\ bill} + \underbrace{p_{i-1}x_{i-1} - \tau_{i-1}}_{net\ CIA\ payment\ to\ supplier} \leq \underbrace{b_i}_{bank\ loan} + \underbrace{p_i x_i - \tau_i}_{net\ CIA\ from\ customer}$$

This constraint states that the amount of cash that firm i is required to have in order to employ n_i units of labor and purchase x_{i-1} units of intermediate good $i-1$, is bounded by the amount of cash that firm i can collect at the beginning of the period. Note that trade credit appears on both sides of the cash-in-advance: a loan from its supplier increases firm i's liquidity, but a loan to its customer reduces its liquidity by reducing the cash-in-advance payment it collects. There is therefore a one-to-one relation between the amount of cash-in-advance a firm can collect from its customer and the size of the trade credit loan it gives its customer.

Firms face borrowing constraints on the size of loans they can obtain from their suppliers and the bank. In particular, firm i can obtain the loan b_i from the bank at the beginning of the period by pledging a fraction B_i of its total end-of-the-period revenue $p_i x_i$, and a fraction α of its accounts receivable τ_{i+1}, where $\alpha \,\epsilon\, (0, 1]$. Thus, firm i faces a bank borrowing constraint of the form

$$b_i \leq B_i p_i x_i + \alpha \tau_i$$

Parameters B_i and α provide an exogenous source of liquidity to each firm, and represent the severity of the agency problem between firm i and the bank. I will later show that α parameterizes the degree of substitutability between bank credit and cash-in-advance payments from customers. I assume that b_i is chosen by firm i, which implies that these bank borrowing constraints will bind in equilibrium as each firm obtains the maximum bank loan possible.

In addition, firm i can pledge a fraction $\theta_{i,i-1}$ of its end-of-the-period revenue to repay its supplier. Then the trade credit loan is bounded by the collateral value of firm i's output

$$\tau_{i-1} \leq \theta_{i,i-1} p_i x_i$$

The parameter $\theta_{i,i-1}$ is a reduced-form representation of the limited ability of firms to delay payment to their suppliers. The precise limited enforcement problem which produces this borrowing constraint is described in detail in Appendix A1. Because firms can collateralize their trade credit (accounts receivable) for bank borrowing (i.e. $\alpha > 0$), they find it optimal to borrow as much as possible from suppliers and the bank. Hence, both the trade credit and bank borrowing constraints bind for all firms in equilibrium. A detailed derivation of this result is given in Appendix A2.[3]

Given the binding borrowing and constraints, we can now re-write firm i's cash-in-advance constraint as

$$wn_i + p_{i-1}x_{i-1} \leq \underbrace{\chi_i p_i x_i}_{liquid\ funds} \tag{3}$$

where

$$\chi_i \equiv \underbrace{\frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i}}_{debt/revenue\ ratio} + \underbrace{1 - \frac{\tau_i}{p_i x_i}}_{cash/revenue\ ratio}$$

This constraint says that a firm's expenditure on inputs is bounded by the value of its liquid funds. The variable χ_i describes the tightness of firm i's cash-in-advance constraint, and will play a key role in the mechanism of the model. The tightness of a firm's cash-in-advance constraint is comprised of the firm's debt-to-revenue ratio and its cash-to-revenue. These describe how much of the firm's revenue is financed by debt, and how much of its revenue is collected as a cash-in-advance payment, respectively. Notice that χ_i is decreasing in $\frac{\tau_i}{p_i x_i}$, the amount of i's output sold on credit: the more credit that i gives its customer, the less cash it collects at the beginning of the period. We can re-write χ_i using firms' binding borrowing constraints to replace τ_i and b_i.

$$\chi_i = \underbrace{B_i + \theta_{i,i-1}}_{debt/revenue\ ratio} + \underbrace{1 - (1-\alpha)\theta_{i+1,i}\frac{p_{i+1}x_{i+1}}{p_i x_i}}_{cash/revenue\ ratio} \tag{4}$$

Notice that the firm's debt-to-revenue ratio is fixed, because firms collateralize their end-of-period revenue for borrowing.

Crucially, equation (4) shows that χ_i is an equilibrium object - it is an endogenous variable which depends on the firm's forward credit linkage $\theta_{i+1,i}$ and the revenue of its customer. Hence, changes

[3]Even if firms could not collateralize their accounts receivables, perfect competition amongst firms in an industry would imply that supplier credit borrowing constraints bind in equilibrium, under some benign assumptions. Suppose that the pledgeability of revenue is common across all firms within an industry. Also suppose that before signing trade and debt contracts, firms anticipate that they may be liquidity-constrained in equilibrium. Finally, suppose also that firms can compete not only in price but also in the length of payment terms. Then competition amongst suppliers in industry i will bid up the trade credit to the maximium that each firm in i can offer their customers in $i + 1$. This maximum is pinned down by the moral hazard problem at $\theta_{i+1,i}p_{i+1}x_{i+1}$. (More than this, and customers would simply pocket the loan). Thus, in equilibrium, firms in a given industry will all sell at the same price and offer the maximum trade credit terms, implying that borrowing constraints bind. In this setting, the allocation of liquidity (cash) across firms would be *ex ante* efficient (at the beginning of the period), but *ex post* inefficient (at the end of the period).

in the price of its customer's good affect the tightness of firm i's cash-in-advance. Note also that the dependence of χ_i on prices p_i and p_{i+1} means that changes a shock will have general equilibrium effects on each χ_i. This a key difference with Bigio and La'O (2015), in which the tightness of each firm's cash-in-advance is an exogenous parameter because there is no inter-firm lending. Here, the endogeneity of χ_i will be a critical determinant of how the economy responds to shocks.

Firm i chooses its input purchases to maximize its profits, subject to its cash-in-advance.

$$\max_{n_i,\, x_{i-1}} \quad p_i x_i - w n_i - p_{i-1} x_{i-1}$$

$$s.t. \;\; w n_i + p_i x_{i-1} \quad \leq \chi_i p_i x_i$$

Firms take the tightness χ_i of their constraints as given when choosing inputs.[4] The solution of each firm's given in detail in Appendix A3. Firm i's optimality condition equates the ratio of expenditure on each type of input with the ratio of their share of production.

$$\frac{w n_i}{p_{i-1} x_{i-1}} = \frac{\eta_i}{\omega_{i,i-1}(1-\eta_i)}$$

This condition pins down the ratio of expenditure on each input. However, the constraint will limit the firm's total expenditure on both inputs.

If firm i's cash-in-advance is not binding in equilibrium, then it simply maximizes its profit function. Its optimal level of expenditure on each input is determined by a condition which equates the marginal cost of the input with its marginal revenue product. The firm's expenditure on labor is therefore given by

$$w n_i = \eta_i p_i x_i \;, \qquad p_{i-1} = \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}}$$

If, on the other hand, the constraint is binding in equilibrium, then the amount of liquidity $\chi_i p_i x_i$ that firm i has limits how much the firm can spend on both inputs. In particular, firm i's expenditure on labor and good $i-1$ is given by

$$w n_i = \chi_i \eta_i p_i x_i \;, \qquad p_{i-1} = \chi_i \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}}$$

I show in Appendix A3 that firm i's cash-in-advance (3) binds in equilibrium if and only if $\chi_i < 1$. Combining the two cases (constrained and unconstrained) yields

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i} \;, \qquad p_{i-1} = \phi_i \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}} \tag{5}$$

where $\phi_i \equiv \min\{1, \chi_i\}$ describes firm i's shadow value of funds.[5] ϕ_i is strictly less than one if and only if firm i's cash-in-advance is binding in equilibrium. Equations (5) says that, if binding, the cash-in-advance inserts a wedge $\phi_i < 1$ between the marginal cost and marginal benefit of each input,

[4] The firm's decisions of how much to borrow and lend, i.e. b_i, τ_i, and τ_{i-1}, are already embedded in χ_i at this point. (See equation (4)). Therfore, given its choice of how much to borrow and lend, the firm's choice of n_i and x_{i-1} is independent of χ_i.

[5] More precisely, the shadow value of funds of firm i is given by $\frac{1}{\phi_i} - 1$.

representing the distortion in the firm's input use created by the constraint. A tighter cash-in-advance (lower χ_i) corresponds to a greater distortion, and lower output.

Through its dependence on the tightness χ_i of the firm's constraint, ϕ_i is endogenous to the model. The credit relationships between firms also imply that firms' shadow value of funds ϕ_i are interdependent. To see this, first recall firm $i+1$'s optimality condition for its intermediate good (5),

$$p_i = \phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})\frac{p_{i+1}x_{i+1}}{x_i} \tag{6}$$

This says that the firm $i+1$ chooses its level of intermediate good use x_i to equate the marginal cost of the good p_i with the marginal revenue product, times times the wedge ϕ_{i+1} created by its cash-in-advance. Re-arranging this and replacing $\frac{p_{i+1}x_{i+1}}{p_i x_i}$ in (6) yields ϕ_i as an increasing function of ϕ_{i+1}.

$$\phi_i = min\left\{1,\ \ B_i + \theta_{i,i-1} - (1-\alpha)\theta_{i+1,i}\frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})}\right\}$$

The positive relationship between ϕ_i and ϕ_{i+1} is a consequence of the fact that firms collateralize their revenue to borrow from suppliers. A tighter constraint of firm $i+1$ implies that every firm upstream of $i+1$ also has a tighter constraint.

1.3 Equilibrium

I close the model by imposing labor and goods market clearing conditions:

$$N = \sum_{i=1}^{M} n_i\,, \qquad C = Y \equiv x_M$$

Definition of Equilibrium: An equilibrium is a set of prices $\{p_{i\,i\epsilon I}\,, w\,\}$, quantities x_i, n_i, $\tau_{i\,i\epsilon I}$ that

i) maximize the representative household's utility, subject to its budget constraint

ii) maximize each firm's profits subject to its cash-in-advance, bank borrowing, and supplier borrowing constraints

iii) clear goods markets and the labor market.

Let $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M}\omega_{j,j-1}$ denote firm i's share in total intermediate good use, and $\tilde{\eta}_i \equiv \eta_i\tilde{\omega}_i$ denote firm i's share of labor in aggregate output. Let $\bar{Y}$ denote the equilibrium aggregate output that would prevail in a frictionless economy (à la Acemoglu et al. (2012)), given by

$$\bar{Y} \equiv \prod_{i=1}^{M}\tilde{\eta}_i^{\tilde{\eta}_i}z_i^{\tilde{\omega}_i}$$

$\bar{Y}$ is log-linear in each firm's productivity z_i and depends on technology parameters η_i and $\omega_{i,i-1}$ for all i. This is equivalent to an Acemoglu et al. (2012) economy in which firms are organized in a vertical production network and face no financial constraints.

In Appendix A3, I show that for my economy, a closed-form expression for equilibrium aggregate output Y is log-linear in the unconstrained aggregate output and some aggregation of each firm's shadow value of funds.

$$Y = \bar{Y}\bar{\Phi} \qquad\qquad \bar{\Phi} \equiv \prod_{i=1}^{M} \phi_i^{\sum_{j=1}^{i} \tilde{\eta}_j} \tag{7}$$

The expression $\bar{\Phi}$ describes how the shadow values aggregate in the input-output network. Thus, equilibrium aggregate output equals $\bar{Y}$ if and only if $\phi_i = 1$ for all i - i.e. if no firm's cash-in-advance is binding in equilibrium.[6] $\bar{\Phi}$ captures the *aggregate liquidity* available to all firms in the economy for trade in inputs. Therefore, (7) says that equilibrium aggregate output is constrained by the aggregate liquidity in the economy at the beginning of the period. Notice that through $\tilde{\eta_j}$, firms who are further downstream have a higher share of total employment through the use of intermediate goods, and therefore have a higher impact on aggregate liquidity.

To summarize the equilibrium, the cash-in-advance constraints faced by firms induces a wedge on their production, which depends on the tightness of their constraints. But in a setting where firms share liqudiity via trade credit, these wedges depend endogenously on the prices of downstream goods and the structure of the credit network. In the next section, I explore the implications of this endogenous relationship between wedges and prices for how aggregate output responds to firm-level shocks.

At this stage, it is worth discussing how this economy compares to that of Bigio and La'O (2015). The novelty of Bigio and La'O (2015) is to show how wedges aggregate in an input-output network. However, in Bigio and La'O (2015), all payments between firms are settled at the end of the period after production takes place. As a result, there is no r5le for trade credit; and χ_i and ϕ_i are fixed exogenously. As I show in the next section, the endogeneity of the wedges means that the economy behaves qualitatively very differently in response to local shocks.

1.4 Aggregate Impact of Firm-Level Shocks

I now examine the response of aggregate output to firm-level financial and productivity shocks. I model a *financial shock* to firm i by a change in B_i, the fraction of firm i's revenue that the bank will accept as collateral for the bank loan. Consider a marginal fall in B_i given by dB_i. This is a reduced-form way to capture a reduction in the supply of bank credit to firm i.[7]

[6] Note that although Y is log-linear in each ϕ_i, it is not globally log-linear in χ_i. (This is reflected in the kink in ϕ_i at $\chi_i = r_i$.) Why is Y not globally log-linear in χ_i? The cash-in-advance creates a kink in the policy function for employment n_i at the point at which the cash-in-advance is no longer binding, i.e. at $\chi_i = r_i$. This kink carries over to Y in aggregation. The kink implies: i) Y is not differentiable with respect to ϕ_i at $\phi_i = 1$; ii) the left derivative of Y with respect to χ_i is strictly positive at $\chi_i = r_i$, and the right derivative is zero; iii) Y is not globally log-linear in χ_i.

[7] In the general network model in the following section, each firm sells some portion of its output directly to the household. In this setting, one could alternatively interpret the fall in B_i as a failed payment by final consumer. In either case, these are idiosyncratic shocks to the firm's liquidity, and are not well-represented by a change in its productivity or technology.

The fall in B_i directly affects the amount of cash firm i can raise at the beginning of the period, causing firm i's cash-in-advance to tighten.

$$\frac{d\,\chi_i}{d\,B_i} = 1 > 0$$

If firm i's cash-in-advance is not binding in equilibrium (i.e. if $\chi_i < 1$), then the marginal drop in liquidity does not affect firm i's its output.

$$\frac{d\,\phi_i}{d\,B_i} = \begin{cases} 1 > 0 & if\;\; \chi_i < 1 \\ 0 & otherwise \end{cases}$$

On the other hand, if firm i's constraint is binding in equilibrium, then the tighter cash-in-advance forces the firm to cut back on production, as it no longer has sufficient beginning-of-the-period funds to finance its original input purchases. This is represented by an increase in firm i's shadow value of funds, i.e. a decrease in ϕ_i. Since the drop in firm i's output is a contraction in the supply of good i, the price p_i of the good rises.

In the absence of any linkages with other firms, the effects of the shock would be contained to firm i. However, the firm is linked to other firms via input-output linkages ω_{cs} and credit linkages θ_{cs}, which transmit the shock to other firms. Indeed, the rise in the price p_i of firm i's good affects other firms in the economy in two ways.

Network Effects: Standard Input-Output Channel

The first channel, which I call the *standard input-output channel*, arises from the input-output linkages between firm i and the other firms in the production network, and is the standard channel analyzed in the input-output literature, including Acemoglu et al. (2012) and Bigio and La'O (2015). This channel affects only firms downstream of firm i. The reduction in firm i's output increases the price p_i of good i. This acts as a supply shock to the customer downstream (firm $i+1$), who is now faced with a higher unit cost of its intermediate good. In response, firm $i+1$ cuts back on its use of both good i and labor.[8] Its output falls, and the price of its owns good p_{i+1}, rises. This, in turn, acts as a supply shock to firm $i+2$, and so on. Thus, as a result of the shock to firm i, all firms downstream experience a supply shock to their intermediate goods, and cut back on labor as a result.

In this way, the initial financial shock to firm i is propagated downstream by the input-output linkages between firms. The effect of the shock on aggregate output is amplified because each time that a firm reduces its output, it cuts back on its employment. The resulting fall in labor demand reduces the wage and therefore reduces the household's demand for the consumption good (and aggregate output). Thus, by propagating the shock from firm to firm, the input-output linkages cause a greater fall in aggregate demand for labor, thereby amplifying the initial effect of the shock on aggregate output.

[8] Firm $i+1$'s optimality condition for its use of intermediate good i implies that a higher p_i will cause the firm to reduce $\frac{x_{i+1}}{x_i}$ in response to the increase marginal cost of the good. This amounts to reducing x_i, its use of the intermediate good. The other optimality condition pins down the ratio of expenditure on each input, implying that the fall in x_i also causes the firm to reduce its employment n_i.

Note that this channel is ultimately driven by the input specificity in each firm's production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good i in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

With Cobb-Douglas production, the price and quantity effects of the shock exactly offset, leaving firm i's demand for inputs unchanged. As a result, upstream firms are unaffected by the input-output channel.[9]

Network Effects: Credit Linkage Channel

In addition to the standard input-output channel, there is a new channel of propagation - which I call the *credit linkage channel* - in which the shock disrupts the flow of cash-in-advance payments upstream. This channel describes the endogenous responses in the tightness χ_j of other firms' cash-in-advance constraints. Recall that when firm i cuts back on production, the price p_i of its good rises. This rise in price increases the collateral value of firm i's revenue, which means that firm i can reduce the cash-in-advance payment it makes to its supplier $i-1$. Thus, with less cash on-hand, the supplier $i-1$ is now faced with a tighter cash-in-advance itself.

More precisely, there are three effects on χ_{i-1}, the tightness of $i-1$'s constraint. Recall the expression for χ_{i-1}.

$$\chi_{i-1} \equiv B_{i-1} + \theta_{i-1,i-2} + 1 - (1-\alpha)\theta_{i,i-1}\frac{p_i x_i}{p_{i-1}x_{i-1}} \tag{8}$$

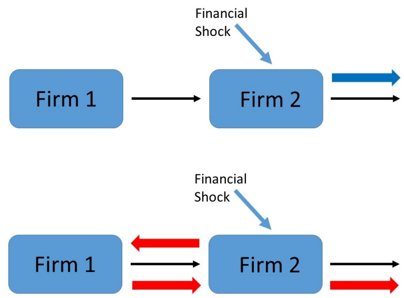
First, the increase in p_i reduces χ_{i-1} due to the lower cash-in-advance payment received from firm i, as discussed above. Second, the fall in firm i's output increases the ratio $\frac{x_i}{x_{i-1}}$ due to the decreasing returns to x_{i-1} (since $(1-\eta_i) < 1$). And third, the fall in i's demand reduces the price p_{i-1} of good $i-1$. Each of these effects reduces the amount of cash that firm $i-1$ has per unit of its revenue, and so the shock to i unambiguously tightens firm $i-1$'s cash-in-advance constraint. Notice from (8) that these effects are increasing in $i-1$'s downstream credit linkage $\theta_{i,i-1}$. In this manner, the credit linkages between i and $i-1$ transmit the financial shock upstream by reducing the cash flow i makes to its supplier.

Thus, χ_{i-1} unambiguously falls in reponse to the shock to i. Faced with a tighter constraint, firm $i-1$ may have to further cut back on its output, represented by a rise in its wedge (i.e. a fall in ϕ_{i-1}). If it does indeed further cut back production, than it also cuts back on employment. This reduces the demand for labor faced by the household, which in turn reduces the wage it earns. In this manner, the initial effect of the shock is amplified. In addition, firm $i-1$ in turn passes the shock on to its own suppliers and customers via both channels. This is discussed in the next section.

Note the role that α plays in mitigating the transmission of the shock via the credit linkage channel. The higher that α is, i.e. the more that firm $i-1$ can collateralize its trade credit $\tau_{i,i-1}$, the less that χ_{i-1} falls in response to the shock to i. Although $i-1$ receives a smaller cash-in-advance payment from its customer, it can collateralize a higher fraction of its trade credit to obtain more credit from the bank. This reduces the loss in liqudity that it suffers due to the smaller cash payment. Therefore,

[9] See Acemoglu et al. (2015) for a detailed discussion.

Figure 2: Feedback Effect



α parameterizes the degree to which each firm can substitute lost cash-in-advance payments for a higher bank loan. The value of α does not effect the qualitative results of the model, but may have a quantitative effect, which I explore later on.

Feedback Effect Created by Transmission Channels

Importantly, the two transmission channels produce a feedback effect which amplifies the shock, as illustrated in Figure 2. To illustrate this, suppose that firm 2 is hit with an adverse financial shock - the bank exogenously reduces the supply of credit to the firm, causing its cash-in-advance constraint to become tighter. If firm 2 is constrained in equilibrium, the shock causes it to reduce its input purchases and cut back on production. This adversely affects its customer downstream in the form of higher prices. This is the standard input-output channel, represented by the blue arrow.

In addition, the presence of credit linkages between firms implies that the shock is also transmitted upstream: firm 2 reduces the cash-in-advance payments it makes to its supplier, firm 1. Faced with a lower cash/revenue ratio, firm 1's constraint becomes tighter, causing it to reduce production by more than it otherwise would. This feeds back to firm 2 again in the form of higher price for the intermediate good. Thus, firm 2 is hit not only with a tighter financial constraint, but also endogenously higher input costs, which it passes on to its customer. This credit linkage channel, represented by the red arrows, creates a feedback loop which exacerbates the initial shock. Thus, a firm-level financial shock to in my model is isomorphic to an *aggregate* financial shock to all firms in a model with fixed constraints, e.g. Bigio and La'O (2015). I explore this point in further detail in the quantitative analysis.

1.4.1 Impact of Firm-Level Shock on Aggregate Output

I now formalize the network effects of the shock on aggregate output. Recall from (7) that equilibrium aggregate output is log-linear in each firm's wedge

$$log\, Y = log\, \bar{Y} + log\, \bar{\Phi}$$

Then the elasticity of aggregate output with respect to firm i's bank borrowing B_i is given by

$$\frac{d\, log\, Y}{d\, B_i} = \frac{d\, log\, \bar{\Phi}}{d\, B_i}$$

$\bar{Y}$ depends only on technology parameters and the productivity of each firm. The financial shock to i therefore affects aggregate output only via $\bar{\Phi}$, which represents the aggregate liquidity available to all firms. Indeed, if no firm's cash-in-advance binds in equilibrium, then a marginal change in any firm's liquidity has no impact on any firm's output.

In Appendix A5, I show that the effect of B_i on aggregate liquidity can be decomposed as follows

$$\frac{d\,log\,\bar{\Phi}}{d\,B_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\,log\,\phi_j}{d\,B_i} \tag{9}$$

The terms $\frac{d\,log\,\phi_j}{d\,B_i}$ capture how the financial shock to firm i affects the shadow value of funds of every other firm j in the network via the credit linkage channel of propagation. The terms $\bar{v}_j$ map these changes in each firm's constraint into aggregate output, and capture the standard input-output channel. $\bar{v}_j$ depends on the share of labor in aggregate output of each firm.

$$\bar{v}_j = \sum_{k=1}^{j} \tilde{\eta}_k$$

The decompostion given by (9) will allow me to quantify the aggregate effects of each channel later on.

Proposition 1 and its corollary, below, constitute the main theoretical result of the paper: firm-level shocks are amplified by the credit network of the economy. Intuitively, stronger credit linkages imply that in response to increases in collateral value, suppliers increase their lending by more, and therefore receive less cash-in-advance; as a result, aggregate liquidity dries up faster in response to shocks. Firms have to cut back on employment and production by more, amplifying the impact of the shock on aggregate output. Notice also that the aggregate impact of a firm-level shock depends on its location in network: shocks to different firms will propagate differently depending on the input-output and credit linkages between firms. Indeed, how central the shocked firm is in both the production and credit networks of the economy will ultimately determine a shock's aggregate impact.

Proposition 1: $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ *and is weakly increasing in* θ_{ij} *for all firms* i *and* j.

Proof: *See Appendix A5.*

Proposition 1 states that a drop in firm i's liquidity B_i causes other firms j to experience an adverse financial shock as well, and that the size of this effect is increasing in the downstream credit linkages between firms, as I discussed in the description of the credit linkage channel. A corollary of this proposition shows how this in turn affects aggregate output.

Corollary: $\frac{d\,log\,Y}{d\,B_i} \geq 0$ *and is weakly increasing in* θ_{jk} *for all firms* i, j, *and* k.

Proof: *This follows from Proposition 1 and (7)*

In the absence of the credit linkage channel, i.e. if the wedges ϕ_j were fixed as in Bigio and La'O (2015), we would have $\frac{d\,log\,\phi_j}{d\,B_i} = 0$ for all $j \neq i$, and (9) would reduce to $\bar{v}_i$. However, since $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ for all j, the endogenous response of the wedges amplifies the aggregate impact of the shock. In addition, the size of this amplification depends on the structure of credit linkages between the firms, θ_{ij}.

Now consider a productivity shock to firm i, represented by a fall in i's total factor productivity (TFP) z_i. What is the effect on aggregate output? Recall the closed-form expression (7) for aggregate output

$$Y = \bar{Y}\bar{\Phi}$$

where

$$\bar{Y} \equiv \prod_{j=1}^{M} \tilde{\eta}_j^{\tilde{\eta}_j} z_j^{\tilde{\omega}_j} \qquad \bar{\Phi} \equiv \prod_{j=1}^{M} \phi_j^{\sum_{k=1}^{j} \tilde{\eta}_k}$$

As it turns out, the aggregation of firm wedges $\bar{\Phi}$ is independent of z_i. To see this, first recall that $\phi_M = min\{1,\, \chi_M\}$, where $\chi_M = \theta_{M,M-1} + B_M$. Firm M's shadow value of funds is thus independent of all z_i. Next, recall that $\phi_{M-1} = min\{1,\, \chi_{M-1}\}$, where

$$\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - (1-\alpha)\frac{\theta_M}{\phi_M \omega_{M,M-1}(1-\eta_M)}$$

Thus, ϕ_{M-1} is also independent of all z_i. Continuing recursively, it follows that all wedges ϕ_j are independent of TFP z_i. Intuitively, changes in a firm's TFP do not affect the severity of agency frictions between the firm and its creditors, and therefore they do not affect the tightness of its cash-in-advance.

Since z_i enters only in $\bar{Y}$, we have

$$\frac{d\,log\,Y}{d\,z_i} = \frac{\tilde{\omega}_i}{z_i}$$

Recall that $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1}$ represents firm i's share in total intermediate good use. A fall in firm i's productivity affects its demand for intermediate goods and its supply of good i. This is the standard input-output channel at work. However, productivity shocks don't affect firms' cash-in-advance constraints ϕ_j. Therefore, the credit network plays no role in propagating productivity shocks.

Because financial shocks directly affect firm wedges while productivity shocks do not, productivity and financial shocks will have differential effects on a firm's output and employment. In the empirical part of the paper, I will use these differential effects to separately identify financial and productivity shocks from the data.

To summarize, three main insights emerge from the model. First, when firms are suppliers of intermediate goods as well as the creditors who finance the transactions of these goods, firm-level shocks can endogenously generate large changes in the aggregate liquidity available for trade in intermediate goods. This creates a multiplier effect which amplifies the aggregate effects of firm-level shocks. Second, the aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

But what precisely is the role of the credit network? Until now, the structure of the networks was assumed to be a straight line, shedding little light on its exact role in generating aggregate fluctuations. And is this mechanism quantitatively relevant? To answer these questions requires a model incorporating more features of the economy which can be taken to the data. To this end, I return to the general network framework in the next section.

2 General Model

I now generalize the model to capture more features of the economy. Specifically, I generalize the network structure of the economy and the household preferences. I assume that each of the M goods can be consumed by the representative household or used in the production of other goods. The household's total consumption C is Cobb-Douglas over the M goods

$$C \equiv \prod_{i=1}^{M} c_i^{\beta_i}$$

The household has Greenwood-Hercowitz-Huffman (GHH) preferences given by

$$U(C, N) = \frac{1}{1-\gamma}\left(C - \frac{1}{1+\epsilon}N^{1+\epsilon}\right)^{1-\gamma} \quad , \qquad C$$

where ϵ and γ respectively denote the Frisch and income elasticity of labor supply. Quantitatively similar results will hold for preferences which are additively separable in aggregate consumption C and labor N. The household maximizes its utility subject to (1), the household budget constraint. This yields optimality conditions equating the ratio of expenditure on each good with the ratio of their marginal utilities, and equating the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \ , \qquad N^{1+\epsilon} = C$$

Each firm can trade with all other firms. Firm i's production function is again Cobb-Douglas over labor and intermediate goods.

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}}\right)^{1-\eta_i}$$

Here, x_i denotes firm i's output and x_{ij} denotes firm i's use of good j. Since ω_{ij} denotes the share of j in i's total intermediate good use, I assume $\sum_{j=1}^{M} \omega_{ij} = 1$, implying that each firm has constant returns to scale. The input-output structure of economy can summarized by the matrix Ω of intermediate good shares ω_{ij}.[10]

[10]This is simply a generalization of the input-output structure in the stylized model. In that case, the Ω would be

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & \omega_{23} & & \\ \omega_{31} & \omega_{32} & \omega_{33} & & \\ \vdots & & & \ddots & \\ \omega_{M1} & & & & \omega_{MM} \end{bmatrix}$$

This matrix describes the structure of the production network. Note that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence, Ω describes how firms would trade with each other in the absence of frictions.

Each firm's cash-in-advance takes the same form as in the stylized model, with the exception that each firm has M suppliers and M customers instead of just one of each. Firm i is required to pay its wage bill wn_i and its intermediate good purchases $p_s x_{is}$ from each supplier s in advance. It receives a loan b_i from the bank and a trade credit loan τ_{is} from each supplier.

$$wn_i + \underbrace{\sum_{s=1}^{M}(p_s x_{is} - \tau_{is})}_{net\ CIA\ payment\ to\ suppliers} \le b_i + \underbrace{p_i x_i - \sum_{c=1}^{M}\tau_{ci}}_{net\ CIA\ received\ from\ customers}$$

Each firm faces a borrowing constraint each of its suppliers, to which it can pledge fractions θ_{is} of its revenue in return for the loans. The borrowing constraints take the form

$$\tau_{is} \le \theta_{is} p_i x_i$$

The structure of the credit network between firms can be summarized by the matrix of θ_{ij}'s.

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & \theta_{23} & & \\ \theta_{31} & \theta_{32} & \theta_{33} & & \\ \vdots & & & \ddots & \\ \theta_{M1} & & & & \theta_{MM} \end{bmatrix}$$

Henceforth, I refer to this matrix as the *credit network* of the economy. As we will see, the structure of this network will play an important role in determing the aggregate impact of idiosyncratic shocks.

Each firm can also borrow b_i from the bank by pledging B_i of its revenue and $1-\alpha$ of its accounts receivable $\sum_{c=1}^{M}\tau_{ci}$, so that its bank borrowing constraint takes the form

$$b_i \le B_i p_i x_i + \alpha \sum_{c=1}^{M} \tau_{ci}$$

$\alpha < 1$ parameterizes the substitutability of cash-in-advance payments and bank credit. If i's customer c reduces its cash-in-advance payment to i by one dollar, then i experiences a net loss in liquidity of

given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.

$1-\alpha$ dollars; it looses 1 dollar in cash, but is able to borrow α more dollars from the bank. Thus, it is able to partially substitute the lost cash payment with more bank credit. $\alpha = 0$ corresponds to the case when the two are not substitutable, and $\alpha = 1$ to the case when they are fully substitutable. The choice of α will have an effect of the quantitative predictions of the model, which I discuss later on.

Each firm chooses the size of the loan to obtain from each creditor, so that the borrowing constraints bind in equilibrium. Plugging the binding borrowing constraints into firm i's cash-in-advance yields a constraint on i's total input purchases

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i$$

where χ_i denotes the tightness of i's cash-in-advance.

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1-\alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

Note that χ_i is again an equilibrium object, depending on the prices customers' goods p_c and forward credit linkages θ_{ci} for all c.

Firms choose labor and intermediate goods to maximize profits subject to their cash-in-advance. This yields optimality conditions of the same form, equating the ratio of expenditure on each good with the ratio of their marginal revenue products.

$$\frac{wn_i}{p_j x_{ij}} = \frac{\eta_i}{(1-\eta_i)\,\omega_{ij}}$$

Again, the cash-in-advance of firm i inserts a wedge ϕ_i between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad\qquad x_{ij} = \phi_i \,(1-\eta_i)\,\omega_{ij} \frac{p_i}{p_j} x_i \tag{10}$$

where the wedge is determined by the firm's shadow value of funds.

$$\phi_i = min \left\{1\,,\ \chi_i\right\} \tag{11}$$

Note that the wedge is still an equilibrium object, depending on collateral value of each customer's output and forward credit linkages. Endogenous wedges imply equilbrium will take same form, and will respond in qualitatively the same way as previously.

Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^{M} n_i \qquad\quad x_i = c_i + \sum_{c=1}^{M} x_{ci}$$

The richness of the model afforded by the general network structure and household preferences will allow me to take the model to the data and examine quantitatively the role of the credit network in generating aggregate fluctuations. The equilibrium conditions take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model. However, the general network structure precludes a closed-form solution.

2.1 Relationship Between Firm Influence and Size

A well-known critique of standard input-output models such as Acemoglu et al. (2012) is that a sufficient statistic for a firm's influence is its share of total sales in the economy. In other words, the size of a firm as measured by its share of aggregate sales is sufficient to determine the aggregate impact of a shock to sector i, and one does not need to know anything about the underlying input-output structure of the economy. All relevant information is captured by the sales share. As a result, an idiosyncratic shock to any firm is isomporphic to an aggregate TFP shock weighted by each firm's share of total value-added. This feature makes it difficult to claim that the origin of aggregate fluctuations is idiosyncratic rather than aggregate shocks, using this class of frictionless models.

Bigio and La'O (2015), however, show that this isomorphism breaks down when the economy has frictions. In particular, the impact on economic aggregates of an idiosyncratic shock to sector i depends on the underlying input-output structure of the economy, and cannot be summarized by the sector's share of aggregate sales.

My model shows that when the constraints faced by firms depends endogenously on their credit relationships and the prices of downstream goods, knowing the input-output structure of the economy is no longer sufficient to measure the aggregate impact of a shock to a sector or firm i. How a financial shock propagates to other firms depends on the credit linkages between them. Therefore, to know how shocks propagate in my economy, one needs to know the underlying structure of credit linkages between firms. Thus, the aggregate impact of an idiosyncratic shock depends on the structure of the input-output network, and the structure of the credit network.

2.2 Solving the General Model

The equilibrium of the general model is the solution to system of $M^2 + 5M + 2$ nonlinear equations in the same number of unknowns, listed in Appendix A6. For any set of model parameters

$$\left\{\left\{z_i,\, B_i,\, \eta_i,\, \beta_i,\, \{\theta_{ij},\, \omega_{ij}\}_{j\epsilon I}\right\}_{i\epsilon I},\, \alpha,\, \epsilon,\, \gamma\right\}$$

there is a unique solution to the system. Since the model is one period, the behavior of the system in response to shocks can be modeled by comparative statics. In particular, I am interested in the *change* in the economy that results form a perturbation of one or more of the model parameters $\{B_i,\, z_i\}_{i\epsilon I}$, representing financial and productivity shocks, respectively. I therefore log-linearize the system of nonlinear equations around a point $\{B_i^*,\, z_i^*\}$. In the quantitative analysis, I calibrate this point (and the remainder of the parameters), to match data for the US economy. I thus obtain a log-linear approximation for the response of the equilibrium variables to firm-level financial and productivity shocks.

It is worth clarifying one point about productivity shocks. It turns out from the Cobb-Douglas specification of firm production functions that the equilibrium is already log-linear in each z_i. Therefore, the log-linearized response of the equilibrium variables to a change in z_i is independent of the level of z_i. Therefore, I do not need to calibrate the parameters $\{z_i\}_{i\epsilon I}$ to approximate a response in the economy to a productivity shock. Indeed, when one log-linearizes the equilibrium system around $\{B_i^*, z_i^*\}$, z_i^* drops out of the log-linear equations.

3 Quantitative Analysis

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

Unfortunately, data on trade credit flows at any level of detail is scarce. While accounts payable and receivable are generally observable at the firm-level from Compustat, flows of trade credit between firm- or industry-pairs is not. In order to overcome this paucity of data, I construct a proxy of industry-level trade credit flows from industry-level input-output data and firm-level balance sheet data, which I now describe.

3.1 Mapping the US Credit Network

The purpose of this section is to construct a proxy for trade credit flows τ_{ij} between industries i and j, from which I can later calibrate the structural parameters θ_{ij}. To build my proxy, I use two sources of data: input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the sample period 1997-2013. The BEA publishes annual data on commodity use by industry (Uses by Commodity Table) at the three-digit level of the North American Industry Classification System (NAICS). At this level, there are 58 industries, exlcuding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to $p_j x_{ij}$ in my model for every industry pair $\{i, j\}$. The BEA also publishes an annual Direct Requirements tables at the same level of detail, which indicate for each industry the amount of a commodity that is required to produce one dollar of that industry's output. These values are quite stable over my sample period. In constructing my proxy, and also in calibrating the model later, I use the input-output tables of the median year in my sample, 2005.

Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm's total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample. Therefore, while I cannot identify from whom each firm receives trade credit or to whom it extends credit, I observe the total stock of trade credit and trade debt that it has in any year.

To construct the proxy of trade credit flows, I partly follow the strategy of Raddatz (2010). I begin with the observation that a trade credit loan from supplier to customer is typically a fraction of the value of the sale from supplier to customer.[11] I therefore assume that the trade credit from industry j to industry i is proportional to the value of the sale.

[11] This has been documented empirically in various studies including Petersen and Rajan (1997).

$$\tau_{ij} = q_{ij} p_j x_{ij}$$

Here, q_{ij} denotes the fraction of i's purchase from j made on trade credit. The value of the total purchase $p_j x_{ij}$ is *directly* observable from the BEA input-output tables. So to construct the proxy for τ_{ij}, it remains to construct an estimate of q_{ij} for each industry-pair.

Appendix A14 describes this procedure in detail. The idea is as follows. Using data on firms' accounts payable and accounts revenue from Compustat, I first construct measures of each industy's payables financing and receivables lending. These respectively descibe, on average, how intensively firms in the industry borrow from their suppliers and lend to their customers. Then to proxy q_{ij}, i.e. how much of j's sales to i are made on credit, I take a weighted average of industry i's payables financing and industry j's receivables lending. My weighting scheme minimizes the mean squared errors in the observed accounts payables of each industry.

Given my proxy $\hat{q}_{ij}$, inter-industry trade credit flows are then proxied as

$$\hat{\tau}_{ij} = \hat{q}_{ij} p_j x_{ij}$$

As such, I have a map of the credit network of the US economy at the three-digit NAICS level of detail, with which I can quantitatively evaluate the model.[12]

4 Calibration

With proxy for trade credit flows at hand, I calibrate the general model of Section 6 to match data on the US economy. My calibration strategy involves using the BEA input-output tables to calibrate technology parameters, and my proxy to calibrate the financial parameters. In this section, I describe this strategy in detail.

4.1 Technology Parameters

I calibrate technology parameters η_i and ω_{ij} to match the BEA input-output tables of the median year in my sample, 2005. At the three-digit level, I have 58 industries after exlcuding financial industries. From firm i's optimality conditions (10), we can write the firm's total expenditure on inputs as

$$\begin{aligned} wn_i + \sum_{j=1}^{M} p_j x_{ij} &= \left(\eta_i + [1 - \eta_i] \sum_{j=1}^{M} \omega_{ij} \right) \phi_i p_i x_i \\ &= \phi_i p_i x_i \end{aligned}$$

where the second equality holds due to the constant returns to scale of i's production technology. This implies that

[12] I omit the financial sector from my analysis.

$$\phi_i = \frac{wn_i + \sum_{j=1}^{M} p_j x_{ij}}{p_i x_i} \tag{12}$$

The right-hand side of (12) is directly observable from the BEA's Direct Requirements table. Therefore I calibrate ϕ_i to match industry i's direct requirements of all commodities and labor.

Looking through the lense of the model, the observed input-output tables reflect both technology parameters and distortions created by the liquidity onstraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm i's optimality conditions for each input and my calibrated ϕ_i's

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \qquad \omega_{ij} = \frac{p_j x_{ij}}{(1-\eta_i)\phi_i p_i x_i}$$

Again the ratios $\frac{wn_i}{p_i x_i}$ and $\frac{p_j x_{ij}}{p_i x_i}$ are directly observable from the Direct Requirements tables for every industry i and j.

4.2 Financial Parameters

I calibrate the parameters θ_{ij}, representing severity of agency problems between industry j and i, to match my proxy of inter-industry trade credit flows $\hat{\tau}_{ij}$. Industry i's binding borrowing constraints pin down its level of borrowing from each of its suppliers j.

$$\theta_{ij} = \frac{\tau_{ij}}{p_i x_i}$$

Industry i's total revenue $p_i x_i$ is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005). I then use this and my proxy for trade credit $\hat{\tau}_{ij}$ to calibrate θ_{ij}.

To calibrate B_i, the parameters reflecting the severity of agency problems between each industry and the bank, recall the definition of ϕ_i given by (11), which depends on the technology parameters (calibrated as described above) and the tightness χ_i of each industry's cash-in-advance, where

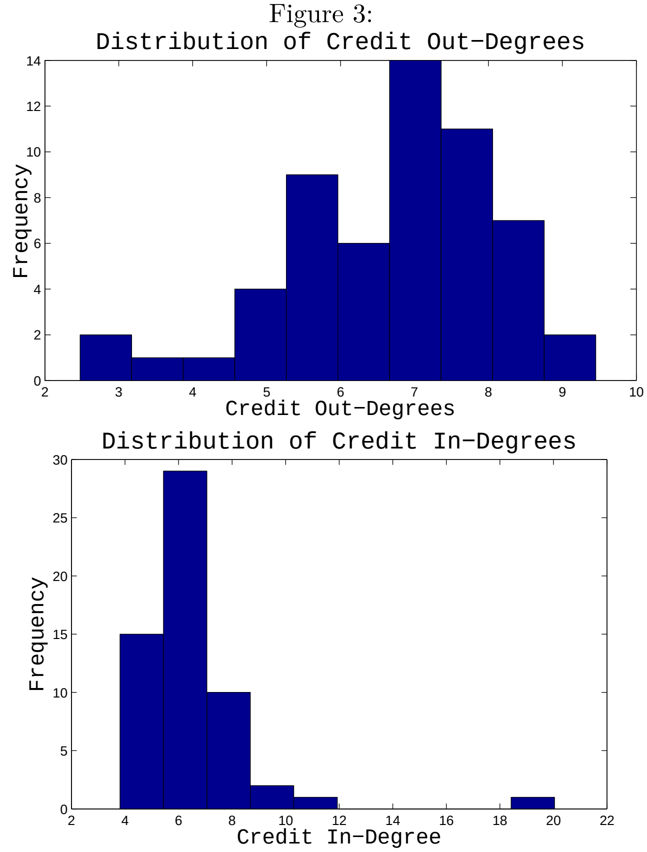
$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1-\alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \tag{13}$$

The total revenue of each industry $p_i x_i$ is observable from the Uses by Commodity tables, and ϕ_i and θ_{is} for all s were calibrated as described above. I therefore use (13) and (11) to back out B_i for each industry.

Appendix A12 plots the calibrated matrix Θ, which represents the credit network of the US economy at the three-digit NAICS level of detail. The matrix is relatively sparse in areas in which indistries do not engage in much trade. Also firms within the same industry are lend to and borrow from one another more intensively, as represented by the red diagonal.

To identify which industries take a more central role in the credit network, I define the *credit out-degree* (COD_i) and *credit in-degree* (CID_i) of industry i as

Figure 3:



$$COD_i \equiv \sum_{c=1}^{M} q_{ci} \qquad CID_i = \sum_{s=1}^{M} q_{is}$$

These two measures respectively measure how much trade credit an industry provides the rest of the economy, and how much it receives from the rest of the economy.. An industry with a high credit out-degree (credit in-degree) makes a high fraction of its total sales (intermediate goods purchases) on credit, *ceteris paribus*. A few industries take particularly central positions in the credit network of the US: the technical services and oil and gas extraction industries provide the rest of the economy with a lot of credit, while the oil and gas auto manufacturing absorb a large amount of credit from the rest of the economy. Figure 3 plots the distribution of the credit out- and in-degrees of the US.

While there is significantly more variation in the credit in-degrees of industries (standard deviation 2.24) than the credit out-degrees (standard deviation 1.48), the distribution of the former is skewed right.

4.3 Remaining Parameters

It remains to calibrate the Frisch and income elasticity parameters ϵ and γ, and α which parameterizes the substitutability of cash-in-advance payments and bank credit. I follow the standard literature and set $\epsilon = 1$ and $\gamma = 2$. Omiccioli (2005) examines how firms collateralize their trade credit for bank borrowing for a sample of Italian firms, and finds that the median firm in the sample collateralizes about 20 percent of its accounts receivable. I therefore set $\alpha = 0.2$.

5 Quantitative Results

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and financial shocks. I first ask how much aggregate fluctuations does the credit network of the US economy generate?

It is instructive to discuss how the transmission mechanism outlined in the stylized model maps into this more general setting. In addition to the feedback effects described in Section 1.4, there are additional spillover effects in this general setting arising from the additional linkages between each industry. To illustrate, consider the petroleum and coal manufacturing industry and the utilities industry in the US. Each is linked by a common supplier, the oil and gas extraction industry. The flow of goods between these industries is represented by the black arrows in Figure 4. Suppose that firms in petroleum and coal manufacturing experience an exogenous tightening of their financial constraints. Unable to finance the same level of inputs, many of these firms may be forced to reduce production as a result. This will adversely affect other firms in the economy who use petroleum and coal products, as they face higher prices for these goods. This is the standard input-output channel of transmission, indicated by the blue arrow in Figure 4. (In addition, the suppliers in the oil and gas industry will face lower demand from their customers, and reduce production accordingly). In the absence of the credit linkage channel of transmission, firms in the utilities industry will remain largely unaffected by the shock (save for some small general equilibrium effects).

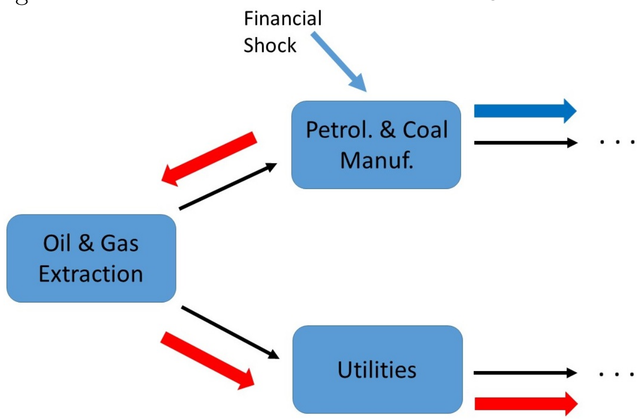
However, the presence of credit linkages implies that the shock causes petroleum and coal manufacturers to reduce the cash payments they make to their oil and gas suppliers. As a result, these oil and gas firms are also faced with tighter financial conditions themselves, and may be forced to cut back on production by more than they otherwise would. This reduction in the supply of oil and gas causes utilities firms to face higher input prices. These utilities firms in turn reduce production and pass the shock on to the rest of the economy in the form of higher energy prices. These additional credit linkage effects, represented in Figure 4 by the red arrows, amplify the initial effect of the financial shock to petroleum and coal manufacturers. How large are these credit linkage effects likely to be?

5.1 Aggregate financial shock

In order to answer this, I perform the following exercise. Suppose that the economy is hit with a one percent aggregate financial shock: each industry i's liquidity falls by one percent. By how much does aggregate output Y fall?

To the guage the maximium effect that the credit network can generate, I first compute the fall in Y for $\alpha = 0$. This corresponds to the case in which industries cannot substitute lost cash-in-advance payments for more bank credit. The propagation of financial shocks is strongest for this case. I then allow for subsitutability by setting α to its baseline calibrated value of 0.2, in order to have a more conservative estimate of the aggregate impact of the shock.

Figure 4: Transmission Mechanism in the General Model



Results for $\alpha = 0$

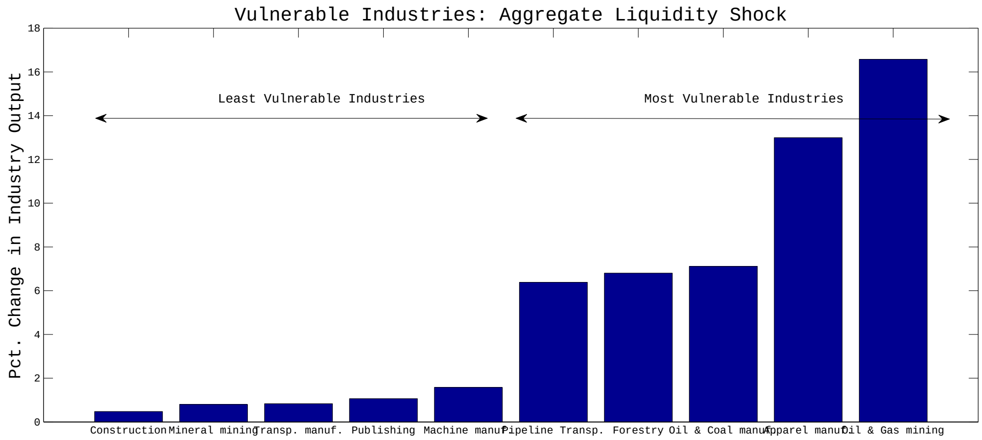
I find that, under this specification, aggregate output falls by 3.70 percent. This is represents a large aggregate effect of the shock. To assess how much of this drop in aggregate output is generated by the propagation of shocks via the credit network, I perform the same exercise, shutting down the credit linkage channel. I leave the detailed technical explanation of how I do this to Appendix A8. The intuitive explanation is as follows. Recall that in the model, changes to firm i's wedge ϕ_i come either from the direct financial shock $\tilde{B}_i$ to firm i, or from shocks to other firms being transmitted to i via its credit linkages. In shutting down the credit linkage channel, I impose that changes in the wedges come only from direct financial shocks to each firm. In this way, the credit linkages play no role in propagating shocks.

With the credit linkage channel shut down, I compute the response in aggregate output to the same aggregate shock, and compare it to the baseline case. I find that, with the credit linkage shut off, GDP falls by only 2.54 percent. Thus, the effect of the credit linkages in propagating the shocks throughout the network increase the response in aggregate output to the shock by 1.38 percentage points. Put differently, the credit network accounts for 31.4 percent of the fluctuation in aggregate output in response to an aggregate financial shock. These are quantitatively significant results, suggesting that the credit network of the US can play an important role in generating aggregate fluctuations in GDP from financial shocks.

Results for for $\alpha = 0.2$

Next, I perform the same exercise for with $\alpha = 0.2$, allowing for substitutability between bank credit and cash-in-advance payments. Even in this more conservative case, the aggregate impact of the shock is quite large: I find that the total fall in GDP is 2.92 percent, and the fall with the credit linkage channel shut off is 2.28 percent. Although the amplification generated by the credit network falls substantially, it is still quantitatively relevant. The credit linkages between industries produce a larger drop in Y by 0.64 percentage points. Put differently, the credit network of the US accounts for 22 percent of the drop in GDP in response to the aggregate financial shock. Therefore, even allowing for firms to substitute lost payments with increased bank borrowing does not substantially diminish

Figure 5:



the effect of credit linkages in generating aggregate fluctuations. The remainder of the paper uses $\alpha = 0.2$.

Which industries are most vulnerable to the aggregate financial shock? Put differently, which experience the largest drop in output?

Figure 5 plots the five most vulnerable and five least vulnerable industries. The figure indicates that there is a large degree of heterogeneity in the response of industries to the aggregate financial shock. While the output of the construction industry falls by less than 1 percent, that of oil and gas mining falls by over 16 percent.

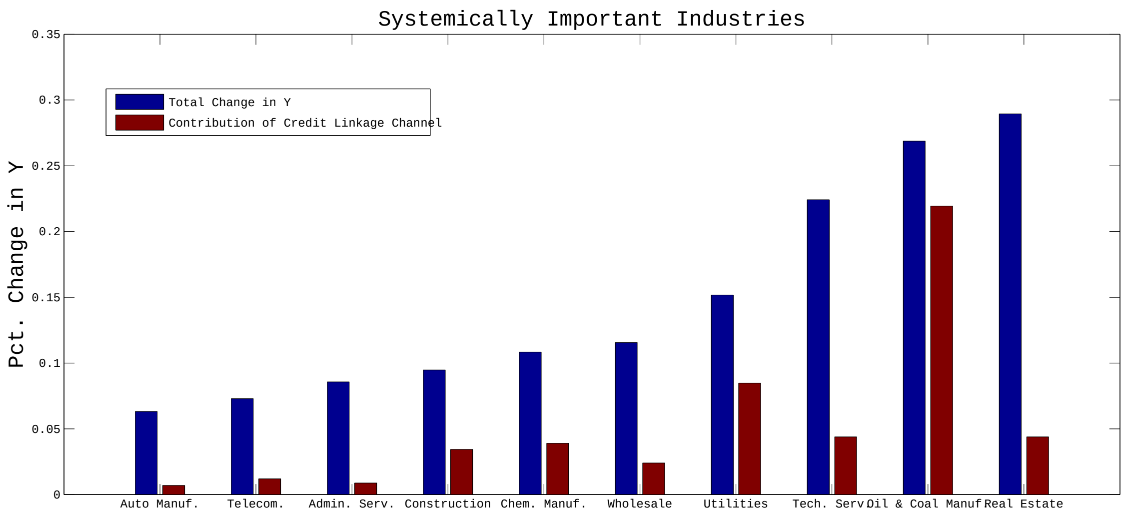
5.2 Industry-Level financial shocks

Next, I ask which industries are most systemically important in the economy, and how this relates to their position in the credit network. I measure the systemic importance of industry i by the elasticity of aggregate output with respect to its liquidity B_i.[13] A higher elasticity implies that an industry-level financial shock to i has a larger impact on aggregate output.

Figure 6 shows a bar graph of the ten most systemically important industries in the US. The blue bars show the elasiticity of aggregate output with respect to each industry's liquidity, or the percentage drop in Y following a 1 percent drop in B_i.

The red bars show the contribution of the full credit network to each elasticity, which is computed by subtracting the drop in Y that occurs with credit linkage channel shut off, from the total drop in Y. To shut off the credit linkage channel, I impose that each industry's wedge ϕ_i changes only in response to a direct financial shock to that industry, and not endogenously through credit linkages with other industries. This gives the drop in aggregate output that would occur in the absence of credit linkages, i.e. if the wedges of industries did not respond endogenously to changes in prices. This is explained in more detail in Appendix A9. In this way, I numerically measure by how much the industry-level

Figure 6:



shock is amplified by the credit network.

Two results emerge from this exercise. First, the model implies that an industry-level financial shock can have a strong impact on US GDP. For example, although the technical services industry accounts for only .069 percent of US GDP, a one percent financial shock this industry causes a fall in GDP of .19 percent, due to its input-output and credit linkages with other industries. This is an enormous response in aggregate output. In the absence of any linkages, the elasticity of GDP to this industry's liquidity would be equal to its share of GDP; i.e. GDP would fall by only .069 percent in response to this shock. Therefore, the network effects generated by input-output and credit linkages greatly amplify the aggregate impact of the industry-level shock.

Second, the credit network of the US plays a quantitatively significant role in amplifying these industry-level shocks. On average, between one fifth to one half of the fall in GDP in response to an industry-level shock is due to the role of credit linkages in propagating the shock across the network. Consider again a one percent financial shock to the technical services industry. In the absence of credit linkages, US GDP would fall by only .16 percent in response this shock. Therefore, the credit network accounts for about one fifth of this industry's systemic importance. (The remainder of the amplification is then caused, of course, by the input-output linkages).

5.3 What Features of the US Economy Contribute to Amplification?

What features of the US economy contribute to the amplification of financial shocks? As it turns out, in the US, industries which are important suppliers of intermediate goods (e.g. manufacturing industries) happen to also be important providers of credit to the rest of the economy. This means that these industries are more vulnerable to nonpayment by customers. As a result of this property, financial shocks to industries downstream will have a larger aggregate impact.

[13] Recall that in the general model precludes analytical expressions for this elasticity. I therefore compute these numerically.

To see this, recall the definition of an industry's credit out-degree (COD), reproduced below, which summarizes how much credit the industry provides the rest of the economy. Industries with a higher credit out-degree are more vulnerable to nonpayment by their customers: they are more constrained *ceteris paribus*, and have extended more credit to their customers. Now define an industry's production out-degree (POD) as

$$COD_i \equiv \sum_{c=1}^{M} q_{ci} \qquad POD_i \equiv \sum_{c=1}^{M} \omega_{ci}$$

The production out-degree of an industry summarizes how important it is as a supplier of intermediate goods to the rest of the economy. A higher production out-degree means that the goods produced by this industry are widely used by other industries.

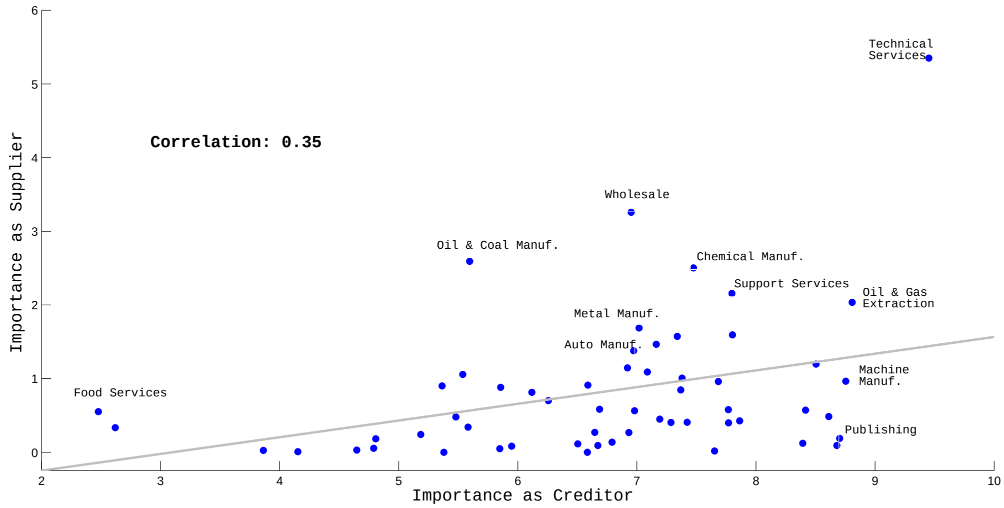
Figure 7 shows a scatter plot of industries' credit and production out-degrees, with a fitted least-squares line. There is a fairly strong positive correlation between the two measures (correlation 0.35), indicating that industries which take a more central position in the economy's input-output network also take a more central position in the credit network, on average.[14]

To see why this positive correlation increases the sensitivity of GDP to financial shocks, consider the following example. The metal manufacturing industry has a high production out-degree - many other industries use metal products intensively. One of its most important customers is auto manufacturing. Metal manufacturing also has a high credit out-degree - these firms sell a lot of output on credit. Suppose that firms in auto manufacturing are hit with a financial shock; i.e. banks reduce the supply of credit to these firms. These firms will then reduce their cash-in-advance payments to their suppliers in metal manufacturing (cia the credit linkage channel). Because these suppliers are quite dependent on receiving these payments from their customers in auto manufacturing, they experience a sharp tightening of liquidity. This forces them to cut back on production. As a result, other industries experience a sharp drop in the supply of metal goods. Because the rest of the economy uses metal products intensively, this fall in production has a large impact on aggregate output. Thus, the financial shock to firms in auto manufacturing will have larger aggregate impact because their suppliers in metal manuf are both more vulnerable to nonpayment and are important suppliers of goods to rest of economy,

The plot indicates that there is a strong positive relationship between the credit out-degree of an industry and its systemic importance. The correlation between the two measures is 0.6. On average, a one standard deviation increase in an industry's credit out-degree corresponds to an increase of 0.13 percentage points in the elasticity of Y with respect to its liquidity, or 0.59 standard deviations. Put differently, a one percent financial shock to an industry will reduce GDP by 0.13 percentage points more than the same shock to an industry which provides less credit to the economy by one standard deviation. Therefore, there is a strong association in the model between an industry's systemic importance and how important that industry is in providing credit to the rest of the economy.

[14] While my paper is agnostic about the source of this correlation, one could conjecture explanations for a positive correlation. For example industries which produce more 'basic' goods, such as mining or manufacturing industries, may have some advantage over others to enforce debt repayment due to the nature of the goods. Although the question of why important suppliers provide more credit is an important and interesting one, it is beyond the scope of this paper.

Figure 7:



5.4 Summary of Quantitative Analysis

The quantitative analysis showed that i) the credit linkages between US industries play a quantitatively significant role in amplifying aggregate and industry-level financial shocks, even when allowing for substitutability between bank credit and cash payments and ii) the systemic importance of an industry depends on how important for the economy its suppliers are in providing credit.

Therfore an understanding of the role that credit linkages play in propagating idiosyncratic shocks introduces a new notion of the systemic importance of firms or industries based on their place in the credit network. The effects of these linkages are quantitatively important. Therefore, by overlooking the importance of credit linkages between nonfinancial firms, the literature has missed an important determinant of what makes an industry or firm systemically important.

6 Empirical Analysis

Now that I have established the role that the credit network plays in propagating shocks, and shown that it can play a quantitatively significant role in generating fluctuations in aggregate output by amplifying financial shocks, I turn to the empirical analysis. I ask, in light of the credit linkages we observe between industries in the US, what role did the credit network of the US play during the Great Recession? How much of observed aggregate volatility can be accounted for by liquidity versus productivity shocks? Have idiosyncratic or aggregate shocks played a more important role in US business cycles? The answers to these questions depends on the nature and magtinude of the shocks that hit the economy over this period. Therefore, to answer these questions, I first need to estimate shocks.

My empirical strategy follows a structural factor analysis approach, similar to that of Foerster et al. (2011), on US industrial production industries at the three-digit NAICS level. In all, I allow for four types of shocks: aggregate financial and productivity shocks, and industry-level (idiosyncratic) financial and productivity shocks. This approach involves a two-step procedure for estimating each type of shock. First, I use the model to back-out the financial and productivity shocks which hit each industry each quarter, using data on each industry's output growth and employment growth. Second, I use dynamic factor methods to decompose these shocks into an aggregate component and an industry-level component. I then feed these estimated shocks into the model to estimate the role of each type of shock, and the credit network of US manufacturing industries, in generating observed aggregate volatility.

6.1 Data

From the Federal Reserve Board's Industrial Production Indexes, I observe the growth rate in output of all manufacturing and mining industries at the three-digit NAICS level, at the quarterly frequency. There are 23 such industries at this level of detail. From the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, I observe the number of workers employed by all industries at the three-digit NAICS level.[15] For each dataset, I take 1997 Q1 -2013 Q4 as my sample period. I seasonally-adjust and de-trend each series.

Looking through the lense of the model, these observed quarterly fluctuations may be driven by:

1. Industry-level liquidity or productivity shocks
2. Aggregate liquidity or productivity shocks
3. Credit and input-output linkages which propagate these shocks

The answers to my questions of interest depend on the relative importance of each of these in driving fluctuations. Since my calibrated model tells me how much industry j's output or employment changes in response to a liquidity or productivity shock to i, I use the model to control for the effect of credit and input-output linkages in propagating shocks across industries. To identify aggregate versus industry-level components of the estimated shocks, I use standard dynamic factor methods. The only remaining challenge is to identify how much fluctuations are driven by changes in productivity versus changes in liquidity.

Most of the literature takes one of two extreme positions on the source of fluctuations: they are assumed to be driven either entirely by productivity shocks (as in Foerster et al. (2011) and Acemoglu et al. (2012)) or entirely by financial shocks (as in Bigio and La'O (2015)). By making use of both employment and output data, I make a weaker assumption and allow for both types of shocks. In the next section, I first describe how I back-out shocks using this data and my model. I then discuss how my model is able to separately identify financial and productivity shocks from the data on output and employment.

[15] Hours worked is not directly available at this level of industry detail and this frequency. However, I will check that hours worked and number workers employed are correlated at lower frequencies and lower levels of industry detail.

6.2 Identification of financial shocks versus Productivity Shocks

What allows the model to identify productivity shocks and financial shocks separately? In other words, how does the model attribute an observed fall in industry i's output x_{it} and employment n_{it} to a financial shock rather than productivity shock? In the model, productivity and financial shocks have differential effects on labor and employment.

Productivity shocks work through the Cobb-Douglas production functions, and directly affect the the amount of labor employed *per unit* of output produced. financial shocks, on the other hand, do not affect production functions. Rather they work by affecting the tightness of industry cash-in-advance constraints, and show up as changes in industry wedges (first-order conditions). The model uses these differential effects to identify the source of fluctuations in observed output and employment.

In short, the model essentially backs-out productivity shocks from industry-level Solow residuals, or unexplained changes in output given changes in the factors of production. Similarly, it backs-out financial shocks from unexplained changes in the ratios of each industry's labor expenditure to revenue. In equilibrium, this ratio is equal to the labor share of production (a constant) times the wedge.[16] Because the model can track how a financial shock or productivity shock to one industry spills over to other industries via their credit and input-output linkages, the model can back out exactly how much of a change in an industry's output and employment is coming from spillover effects versus a direct shock, and can identify the industry which was shocked. A more detailed discussion of this identification procedure is relegated to Appendix A10.

6.3 Using the Model to Back Out Shocks from the Data

Recall that the model yields a system of log-linear equations describing the (first-order approximated) elasiticity of each equilibrium variable to the liquidity B_i and productivity z_i of each industry i. Suppose that the static model is extended to be a repeated cross-section. Then these same equations describe the evolution of the equilibrium variables that occurs each period in response to financial and productivity shocks, to a first-order approximation. I obtain a closed-form solution for this evolution, which is derived in the Appendix.

Let X_t and N_t denote the M-by-1 dimensional vectors of industry output and employment growth at time t, $\tilde{x}_{it}$ and $\tilde{n}_{it}$, respectively. And let B_t and z_t similarly denote the M-by-1 dimensional vectors of industry financial and productivity growth (i.e. shocks) at time t, $\tilde{B}_{it}$ and $\tilde{z}_{it}$, respectively. The closed-form solutions for X_t and N_t yield

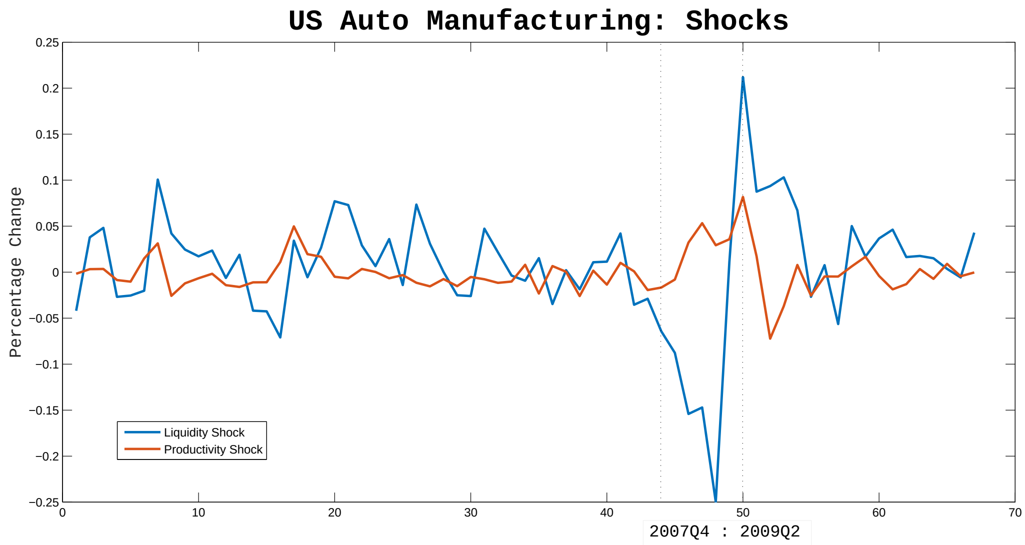
$$X_t = G_X B_t + H_X z_t$$

$$N_t = G_N B_t + H_N z_t$$

These respectively describe how each industry's output and employment changes each period in response to the financial and productivity shocks to every industry. Here, the M-by-M matrices G_X, G_N, H_X and H_N are functions of the economy's input-output and credit networks Ω and Θ,

[16] See equation (5).

Figure 8:



and capture the effects of the input-output and credit linkages in propagating either type of shock across industries, as was described in the theoretical analysis. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct X_t and N_t for US industrial production industries (at the three-digit NAICS level) from the output and employment data described above. Let $\hat{X}_t$ and $\hat{N}_t$ denote these observed fluctuations. I then have a system of $2M$ equations in as many unknowns for each quarter, and can invert the system to back-out shocks B_t and z_t each quarter from 1997 Q1 to 2013 Q4.

$$B_t = G_N^{-1}\left(\hat{N}_t - H_N z_t\right)$$

$$z_t = Q^{-1}\hat{X}_t - Q^{-1}G_X G_N^{-1}\hat{N}_t$$

where

$$Q \equiv H_X - G_X G_N^{-1} H_N$$

Thus, I construct financial and productivity shocks as the industry-level fluctuations in output and employment, filtered for the effects of credit and input-output linkages in propagating them from industry to industry.

Figure 8 shows the time series of the estimated financial and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

From the figure, we can see that the changes in auto manufacturing's financial and productivity both fluctuate moderately around zero for most of the sample period. Between 2007 and 2009, the liquidity available to this industry took a sharp drop for a number of consecutive quarters, reaching up to a 25 percent decline. Over this period, the industry's output and employment experienced a large drop attributable to changes in the labor wedge of the industry. Given the credit linkages, the model is able to trace how much of the drop in th ewedge is due to a direct financial shock to auto manufacturing versus shocks to other industries being transmitted to it. The blue line plotted in the figure reflect the direct financial shocks experienced each quarter by the industry.

In addition, the TFP of the industry seems to have not fluctuated greatly over this recessionary period; in fact, it increased slightly. These features broadly hold across most industries in industrial production. The aggregate effects of these features and their interpretation will be discussed in subsequent sections.

6.4 Dynamic Factor Analysis

Next, I decompose the financial and productivity shocks, B_t and z_t, into an aggregate and industry-level shock. I assume that each may be described by a common component and a residual idiosyncratic component.

$$B_t = \Lambda_B F_t^B + u_t$$

$$z_t = \Lambda_z F_t^z + v_t$$

Here, F_t^B and F_t^z are scalars denoting the common factors affecting the output and employment growth of each industry, respectively, at quarter t. I interpret these factors as aggregate liquidty and productivity shocks, respectively. The M-by-1 vectors Λ_B and Λ_z denote the factor loadings, and map the aggregate shocks into each industry's financial and productivity shocks. Together, $\Lambda_B F_t^B$ and $\Lambda_z F_t^z$ comprise the aggregate components of B_t and z_t.

The residual components, u_t and v_t, unexplained by the common factors, are the idiosyncratic or industry-level shocks affecting each industry's financial and productivity growth. I assume that $\left(F_t^B, u_t\right)$ and $\left(F_t^B, u_t\right)$ are each serially uncorrelated, F_t^B, u_t, F_t^z, and v_t are mutually uncorrelated, and the variance-covariance matrices of u_t and v_t, Σ_{uu} and Σ_{vv}, are diagonal.

I suppose further that the factors follow an AR(1) process such that

$$F_t^B = \gamma_B F_{t-1}^B + \psi_t^B$$

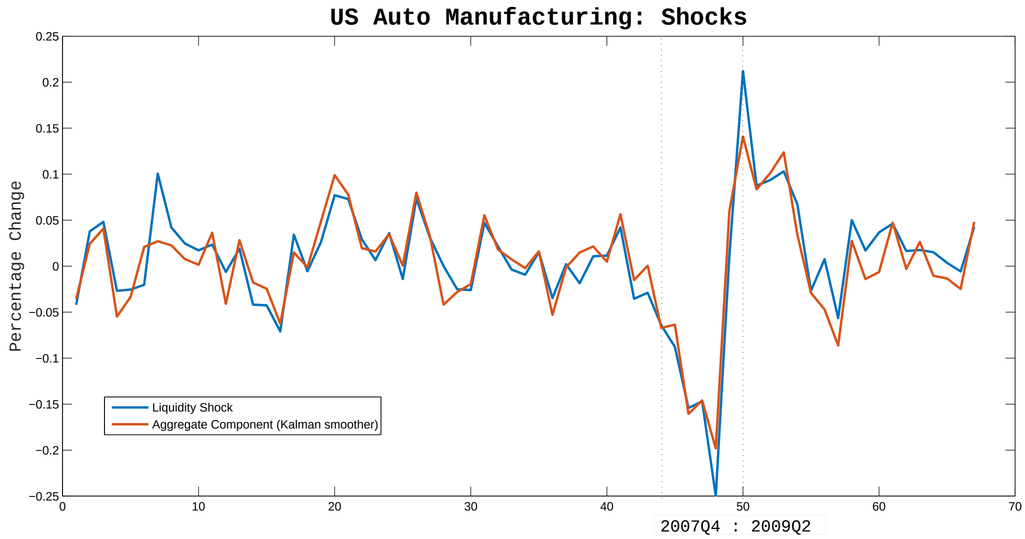
$$F_t^z = \gamma_z F_{t-1}^z + \psi_t^z$$

Here, ψ_t^B and ψ_t^z are independently and identically distributed. Hence, I have two dynamic factor models; one for the financial shocks B_t and one for the productivity shocks z_t.

Use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more infortmation in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

Figure 9 plos the time series for the estimated financial shocks and their aggregate components

Figure 9:



for the auto manufacturing industry over the full sample period. The aggregate component explain most of the financial shocks suffered by auto manufacturing. These features are fairly representative of those in other industries. A similar decomposition for the productivity shocks to auto manufacturing is given in Appendix A13.

7 Aggregate financial shocks and the Excess Bond Premium

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks estimated using the model with a measure of the risk-bearing capacity of the US financial sector - namely, the excess bond premium of Gilchrist and Zakrajsek (2012). If the aggregate shocks estimated using my model are indeed picking up financial shocks to each industry, then the two time series should exhibit a negative correlation. The correlation between these two time series turns out to be -0.51.[17] This is suggestive evidence that the aggregate financial shocks picked up by the model are indeed reflective of changes in the supply of credit from the financial sector to the IP industries.

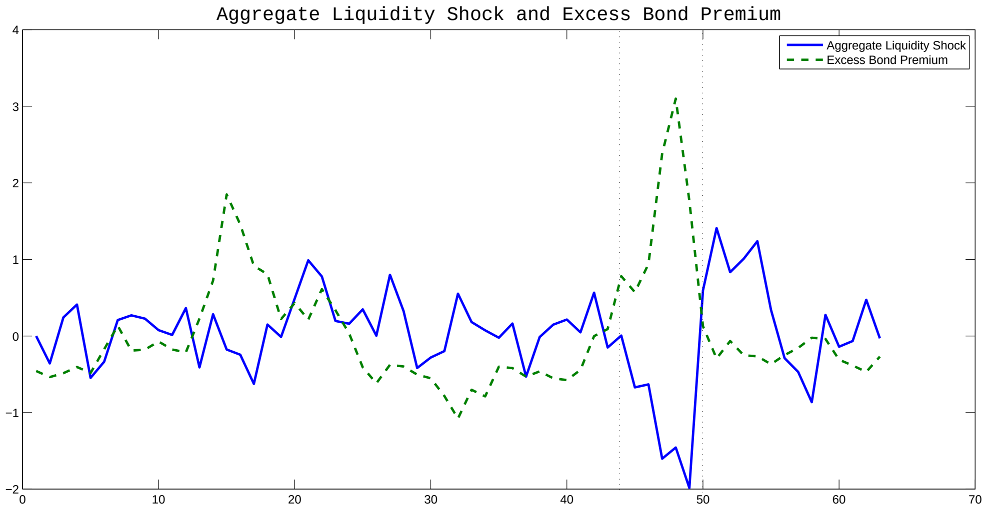
From Figure 10, we can see that, toward the end of 2007, the model picks up a huge aggregate financial shock, of about -20 percent. Simultaneously, the excess bond premium exhibits a huge spike, suggesting that the estimated aggregate liqudity shock captures the severe credit crunch that occurred during the financial crisis.

8 Empirical Results

I now present and discuss the empirical results using the shocks estimated in the previous sections.

[17] This finding is robust to using different methods to estimate the factors (e.g. Kalman smoother, one-step prediction procedure). The correlation ranges between -0.44 and -0.51.

Figure 10:



8.1 Aggregate Volatility Before the Great Recession

In this section, I use the shocks estimated in the previous section to estimate how much of observed volatility in aggregate industrial production from 1997Q1:2013Q4 can be explained by each type of shock. In addition, I estimate the contribution of the credit network of the US industrial production industries to aggregate volatility. What follows is a brief summary of the procedure; a more detailed description is given in Appendix A11.

Let the variance-covariance matrix of industry output growth X_t be denoted by Σ_{XX}. In addition, let $\bar{s}$ denote the M-by-1 vector of industry shares of aggregate output during the median year of my sample, 2005. Since these shares are close to constant across the quarters in my sample, the volatility of aggregate industrial output - henceforth *aggregate volatility* - can be approximated by σ^2, where

$$\sigma^2 \equiv \bar{s}'\Sigma_{XX}\bar{s}$$

The factor model described above implies the following identities for the variance-covariance matrices of output growth X_t and those of the shocks B_t and z_t.

$$\Sigma_{XX} = G_X\Sigma_{BB}G_X' + H_X\Sigma_{zz}H_X'$$

$$\Sigma_{BB} = \Lambda_B\Sigma_{FF}^B\Lambda_B' + \Sigma_{uu} \qquad \Sigma_{zz} = \Lambda_z\Sigma_{FF}^z\Lambda_z' + \Sigma_{vv}$$

The fraction of observed aggregate volatility generated by aggregate financial shocks can be computed as the ratio of volatility generated by the aggregate component of B_t to σ^2.

$$\frac{\bar{s}'G_X\left(\Lambda_B\Sigma_{FF}^B\Lambda_B'\right)G_X'\bar{s}}{\sigma^2}$$

Table 1: Pre-Recession Composition of Agg. Vol.: 1997Q1:2006Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	.365
Agg. Component	.133
Idios. Component	.232
financial shocks	.635
Agg. Component	.45
Idios. Component	.185

I estimate the above variance-covariance matrices Σ_{BB} and Σ_{zz} using the estimated financial and productivity shocks B_t and z_t. Similarly, I estimate the variance-covariance matrices of the factors and idiosyncratic shocks using the predicted factors from my factor esimtation, imposing that Σ_{uu} and Σ_{vv} are diagonal matrices. I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19%.[18]

The results of this analysis are summarized in Table (3). The results indicate that, before the Great Recession, aggregate volatility was driven primarily by aggregate financial shocks and idiosyncratic productivity shocks; aggregate financial shocks account for nearly a half of aggregate volatility. On the other hand, there appears to be only a minor role for aggregate productivity shocks in generating aggregate fluctuations, accounting for only about 13 percent. Nevertheless, idiosyncratic productivity shocks still play an important role, accounting for a quarter of aggregate volatility. Note that idiosyncratic productivity shocks do not average out precisely because of the input-output linkages connecting industries. Together, idiosyncratic productivity shocks and aggregate financial shocks account for nearly three-quarters of aggregate volatility during this period.

Next, I evaluate the role of the credit network of industrial production in aggregate volatility. Recall from the quantitative analysis that the credit network amplifies shocks by transmitting them across industries. How much of the observed aggregate volatility in industrial production can be accounted for by the credit network amplifying the estimated shocks? The results are summarized in Table (4). Overall, the credit network accounts for nearly one-fifth of aggregate volatility. Put differently, in the absence of the credit linkage channel of propagation, aggregate volatility from 1997-2006 would be 17 percent lower. As was discussed in the theoretical analysis, the credit network primarily propagates financial shocks. Indeed, most of the effect of the credit network is in amplifying the aggregate financial shock.

In summary, the main results of this analysis are that, when taking into account the credit linkages between industries,

[18]This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.

Table 2: Contribution of Credit Network

	Contribution of Credit Network
Effect of Prod. Shocks On Agg. Vol.	.019
Effect of Liq. Shocks On Agg. Vol.	.211
Total Agg. Volatility	.171

1. Aggregate productivity shocks *do not* play an important role in aggregate fluctuations in industrial production
2. Aggregate volatility is driven primarily by *idiosyncratic productivity* shocks and *aggregate financial shock*s
3. The credit network of the economy plays an important role in amplifying fluctuations in aggregate output

How does this compare to the findings of studies? Foerster et al. (2011) show that, when accounting for the effects of input-output linkages in propagating shocks across industries, the role of aggregate productivity shocks in driving the business cycle is diminished; more of aggregate volatility in IP can be explained by industry-level productivity shocks. Nevertheless, they still find a quantitatively large role for aggregate productivity shocks. On the other hand, my analysis shows that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate *financial shock*s seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented.

8.2 Great Recession

In this section, I perform an accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production during the Great Recession each type of shock can explain. To perform this accounting exercise, I do the following. I first restrict the sample to the time in which the peak-to-trough decline in aggregate IP occurred: 2007Q4: 2009Q2. For each quarter during this period, I use the estimated shocks to decompose the drop in aggregate IP into components arising from each type of shock. For each quarter, this yields a breakdown of the quarterly decline in aggregate IP across each shock. I then take a weighted sum of these breakdowns across quarters. I weight each quarterly breakdown by the fraction of the peak-to-trough decline in aggregate IP accounted for by each quarter. This yields a weighted average breakdown, describing, on average, how much of the total peak-to-trough decline in aggregate IP that occurred during the Great Recession can be accounted for by each type of shock.

I find that both aggregate and idiosyncratic productivity shocks were on average slightly positive during this period. As such, changes in productivity did not contribute to the decline in aggregate IP during the recession. On the contrary, the observed movements in aggregate IP can be accounted for by financial shocks. I find that 73 percent of the drop in aggregate IP is due to an adverse aggregate financial shock. This is natural given the financial crisis that occurred during the beginning of the recession.

Of the remaining 27 percent not explained by the aggregate financial shock, idiosyncratic financial shocks to the three most systemically important industries can account for a sizable fraction. Idiosyncratic shocks to the oil and coal products manufacturing, chemical products manufacturing, and auto manufacturing industries account for between one-third and all of the remaining decline in aggregate IP, despite comprising only about 25 percent of aggregate IP. This suggests that idiosynratic financial shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for such shocks. This paper, however, finds a minimal role for aggregate productivity shocks in the US business cycle, but a vital one for aggregate financial shocks. Hence, my results suggest that when one accounts for the effects of credit linkages between industries, aggregate financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle. This finding is in line both with growing evidence on the importance of the financial sector for real activity, and with standard interpretations of the causes of the Great Recession. Thus, by explicitly accounting for the propagation generated by credit linkages, this paper captures the importance of financial shocks for aggregate fluctuations in the real economy.

9 Conclusion

In this paper, I showe that inter-firm lending plays an important role in business cycle fluctuations. First, I introduced supplier credit into a network model of the economy. In this model, a shock to one firm's liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects. Thus, the endogenous response in cash-in-advance constraints to financial shocks is crucial for how the economy behaves in response to financial shock.

To evaluate the model quantitatively, I constructed a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data. I then calibrated the model to assess the quantitative importance of this propagation mechanism. I found that, in response to an aggregate financial shock, the credit network of the US economy accounts for 22 percent of the fall in GDP. I also showed that US industries which are important suppliers of intermediate goods are also more vulnerable to nonpayment by their customers. This feature of the US economy exacerbates the aggregate impact of liqudity shocks.

Finally, I investigated which shocks drive the US business cycle when we account for the credit linkages between industries. To do so, I used a structural factor approach to estimate the contribution of productivity and financial shocks to observed aggregate fluctuations in US industrial production

(IP) from 1997-2013, and find that these fluctuations were driven primarily by aggregate financial shocks and idiosyncratic productivity shocks. During the Great Recession, productivity shocks played a minimal role; rather, most of the drop in aggregate IP was driven by an aggregate financial shock. Thus, by explicitly accounting for the credit linkages between industries, this paper quantitatively and empirically captures the importance of financial shocks for US business cycle fluctuations.

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Appendix

A1. Agency Problem

Each firm has some ability to enforce debt repayment. Firm i can pledge at most $\theta_{i,i-1}$ fraction of its end-of-period revenue repay the trade credit τ_{i-1} from its supplier. If the firm repudiates the trade credit contract, it loses $\theta_i p_i x_i$, and keeps the remainder for itself. To protect itself from this possibility, the supplier $i-1$ takes care that the loan size does not incentivize its customer to repudiate the contract. Thus, there is an incentive constraint on the trade credit from $i-1$ to i, which states that the payoff to i of repudiating the contract does not exceed the payoff of collecting all of its receivables and paying the trade debt.

$$(1-\theta_i)p_i x_i \leq p_i x_i - \tau_{i-1}$$

Rearranging terms, I call this firm i's *borrowing constraint.*

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{14}$$

Since trade credit must satisfy this constraint, firms never have enough incentive to default on their trade credit contracts in equilibrium.

A2. Borrowing and Lending Decisions

(The following results hold for an infinite horizon setting). Let δ denote the rate at which the firm discounts end-of-period payouts, and let r^R, r^P, and r^B respectively denote the interest rates on accounts receivables, accounts payables, and bank debt. Let d_{i0} and d_{i1} respectively denote the payout to the firm at the beginning and end of the period. It follows that

$$d_{i0} = p_i x_i - \tau_i - (p_{i-1} x_{i-1} - \tau_{i-1}) - w n_i + b_i$$

$$d_{i1} = \tau_i \left(1 + r^R\right) - \tau_{i-1} \left(1 + r^P\right) - b_i \left(1 + r^B\right)$$

At the beginning of the period, the firm must make a net cash payment of $p_{i-1} x_{i-1} - \tau_{i-1}$ to its supplier, and $w n_i$ to the household. In addition, it receives a cash-in-advance payment of $p_i x_i - \tau_i$ from its customer along with a cash loan b_i from the bank. At the end of the period, firm i must repay its loans with interest, and receives repayment for its trade credit with interest.

The firm's objective is to choose how much to borrow from its supplier and the bank, and how much to lend to its customer to maximize its discounted payout. Because the firm is a price-taker, it does not internalize the effect that its borrowing and lending decisions have on the demand it faces from its customer, nor does it internalize its supplier's liquidity needs. Therefore, the problem of the firm is to choose τ_{i-1}, τ_i, and b_i taking prices and quantities of output as given, to maximize its discounted payouts subject to its borrowing constraints.

$$\max_{\tau_{i-1}, \tau_i, b_i} d_{i0} + \delta d_{i1}$$

$$s.t. \;\; \tau_{i-1} \leq \theta_i p_i x_i$$

$$b_i \leq B_i p_i x_i + \alpha \tau_i$$

The firm's first order conditions are

$$\delta \left(1 + r^R\right) + \alpha \lambda^B = 1$$

$$\delta \left(1 + r^P\right) + \lambda^\tau = 1$$

$$\delta\left(1+r^B\right)+\lambda^B=1$$

where λ^τ and λ^B denote the shadow values of trade and bank credit, respectively. Suppose that the interest rates for accounts payables and receivables are equal, so that $r^R=r^P$. Then it follows that

$$\lambda^\tau=\alpha\lambda^B$$

Because $\alpha\geq 0$, we have that $\lambda^\tau>0$ if and only if $\lambda^B>0$. In words, firm i's trade credit borrowing constraint is binding if and only if its bank credit constraint is binding.

Now suppose that $\frac{1}{\delta}>1+r^B$. (A sufficient condition for this is that $r^B=0$ and $\delta<1$.) This implies that $\lambda^B>0$. Intuitively, if the interest rate on bank debt is sufficiently low, then the firm will always want to borrow more from the bank, and the bank borrowing constraint always binds. In equilibrium, all firms are maxing out their bank credit, and borrowing the maximum from their suppliers and lending the maximum to their customers. Under these conditions, each firm will want to borrow the maximum from its supplier and lend the maximum to its customer because the interest paid on trade debt is same as that received on trade credit, and the firm can relax its bank borrowing constraint because trade credit is collateralizable.

In reality, firms often collateralize their accounts receivable to borrow from financial intermediaries. Burkart and Ellingsen (2004) also find that this assumption is critical to explain why liquidity-constrained firms often grant delayed payment terms to their customers. Omiccioli (2005) examines empirically the prevalence of collateralizing trade credit using data on Italian firms. In the remainder of the paper, I assume that $\delta<1$ and $r^R=r^P=r^B=0$ for simplicity. These conditions are sufficient that all firm borrowing constraints bind in equilibrium.

Nevertheless, binding borrowing constraints are not critical for the results. The qualitative results go through for sufficiently large financial shocks, even when borrowing constraints are not binding in equilibrium. As for the quantitative results, I assess how sensitive they are to these assumptions by checking their robustness to varying the value of α. The results are summarized in the text.

A3. Simple Model Solution

Solved in closed-form recursively, starting with the final firm in the chain, firm M.

Firm M

Recall that firm M collects none of its sales from the household up front (does not give the household any trade credit, \tau_M=0). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from M-1, to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

$$\max_{n_M,x_{M-1},b_M,\tau_{M-1}}\quad p_Mx_M-wn_M-p_{M-1}x_{M-1}$$

$$s.t.\;\; wn_M+p_{M-1}x_{M-1}\leq b_M+\tau_{M-1}+p_Mx_M-\tau_M$$

$$b_M \leq B_M p_M x_M + \alpha \tau_M$$

$$\tau_{M-1} p_{M-1} x_{M-1} \leq \theta_{M,M-1} p_M x_M$$

Recall that the firm does not collect any cash-in-advance from the household, so that its trade credit $\tau_M = 0$. Also recall that its borrowing constraints () and () bind in equilibrium, so that the problem can be rewritten

$$\max_{n_M, x_{M-1}, b_M, \tau_{M-1}} \; p_M x_M - w n_M - p_{M-1} x_{M-1}$$

$$s.t. \;\; w n_M + p_{M-1} x_{M-1} \leq \chi_M p_M x_M$$

where

$$\chi_M = \theta_{M,M-1} + B_M$$

Notice that χ_M is given by exogenous parameters.

If firm M is unconstrained in equilibrium, then the optimality conditions equate the marginal cost of each type of input with the marginal revenue.

$$w = \eta_M \frac{p_M x_M}{n_M} \tag{15}$$

$$p_{M-1} = (1 - \eta_M) \frac{p_M x_M}{x_{M-1}} \tag{16}$$

Firm M's expenditure in inputs is then

$$w n_M + p_{M-1} x_{M-1} = (\eta_M + (1 - \eta_M))\, p_M x_M \tag{17}$$

Then firm 3 is then unconstrained in equilibrium if and only if its expenditure at its unconstrained optimum is less than its liquidity at this optimum.

$$p_M x_M < \chi_M p_M x_M \tag{18}$$

i.e.

$$\chi_M > 1$$

If firm M is constrained in equilibrium, then its binding cash-in-advance pins down its level of output. The only choice left to make is how much labor to hire n_M versus how much intermediate goods x_{M-1} to purchase, given its level of output x_M. Because χ_M is independent of M's choice of n_M and x_{M-1}, the problem of maximizing profits subject to the binding cash-in-advance is equivalent to minimizing its expenditure $n_M + x_{M-1}$ subject to producing x_M. Thus, it solves the following cost-minimization problem.

$$min_{n_M,\, x_{M-1}}\, wn_M + p_{M-1}x_{M-1}$$

$$s.t.\ x_M = z_M n_M^{\eta_M} x_{M-1}^{(1-\eta_M)}$$

Then firm M's optimality condition equates the ratio of expenditure on each input with the ratio of each input's share in production.

$$\frac{wn_M}{p_{M-1}x_{M-1}} = \frac{\eta_M}{(1-\eta_M)} \tag{19}$$

Using this, we can rewrite M's binding cash-in-advance as

$$wn_M\left(1 + \frac{(1-\eta_M)}{\eta_M}\right) = \chi_M p_M x_M \tag{20}$$

Rearranging yields

$$w = \eta_M \chi_M \frac{p_M x_M}{n_M}$$

Combining () with its analog () in the unconstrained case, we can see that

- if $\chi_M > r_M$ (i.e. if firm i is unconstrained in equilibrium)

$$w = \eta_M \frac{p_M x_M}{n_M}$$

- otherwise

$$w = \eta_M \chi_M \frac{p_M x_M}{n_M}$$

These two cases imply that we can write

$$w = \phi_M \eta_M \frac{p_M x_M}{n_M} \tag{21}$$

where

$$\phi_M \equiv min\,\{1,\ \chi_M\}$$

ϕ_M represents the distortion in firm M's optimal labor usage due to its cash-in-advance. Financial frictions introduce wedge between firm's marginal benefit and cost of production. The wedge between these two objects is increasing in the tightness χ_M of M's constraint, and decreasing in the returns-to-scale of firm M's production function.

Firm M-1

Given firm M's solution, we can proceed to firm M-1's problem.

$$max_{n_{M-1},x_{M-2},\tau_{M-2}}\ p_{M-1}x_{M-1} - wn_{M-1} - p_{M-2}x_{M-2}$$

$$s.t.\ \ wn_{M-1} + p_{M-2}x_{M-2} \leq \chi_{M-1}p_{M-1}x_{M-1}$$

where

$$\chi_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - (1-\alpha)\frac{\tau_M}{p_{M-1}x_{M-1}}$$

The binding borrowing constraint implies

$$\chi_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - (1-\alpha)\frac{\theta_{M,M-1}p_M x_M}{p_{M-1}x_{M-1}}$$

And () and () imply that $\frac{p_M x_M}{p_{M-1}x_{M-1}} = \frac{1}{\phi_M \omega_{M,M-1}(1-\eta_M)}$. Therefore,

$$\chi_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - \alpha\frac{\theta_{M,M-1}}{\phi_M(1-\eta_M)}$$

Since ϕ_M is given by (), this is a closed-form expression for χ_{M-1}. Note that, since ϕ_M depends on χ_M, χ_{M-1} is an increasing function of χ_M; this interdependence of cash-in-advances comes from the trade credit relationship between M and M-1.

Given χ_{M-1}, the solution to firm M-1's problem takes the same form as that of firm M. (Note that χ_{M-1} does not depend directly on M-1's choice of n_{M-1} versus x_{M-2}. Therefore, when constrained in equilibrium, M-1 will solve the analogous cost-minimization problem as M to maximize profits.) The cash-in-advance places a wedge ϕ_{M-1} between the marginal benefit of hiring labor and the marginal cost

$$w = \phi_{M-1}\eta_{M-1}\frac{p_{M-1}x_{M-1}}{n_{M-1}}$$

Given the above expressions for χ_{M-1} and χ_M, the the wedge $\phi_{M-1} = min1, \chi_{M-1}$ is a closed-form expression.

Equilibrium: Each other firm's problem is symmetric. Continuing recursively, I obtain the closed-form solution for each firm. To summarize, I have, for each firm i

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}$$

where

$$\phi_i = min\{1,\, \chi_i\} \quad and \quad \chi_i = B_i + \theta_i + 1 - \theta_{i+1}\frac{1}{\phi_{i+1}\omega_{i,i-1}(1-\eta_i)}$$

Market clearing conditions are given by

$$C = Y \equiv x_M \,, \qquad N = \sum_{i=1}^{M} n_i$$

Given these expressions, the task is to write each n_i as a function of aggregate output x_M, starting with firm M-1. From the firm optimality conditions, we have the following three expressions:

$$wn_{M-1} = \phi_{M-1}\eta_{M-1}p_{M-1}x_{M-1}, \qquad wn_M = \phi_M \eta_M p_M x_M, \qquad p_{M-1}x_{M-1} = wn_M \frac{(1-\eta_M)}{\eta_M}$$

Combining these yields n_{M-1} as a function of x_M.

$$wn_{M-1} = \phi_M \phi_{M-1}\eta_{M-1}\omega_{M,M-1}(1-\eta_M)p_M x_M$$

Continuing recursively, we can write n_i as a function of x_M, for each i (LEFT OFF HERE)

$$wn_i = p_M x_M \left(\prod_{j=i}^{M}\phi_j\right)\left(\prod_{j=i}^{M-1}\omega_{j+1,j}(1-\eta_j)\right)\eta_i$$

The household's preferences and optimality conditions imply

$$w = \frac{V'(N)}{U'(x_M)} = x_M$$

Let good M be the numeraire. Combining () with () yields a closed-form expression for each firm's labor use.

$$n_i = \eta_i \prod_{j=i+}^{M} \omega_{j,j-1}(1-\eta_j)\phi_j$$

Recall that the production functions imply that aggregate output can be written

Then () and () yield a closed-form expression for aggregate output.

A4. Production Influence Vector

$$\bar{v} = \begin{bmatrix} v_1 & v_2 & v_3 & \cdots & v_M \\ 0 & v_1 & v_2 & & \\ 0 & 0 & v_1 & & \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & & v_1 \end{bmatrix} 1_{Mx1}$$

v_i=$\tilde{\eta}_i$ captures downstream propagation (supply effects). But misses upstream demand effects. Total effect is sum $\sum_{j=1}^{i} v_i$

$$v' = \left[\eta_1 \prod_{k=2}^{M} (1-\eta_k)\, \omega_{k,k-1} \quad \cdots \quad \eta_j \prod_{k=j+1}^{M} (1-\eta_k)\omega_{k,k-1} \quad \cdots \quad \eta_M \right] =$$

A5. Proof of Proposition 1

Proof: *From the definition of χ_i (4) and the unnumbered equation after (6), we have*

$$\phi_i = min \left\{ 1,\ \frac{1}{r_i} \left(B_i + \theta_{i,i-1} - \theta_{i+1,i} \frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})} \right) \right\}$$

Here, $r_i = 1$ denotes firm i's returns-to-scale. It follows that

$$\frac{d\,\phi_{i-1}}{d\,B_i} = \begin{cases} \frac{1}{r_i} \frac{\alpha\theta_{i,i-1}}{\phi_i\omega_{i,i-1}(1-\eta_i)} > 0 & if\ \phi_{i-1} < 1 \\ 0 & otherwise \end{cases}$$

$$\frac{d\,\phi_j}{d\,B_i} = 0\ \forall\, j > i \quad and \quad \frac{d\,\phi_j}{d\,B_i} = \frac{1}{r_i} > 0\ for\ j = i$$

Putting these cases together, we can write $\frac{d\,log\,\phi_j}{d\,B_i}$ for any j.

$$\frac{d\,log\,\phi_j}{d\,B_i} = \begin{cases} \frac{1}{r_i} > 0 & if\ j = i \\ \frac{1}{\phi_j} \frac{1}{r_j} \frac{\theta_{kj}}{\phi_k\omega_{kj}(1-\eta_k)} \frac{d\,\phi_k}{d\,B_i} \geq 0\ \forall\, k & if\ j < i \\ 0 & otherwise \end{cases}$$

It follows that $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ and $\frac{d}{d\,\theta_{ij}} \left(\frac{d\,log\,\phi_j}{d\,B_i} \right) \geq 0$.

A6. Solution Procedure in General Model

Claim: solution procedure takes same form in general model as in stylized.

Firm i's problem is to maximize profits subject to its cash-in-advance.

$$max_{n_i, \{x_{is}\}_{s \epsilon I}}\ \ p_i x_i - w n_i - \sum_{s=1}^{M} p_s x_{is}$$

$$w n_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i$$

where χ_i denotes the tightness of i's cash-in-advance.

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

If firm i is unconstrained in equilibrium, i . Consider the case when i is constrained in equilibrium. For profit maximization to be equivalent to minimizing its expenditure subject to producing x_i, we must have that χ_i is independent of i's choice of n_i and x_{is} for each s (or that firm i does not internalize these effects). First, suppose that χ_i is independent of this choice. I will later verify that this indeed the case.

Firm i's solution takes the same form as in the simple version of the model. The equilibrium system of $M^2 + 5M + 2$ nonlinear equations (for every i and j)

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i}$$

$$\phi_i = min \left\{ 1 \, , \; \frac{1}{r_i} \left(B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \right) \right\}$$

$$\sum_{i=1}^{M} c_i^{\beta_i} = N^{1+\epsilon}$$

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad\qquad x_{ij} = \phi_i \left(1 - \eta_i\right) \omega_{ij} \frac{p_i}{p_j} x_i$$

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \qquad p_1 = 1$$

$$N = \sum_{i=1}^{M} n_i \qquad\qquad x_i = c_i + \sum_{c=1}^{M} x_{ci}$$

$M^2 + 5M + 2$ unknowns

$$\left\{ \left\{ n_i, \, c_i \, , x_i, \, \left\{ x_{ij} \right\}_{j \epsilon I}, \, \phi_i, \, p_i \right\}_{i \epsilon I} , \, N, \, w \right\}$$

I now verify that χ_i is independent of i's choice of n_i and x_{is} for all s. Note that

$$\frac{p_c x_c}{p_i x_i} = \frac{p_c x_c}{p_i x_{ci}} \frac{p_i x_{ci}}{p_i x_i} = \frac{\theta_{ci}}{\phi_c \left(1 - \eta_c\right) \omega_{ci}} \nu_{ci}$$

where the second equality follows from firm c's optimality condition for intermediate good i, and from the definition of ν_{ci}. The term $\frac{1}{\phi_c (1-\eta_c) \omega_{ci}}$ represents the inverse of firm c's demand for good i, and is independent of i's choice of n_i versus x_{is}. The term ν_{ci} represents firm c's share of i's total output, and is determined by each customer c's optimal behavior. Thus, firm i's choice of intermediates vs labor doesn't (directly) affect χ_i. This verifies the conjecture that, when constrained, profit maximization is equivalent to expenditure minimization.

A7. Log-Linearized System

Stars are point around which system is approximated. Calibrated equilibrium values.

For all i and j

In order: firm i's optimality condition for input j, firm i's optimality condition for labor, definition of wedge phi_i, household optimality condition for consumption of each good, market clearing for good i, production function for firm i, household budget constraint, labor market clearing condition, household optimality for labor versus aggregate consumption.

$$\tilde{p}_j + \tilde{x}_{ij} = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \qquad \tilde{w} + \tilde{n}_i = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \qquad \tilde{\phi}_i = \begin{cases} \tilde{\phi_i}^c & if\ \phi_i < 1 \\ 0 & otherwise \end{cases}$$

$$\tilde{\phi_i}^c = \frac{B_i}{r_i\phi_i}\tilde{B}_i + \frac{\alpha}{r_i\phi_i}\sum_{c=1}^{M}\frac{\theta_{ci}\nu_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\tilde{\phi}_c - \frac{\alpha}{r_i\phi_i}\sum_{c=1}^{M}\frac{\theta_{ci}\nu_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\tilde{\nu}_{ci}$$

$$\tilde{p}_i + \tilde{c}_i = \tilde{p}_j + \tilde{c}_j \qquad \tilde{x}_i = \left(\frac{p_i c_i^*}{p_i x_i^*}\right)\tilde{c}_i + \sum_c \left(\frac{p_i x_{ci}^*}{p_i x_i^*}\right)\tilde{x}_{ci} \qquad \tilde{x}_i = \tilde{z}_i + \eta_i\tilde{n}_i + (1-\eta_i)\sum_s \omega_{is}\tilde{x}_{is}$$

$$\tilde{w} = \sum_i \beta_i\left(\tilde{c}_i + \tilde{p}_i\right) \qquad \sum_i \left(\frac{n_i^*}{N}\right)\tilde{n}_i = 0 \qquad (1+\epsilon)\,\tilde{N} = \sum_i \beta_i\tilde{c}_i$$

A8. Counterfactual

Recall the definition of ϕ_i

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \sum_{s=1}^{M}\theta_{is} + 1 - \alpha\sum_{c=1}^{M}\theta_{ci}\frac{p_c x_c}{p_i x_i}\right)\right\}$$

Replace $\frac{p_c x_c}{p_i x_i}$ with firm c's optimality conditions for good i yields

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \sum_{s=1}^{M}\theta_{is} + 1 - \alpha\sum_{c=1}^{M}\frac{\theta_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\nu_{ci}\right)\right\}$$

Log-linearizing ϕ_i yields

$$\tilde{\phi}_i = \begin{cases} \left(\frac{B_i^*}{r_i^*\phi_i^*}\right)\tilde{B}_i + \frac{\alpha}{r_i^*\phi_i^*}\sum_{c=1}^{M}\left(\frac{\theta_{ci}}{\phi_c^*(1-\eta_c)\omega_{ci}}\right)\tilde{\phi}_c & if\ \phi_i^* < 1 \\ 0 & otherwise \end{cases}$$

Thus, in the full model wedges respond endogenously to direct financial shocks B_i and to changes in its customers' wedges ϕ_c through the credit linkage channel. This second term captures the propagation due to the credit linkages between firms. In performing my counterfactual, I compute the response in GDP to athe aggregate financial shock B_tilde=.01 for all i, and then do the same by after imposing

$$\tilde{\phi}_i = \begin{cases} \left(\frac{B_i^*}{r_i^*\phi_i^*}\right)\tilde{B}_i & if\ \phi_i^* < 1 \\ 0 & otherwise \end{cases}$$

This latter exercise gives me the model's response without propagation via the credit network. Then the marginal contribution to the change in GDP of including the credit linkages is given by the difference in ...

A9. Effect of Credit Linkages in General Model

Effect of Credit Linkages in General Model. In the model the trade credit parameters θ_{cs} show up only in the wedges ϕ_i. Therefore, to see effect of credit network in propagating financial and productivity shocks, it suffices to show how ϕ_i responds to shocks to other industries. Recall

$$\phi_i = min\left\{1,\ \frac{\chi_i}{r_i}\right\}$$

where

$$\chi_i = B_i + \sum_{s=1}^{M}\theta_{is} + 1 - \alpha\sum_{c=1}^{M}\theta_{ci}\frac{p_c x_c}{p_i x_i}$$

$$= B_i + \sum_{s=1}^{M}\theta_{is} + 1 - \alpha\sum_{c=1}^{M}\theta_{ci}\frac{p_c x_c}{p_i x_{ci}}\frac{x_{ci}}{x_i}$$

Let $\nu_{ci} \equiv \frac{x_{ci}}{x_i}$ represent the share of c in i's total revenue. Substituting c's optimality condition for good i in for $\frac{p_c x_c}{p_i x_{ci}}$ yields

$$\chi_i = B_i + \sum_{s=1}^{M}\theta_{is} + 1 - \alpha\sum_{c=1}^{M}\frac{\theta_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\nu_{ci}$$

The response in ϕ_i to some shock can be summarized by the log-linearized expression for ϕ_i.

$$\tilde{\phi}_i = \begin{cases}\tilde{\phi}_i^{\ c} & if\ \phi_i < 1\\ 0 & otherwise\end{cases}$$

where

$$\tilde{\phi}_i^{\ c} = \frac{B_i}{r_i\phi_i}\tilde{B}_i + \frac{\alpha}{r_i\phi_i}\sum_{c=1}^{M}\frac{\theta_{ci}\nu_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\tilde{\phi}_c - \frac{\alpha}{r_i\phi_i}\sum_{c=1}^{M}\frac{\theta_{ci}\nu_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\tilde{\nu}_{ci}$$

and

$$\tilde{\nu}_{ci} = \tilde{x}_{ci} - \tilde{x}_i$$

This expression says that industry i's wedge can change either from direct financial shock to i (given by $\tilde{B}_i$), changes in the wedges of customers (given by $\tilde{\phi}_i$) through credit linkages θ_{ci}, or changes in the composition of industry i's sales (given by $\tilde{\nu}_{ci}$ for all customers c), also through credit linkages.

Consider first a financial shock to industry j, given by $\tilde{B}_j < 0$. How does this affect ϕ_i, and how does this effect depend on i's credit linkages with j? From (), we can see that there are two effects. First, the shock reduces ϕ_j, so that $\tilde{\phi}_j < 0$. This pushes ϕ_i down. Second, because i has M customers,

x_{ji} falls by more than x_i falls. Therefore, j's share of i's output ν_{ji} falls, and $\tilde{\nu}_{ji} < 0$. This pushes ϕ_i up. The stronger is j's downstream credit linkage θ_{ji} with i, the stronger are both of these effects.

But there is a more indirect way by which ϕ_i changes in response to $\tilde{B}_j < 0$. The initial fall in ϕ_i is transmitted to each of i's customers c as a supply shock, causing all c to cut back on output. Then the fall in $p_c x_c$ causes ϕ_c to fall, as the amount of credit c is giving per unit of its revenue is lower. Since all industries are interconnected, industry c is also industry i's customer. As a result, the fall in ϕ_c causes ϕ_i to fall via the credit linkage from i to c. This fall in ϕ_i effect is increasing in i's downstream linkage with c θ_{ci}. Thus, the greater θ_{ci} for all c, i.e. the larger i's credit out-degree, the more that ϕ_i will respond to the shock to j, and the larger will be the aggregate impact.

Now consider an adverse productivity shock to industry j, given by $\tilde{z}_j < 0$. This shock affects neither ϕ_j nor ϕ_i directly. However, it has an indirect affect on ϕ_i through the composition of i's sales ν_{ji}. In particular j's share of i's total output ν_{ji} falls, and so $\tilde{\nu}_{ji} < 0$. This reduces the amount of trade credit per dollar of revenue that i is giving its customers, and so i's wedge increases: $\tilde{\phi}_i > 0$. This effect is increasing in i's downstream credit linkage with j, θ_{ji}. Therefore, stronger credit linkages mitigate the impact of the productivity shock. This effect is not present in the stylized model, because $\nu_{ji} = 1$ for $j = i + 1$ and 0 for all other j; there is no change in the composition of i's sales. Nevertheless, this mitigation effect is quantitatively small, as discussed in the quantitative analysis.

A10. Identification of Productivity vs. financial shocks

Recall the production functions, optimality conditions for labor use, and definition of the wedges. First, the employment and output of an industry are linked by the industry production function $x_{it} = z_{it} n_{it}^{\eta_i} \left(\prod_{s=1}^{M} x_{ist}^{\omega_{is}}\right)^{1-\eta_i}$. Therefore, a change in the TFP of industry i is given by

$$\tilde{z}_{it} = \tilde{x}_{it} - \eta_i \tilde{n}_{it} - (1 - \eta_i) \sum_{s=1}^{M} \omega_{it} \tilde{x}_{ist}$$

The constant returns-to-scale of industry i's production function implies that if an observed change in industry i's output $\tilde{x}_{it}$ from period $t-1$ to t exceeds that of $n_{it}^{\eta_i} \left(\prod_{s=1}^{M} x_{ist}^{\omega_{is}}\right)^{1-\eta_i}$, then there must have been an increase increase in i's TFP such that $\tilde{z}_{it} > 0$.

Industry i's optimality condition for labor equates the ratio of its wage bill to revenue with labor's marginal product, times the wedge, i.e. $\frac{w n_i}{p_i x_i} = \eta_i \phi_i$. In log-changes from period $t-1$ to t, this can be written as

$$\tilde{w}_t + \tilde{n}_{it} - \tilde{p}_{it} - \tilde{x}_{it} = \tilde{\phi}_{it}$$

This says that an observed change in industry i's ratio of labor expenditure to revenue from time $t-1$ to t, must have come from a change in the firm's wedge $\tilde{\phi}_{it}$ from $t-1$ to t.

Finally, recall the definition of industry i's wedge.

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \sum_{s=1}^{M}\theta_{is} + 1 - (1-\alpha)\sum_{c=1}^{M}\frac{\theta_{ci}}{\phi_c(1-\eta_c)\omega_{ci}}\nu_{ci}\right)\right\}$$

This implies that a change in industry i's wedges must be driven by changes in liquidity, either directly shock to B_i, or through credit linkages via ϕ_c. In this way, the model attributes a change in the ratio of industry i's wage bill to revenue to a financial shock. In a later section, I discuss the extent to which the model's predicted financial shocks are correlated with some industry-level measures of credit spreads, an indication of changes in liquidity conditions computed from an independent dataset.

Because the model can track how a financial shock or productivity shock to one industry spills over to other industries via their credit and input-output linkages, the model can back out exactly how much of a change in an industry's output and employment is coming from spillover effects versus a direct shock, and can identify the industry which was shocked. In this manner, for any combination of $2M$ observations $\tilde{x}_{it}$ and $\tilde{n}_{it}$, the model exactly identifies the sequence of financial and productivity shocks $\tilde{B}_{it}$ and $\tilde{z}_{it}$ faced by each industry between periods $t-1$ and t.

A11. Aggregate Volatility

Recall that the growth in industry output can be written as a function of the industry financial and productivity shocks. Recall that X_t is a vector of the percentage change $\tilde{x}_{it}$ in each industry's output at time t.

$$X_t = G_X B_t + H_X z_t$$

And the shocks B_t and z_t, in turn, are composed of an aggregate and idiosyncratic components.

$$B_t = \Lambda_B F_t^B + u_t \qquad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B$$

$$z_t = \Lambda_z F_t^z + v_t \qquad F_t^z = \gamma_z F_{t-1}^z + \iota_t^z$$

Then letting Σ_{XX} denote the variance-covariance matrix of X_t (and similarly for the other variables), we have

$$\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X'$$

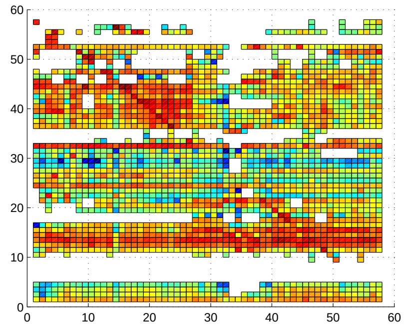
$$\Sigma_{BB} = \Lambda_B \Sigma_{FF}^B \Lambda_B' + \Sigma_{uu}$$

$$\Sigma_{zz} = \Lambda_z \Sigma_{FF}^z \Lambda_z' + \Sigma_{vv}$$

where Σ_{uu} and Σ_{vv} are diagonal matrices.

Aggregate manufacturing output at time t is defined as $\Sigma_i x_{it}$. Let $\bar{s}_t$ denote the vector of industry

Figure 11:



shares of aggregate output at time t. Then the growth of aggregate output at time t is given by

$$\bar{s}_t X_t$$

Suppose that industry shares don't fluctuate much over time, so that $\bar{s}_t \approx \bar{s}$ for all t. Then growth in aggregate output at time t can be approximated by $\bar{s}X_t$. Then the variance of aggregate output, i.e. *aggregate volatility* in the economy, is approximately given by

$$\sigma^2 \equiv \bar{s}'\Sigma_{XX}\bar{s} = \bar{s}'G_X\Sigma_{BB}G_X'\bar{s} + \bar{s}'H_X\Sigma_{zz}H_X'\bar{s}$$

Then the contribution of aggregate financial shocks to aggregate volatility is given by

$$\frac{\bar{s}'G_X\left(\Lambda_B\Sigma_{FF}^B\Lambda_B'\right)G_X'\bar{s}}{\sigma^2}$$

And the aggregate volatility generated by the credit network in propagating aggregate financial shocks is then given by

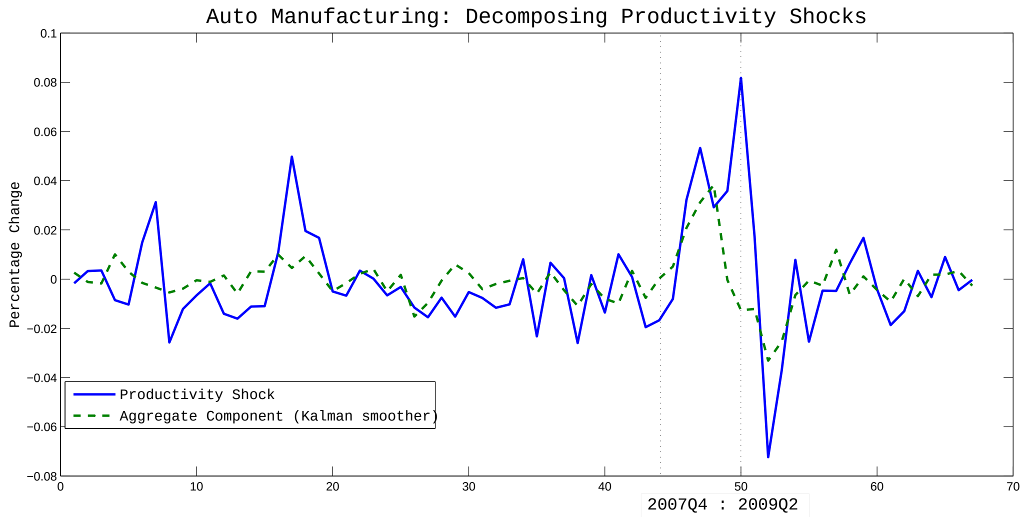
where G_{NoTC} maps B_t into X_t when the credit linkage channel is shut-off. Similar expressions can be derived for the contribution to aggregate volatility of idiosyncratic financial shocks, and aggregate and idiosyncratic financial shocks.

A12. Credit Network of US Economy

A13. Decomposition of Estimated Productivity Shocks

A14. Construction of Proxy for Inter-Industry Credit Flows

For each firm in the sample, I want a measure of its cost of goods sold (COGS) financed with accounts payable (AP) in each year t, which I call its *payables financing* ($PayFin$) at time t. Since a firm may



repay its accounts payable irregularly, simply taking the ratio $\frac{AP_t}{COGS_t}$ may in part reflect a spuriously high or low repayment of its accounts payable in that year. Therefore, I take a take a moving average of AP to smooth it over time. Thus, I compute firm f's payables financing at time t as

$$PayFin_{f,t} = \frac{.5\,(AP_{f,t-1} + AP_{f,t})}{COGS_{f,t}}$$

I do this only for years in which there is data for both AP and COGS for each firm. I obtain a firm-level measure of payables financing by taking the median of $PayFin_{f,t}$ across time, to minimize effect of outliers and get a representative firm-level estimate of the average COGS financed with trade credit. Then to get an industry-level measure of payables financing, I take the median of $PayFin_f$ across all firms f in each three-digit level NAICS industry. In this way, I obtain a measure of payables financing for each of my industries.

Raddatz (2010) uses this industry-level measure of PayFin to construct q_{ij}. However, since he only uses AP data, he must impose that $q_{ij} = q_{ik}$ for all j, k. In other words, he assumes that each industry finances the same fraction of purchases with trade credit, across all of its suppliers. This is a fairly strong assumption that he is forced to make due to the paucity of data on trade credit. However, I improve on this proxy by making use of additional data on accounts receivables to obtain a more precise and industry-pair-specific measure of q_{ij}.

In particular, I construct an industry-level measure of the fraction of total sales made on credit to customers, which I call the industry's *receivables lending* ($RecLend$), using each firm's accounts receivable (AR) and sales each year.

$$RecLend_{f,t} = \frac{.5\,(AR_{f,t-1} + AR_{f,t})}{Sales_{f,t}}$$

I then aggregate across time and across firms in each industry to obtain an industry-level measure of receivables lending.

The measure $PayFin_i$ tells me how much trade credit each industry i receives from all of its suppliers collectively; it does not tell me how this breaks down across each of its suppliers. Similarly, $RecLend_i$ tells me how much trade credit each industry i gives to all of its customers collectively; it does not tell me how this breaks down across each of its customers. Therefore, to construct q_{ij} the fraction of industry j's sales to industry i made on trade credit, I take a weighted average of $PayFin_i$ and $RecLend_j$. In the next section, I consider two weighting schemes and compare their aggregate accuracy. My baseline proxy uses weights given by each industry's total sales.

$$\hat{q}_{ij} = k_{ij} PayFin_i + k_{ji} RecLend_j \ , \qquad k_{ij} \equiv \frac{p_i x_i}{p_i x_i + p_j x_j}$$

Therefore, a larger industry will carry more weight in determining the trade credit flows to and from it. Alternative weighting schemes, such as equal weights to both cusomer and supplier, do not significantly alter the results. Given my proxy $\hat{q}_{ij}$, inter-industry trade credit flows are then proxied as

$$\hat{\tau}_{ij} = \hat{q}_{ij} p_j x_{ij}$$

In the Appendix, I discuss the conditions under which the weighting scheme k_{ij} described above is optimal for constructing the proxy of trade credit flows. Intuitively, this weighting scheme minimizes the mean squared errors in the observed accounts payables of each industry, when I impose the restriction that any given industry's lending or borrowing not vary greatly across suppliers or customers.