Technical Appendix

TA.1. System Reduction

Here, I derive the closed-form vector equations (28), describing how the endogenous variables respond to financial and productivity shocks to each industry, to a first-order approximation. Given system of log-linearized equations, we want to reduce the system of $3M^2 + 5M + 2$ equations to one of M flows and M driving processes.

First, define some notation.$\forall\, z_{it}$, let z_t be an M-by-1 vector $z_t \equiv \begin{bmatrix} \tilde{z}_{1t} \\ \vdots \\ \tilde{z}_{Mt} \end{bmatrix}$. And $\forall\, z_{ijt}$ let z_t by a M^2-by-1 vector such that

$$z_t \equiv \begin{bmatrix} \tilde{z}_{11t} \\ \tilde{z}_{21t} \\ \vdots \\ \tilde{z}_{M1t} \\ \tilde{z}_{12t} \\ \tilde{z}_{22t} \\ \vdots \\ \vdots \\ \tilde{z}_{MMt} \end{bmatrix}$$

Let industry output be denoted by y_i instead of x_i, so that x_{ij} unambiguously denotes intermediate goods use. Let I_M denote the M-by-M identity matrix, and $1_{Mx1} \bigotimes I_M$ and $I_M \bigotimes 1_{Mx1}$ be M^2-by-M matrices of zeros and ones such that

$$1_{Mx1} \bigotimes I_M \equiv \begin{bmatrix} I_M \\ \vdots \\ I_M \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \\ \\ \\ \\ 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_M \bigotimes 1_{Mx1} \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 1 & & 0 \\ & \vdots & \vdots & \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 1 \end{bmatrix}$$

Let p_{num} and c_{cum} respectively denote 1-by-1 values for the price and household consumption of the numeraire good in equilibrium at t. (Note that by definition, the price of the numeraire doesn't change; hence $p_{num,t} = 0 \; \forall \, t$). Let $diag(\cdot)$ be the diagonal operator, putting its vector argument into the diagonal of a square matrix of zeros. Let $x \bullet y$ denote the element-wise product of matrices x and y. Let starred variables denote observed or calibrated coefficients; e.g. $\frac{p_i c_i^*}{p_i x_i^*}$ denotes the share of consumption of industry i's total output, observed from input-output tables.

Re-write equilibrium system of log-linear equations using the above notation.

$$x_t = \Omega_{x\phi}\phi_t + \Omega_{xy} y_t + \Omega_{xp} p_t \tag{1}$$

$$\tilde{w}_t \vec{1}_{Mx1} + n_t = \phi_t + p_t + y_t \tag{2}$$

$$p_t + c_t = p_{num,t}\vec{1}_{Mx1} + c_{num,t}\vec{1}_{Mx1} \tag{3}$$

$$\ell_t = \Omega_{\ell p} p_t + \Omega_{\ell y} y_t - x_t \tag{4}$$

$$\nu_t = x_t - \Omega_{\nu y} y_t \tag{5}$$

$$\phi_t = \Omega_{\phi B} B_t - \Omega_{\phi \nu}\nu_t - \Omega_{\phi \ell}\ell_t \tag{6}$$

$$y_t = \Omega_{yc} c_t + \Omega_{yx} x_t \tag{7}$$

$$y_t = z_t + \Omega_{yn} n_t + J_{yx} x_t \tag{8}$$

$$. \tag{9}$$

$$\tilde{N}_t = \Omega_{Nn} n_t \tag{10}$$

$$(1+\epsilon)\,\tilde{N}_t = \Omega_{Nc} c_t \tag{11}$$

where matrix and vector coefficients are given by

$$\Omega_{x\phi} \equiv 1_{Mx1} \bigotimes I_M \quad \Omega_{xy} \equiv 1_{Mx1} \bigotimes I_M \quad \Omega_{xp} \equiv \left(1_{Mx1} \bigotimes I_M - I_M \bigotimes 1_{Mx1}\right)$$

$$\Omega_{\ell p} \equiv \left(1_{Mx1} \bigotimes I_M - I_M \bigotimes 1_{Mx1}\right) \quad \Omega_{\ell y} \equiv 1_{Mx1} \bigotimes I_M$$

$$\Omega_{\nu y} \equiv I_M \bigotimes 1_{Mx1}$$

$$\Omega_{\phi B} \equiv diag\left(\frac{B_i}{r_i \phi_i}^*\right) \qquad \Omega_{\phi \ell} = \Omega_{\phi \nu} \equiv \left[\alpha diag\left(\frac{\ell_{ci}\nu_{ci}}{r_i \phi_i}^*\right)\left(I_M \bigotimes 1_{Mx1}\right)\right]^T$$

$$\Omega_{yc} \equiv diag\left(\frac{p_i c_i^*}{p_i x_i^*}\right) \quad \Omega_{yx} \equiv \left[diag\left(\frac{p_i x_{ci}^*}{p_i x_i^*}\right)\left(I_M \bigotimes 1_{Mx1}\right)\right]^T$$

$$\Omega_{yn} \equiv diag\left(\eta\right) \qquad J_{yx} \equiv \left[\left(diag\left(\vec{1}_{M^2x1} - \eta^I\right) \bullet diag\left(\omega\right)\right)\left(I_M \bigotimes 1_{Mx1}\right)\right]^T$$

where $\omega = [\omega_{11}\ \omega_{21} \cdots \omega_{M1}\ \omega_{12}\ \omega_{22} \cdots\cdots \omega_{MM}]^T$ is a M^2-by-1 vector, and $\eta^I \equiv (\eta_1 \cdots \eta_M\ \eta_1 \cdots \eta_M \cdots\cdots \eta_M)^T$ is a M^2-by-1 vector ($\vec{\eta}$ stacked on itself M times).

$$\Omega_{wc} \equiv \left[diag\left(\beta\right)\vec{1}_{Mx1}\right]^T$$

$$\Omega_{Nn} \equiv \left[diag\left(\frac{n_i}{N}^*\right)\vec{1}_{Mx1}\right]^T$$

$$\Omega_{Nc} = \Omega_{wc}$$

Now reduce the system by solving it algebraically. Plug (4) and (5) into (6):

$$\phi_t = \Omega_{\phi B} B_t - \Omega_{\phi\nu}\left(x_t - \Omega_{\nu y} y_t\right) - \Omega_{\phi\ell}\left(\Omega_{\ell p} p_t + \Omega_{\ell y} y_t - x_t\right)$$

$$= \Omega_{\phi B} B_t + \left(\Omega_{\phi\ell} - \Omega_{\phi\nu}\right) x_t + \left(\Omega_{\phi\nu}\Omega_{\nu y} - \Omega_{\phi\ell}\Omega_{\ell y}\right) y_t - \Omega_{\phi\ell}\Omega_{\ell p} p_t \tag{12}$$

Noting that $p_{num,t} = 0$, replace (3) and (12) into (1).

$$x_t = \Omega_{x\phi}\Omega_{\phi B} B_t + \Omega_{x\phi}\left(\Omega_{\phi\ell} - \Omega_{\phi\nu}\right) x_t + \left[\Omega_{x\phi}\left(\Omega_{\phi\nu}\Omega_{\nu y} - \Omega_{\phi\ell}\Omega_{\ell y}\right) + \Omega_{xy}\right] y_t + \left(\Omega_{xp} - \Omega_{x\phi}\Omega_{\phi\ell}\Omega_{\ell p}\right) c_{num,t}\vec{1}_{Mx1} + \left(\Omega_{x\phi}\Omega_{\phi\ell}\Omega_{\ell p} - \Omega_{xp}\right) c_t \tag{13}$$

So

$$x_t = AB_t + Dy_t + Ec_{num,t}\vec{1}_{Mx1} + Fc_t \tag{14}$$

where

$$A \equiv \left[I_{M^2} - \Omega_{x\phi}\left(\Omega_{\phi\nu} - \Omega_{\phi\ell}\right)\right]^{-1} \Omega_{x\phi}\Omega_{\phi B} \quad D \equiv \left[I_{M^2} - \Omega_{x\phi}\left(\Omega_{\phi\nu} - \Omega_{\phi\ell}\right)\right]^{-1}\left[\Omega_{x\phi}\left(\Omega_{\phi\nu}\Omega_{\nu y} - \Omega_{\phi\ell}\Omega_{\ell y}\right) + \Omega_{xy}\right]$$

$$E \equiv \left[I_{M^2} - \Omega_{x\phi}\left(\Omega_{\phi\nu} - \Omega_{\phi\ell}\right)\right]^{-1}\left(\Omega_{xp} - \Omega_{x\phi}\Omega_{\phi\ell}\Omega_{\ell p}\right) \quad F \equiv \left[I_{M^2} - \Omega_{x\phi}\left(\Omega_{\phi\nu} - \Omega_{\phi\ell}\right)\right]^{-1}\left(\Omega_{x\phi}\Omega_{\phi\ell}\Omega_{\ell p} - \Omega_{xp}\right)$$

Also plug (3) and (12) into (2). This yields

$$\tilde{w}_t\vec{1}_{Mx1} + n_t = \Omega_{\phi B} B_t + \left(\Omega_{\phi\ell} - \Omega_{\phi\nu}\right) x_t + \left[\left(\Omega_{\phi\nu}\Omega_{\nu y} - \Omega_{\phi\ell}\Omega_{\ell y}\right) + I_M\right] y_t + \left[I_M - \Omega_{\phi\ell}\Omega_{\ell p}\right] c_{num,t}\vec{1}_{Mx1} + \left(\Omega_{\phi\ell}\Omega_{\ell p} - I_M\right) c_t$$

which implies

$$\tilde{w}_t\vec{1}_{Mx1} + n_t = \Omega_{\phi B} B_t - Gx_t + Hy_t + Jc_{num,t}\vec{1}_{Mx1} + Kc_t \tag{15}$$

where

$$G \equiv -\left(\Omega_{\phi\nu} - \Omega_{\phi\ell}\right) \quad H \equiv \left(\Omega_{\phi\nu}\Omega_{\nu y} - \Omega_{\phi\ell}\Omega_{\ell y} + I_M\right) \quad J \equiv \left(I_M - \Omega_{\phi\ell}\Omega_{\ell p}\right) \quad K \equiv \left(\Omega_{\phi\ell}\Omega_{\ell p} - I_M\right)$$

Plugging (3) into (9) yields

$$(IGNORE) \quad \tilde{w}_t = \Omega_{wc} c_{num,t} \vec{1}_{Mx1} \tag{16}$$

And plugging (10) into (11) yields

$$(1+\epsilon)\,\Omega_{Nn} n_t = \Omega_{Nc} c_t \tag{17}$$

So far, the reduced system is given by (7), (8), and (14)-(17). Now plug (14) into (7) to obtain

$$y_t = Lc_t + OB_t + Pc_{num,t}\vec{1}_{Mx1} \tag{18}$$

where

$$L \equiv [I_M - \Omega_{yx} D]^{-1} (\Omega_{yc} + \Omega_{yx} F) \quad O \equiv [I_M - \Omega_{yx} D]^{-1} \Omega_{yx} A \quad P \equiv [I_M - \Omega_{yx} D]^{-1} \Omega_{yx} E$$

Now plug (15) into (17).

$$(\Omega_{Nc} - (1+\epsilon)\,\Omega_{Nn} K)\, c_t = (1+\epsilon)\,\Omega_{Nn} \left(\Omega_{\phi B} B_t - Gx_t + Hy_t + Jc_{num,t}\vec{1}_{Mx1} - \tilde{w}_t \vec{1}_{Mx1}\right) \tag{19}$$

Plug (14) into (15).

$$n_t = (\Omega_{\phi B} - GA)\, B_t + (H - GD)\, y_t + (J - GE)\, c_{num,t}\vec{1}_{Mx1} + (K - GF)\, c_t - \tilde{w}_t \vec{1}_{Mx1} \tag{20}$$

Then plug (14) and (20) into (8).

$$y_t = Qz_t + RB_t + Sc_{num,t}\vec{1}_{Mx1} + Tc_t - U\tilde{w}_t\vec{1}_{Mx1} \tag{21}$$

where

$$Q \equiv [I_M - J_{yx} D - \Omega_{yn} (H - GD)]^{-1} \quad R \equiv Q\,[\Omega_{yn} (\Omega_{\phi B} - GA) + J_{yx} A] \quad S \equiv Q\,[\Omega_{yn} (J - GE) + J_{yx} E]$$

$$T \equiv Q\,[\Omega_{yn} (K - GF) + J_{yx} F] \quad U \equiv Q\Omega_{yn}$$

Combine (21) with (18).

$$c_t = Vz_t + WB_t + Xc_{num,t}\vec{1}_{Mx1} - A_2 \tilde{w}_t \vec{1}_{Mx1} \tag{22}$$

where

$$V \equiv [L-T]^{-1} Q \quad W \equiv [L-T]^{-1} (R-O) \quad X \equiv [L-T]^{-1} (S-P) \quad A_2 \equiv [L-T]^{-1} U$$

The goal is to make use of (22), (16), and (19) to solve for $c_{num,t}, c_t$, $\tilde{w}_t$. But first, we need to replace x_t and y_t in (19) using (18) and (14).

Now we want to re-write (19) replacing x_t and y_t using (21) and (14). First get rid of y_t in (14) using (18). This yields

$$x_t = (A + DO) B_t + (DL + F) c_t + (DP + E) c_{num,t} \vec{1}_{Mx1} \tag{23}$$

Plugging this into (19) yields

$$(\Omega_{Nc} - (1+\epsilon) \Omega_{Nn} K) c_t = (1+\epsilon) \Omega_{Nn} \left(\left[\Omega_{\phi B} - G (A + DO) \right] B_t - G (DL + F) c_t + H y_t + [J - G (DP + E)] c_{num,t} \vec{1}_{Mx1} - \tilde{w}_t \vec{1}_{Mx1} \right) \tag{24}$$

And now plugging (18) into this yields

$$(\Omega_{Nc} - (1+\epsilon) \Omega_{Nn} K) c_t = (1+\epsilon) \Omega_{Nn} \left(\left[\Omega_{\phi B} - G (A + DO) + HO \right] B_t + [HL - G (DL + F)] c_t + [HP + J - G (DP + E)] c_{num,t} \vec{1}_{Mx1} - \tilde{w}_t \vec{1}_{Mx1} \right) \tag{25}$$

This implies

$$(B_2 - E_2) c_t = D_2 B_t + F_2 c_{num,t} \vec{1}_{Mx1} - G_2 \tilde{w}_t \vec{1}_{Mx1} \tag{26}$$

where

$$G_2 \equiv (1+\epsilon) \Omega_{Nn} \quad B_2 \equiv (\Omega_{Nc} - G_2 K) \quad D_2 \equiv G_2 [\Omega_{\phi B} - G (A + DO) + HO]$$

$$E_2 \equiv G_2 [HL - G (DL + F)] \quad F_2 \equiv G_2 [HP + J - G (DP + E)]$$

I now use (22) and (26), which contain M+1 separate equations, to solve for c_t and $\tilde{w}_t$, which contain M+1 unknowns. (Recall that $c_{num,t}$ is contained in c_t). First, suppose that the numeraire good is good M, so that $\tilde{p}_{M,t} = 0$ and $\tilde{c}_{num,t} = \tilde{c}_{M,t}$. Also let V_M, $A_{2,M}$, W_M, and X_M, denote the M'th rows of the M-by-M matrices V, A_2, W, and X, respectively. Then the M'th row of (22) is given by

$$\tilde{c}_{M,t} = V_M z_t + W_M B_t + X_M \vec{1}_{Mx1} \tilde{c}_{M,t} - A_{2,M} \vec{1}_{Mx1} \tilde{w}_t$$

which implies

$$\tilde{w}_t = \frac{1}{A_{2,M}\vec{1}_{Mx1}}\left(V_M z_t + W_M B_t + \left[X_M\vec{1}_{Mx1} - 1\right]\tilde{c}_{M,t}\right) \tag{27}$$

Then (26) implies

$$(B_2 - E_2)\, c_t = D_2 B_t + \left(F_2\vec{1}_{Mx1}\right)\tilde{c}_{M,t} - \frac{G_2\vec{1}_{Mx1}}{A_{2,M}\vec{1}_{Mx1}}\left(V_M z_t + W_M B_t + \left[X_M\vec{1}_{Mx1} - 1\right]\tilde{c}_{M,t}\right)$$

Define the scalar Q_3 as

$$Q_3 \equiv \frac{G_2\vec{1}_{Mx1}}{A_{2,M}\vec{1}_{Mx1}}$$

Then the above implies

$$(B_2 - E_2)\, c_t = (D_2 - Q_3 W_M)\, B_t - Q_3 V_M z_t + \left(F_2\vec{1}_{Mx1} - Q_3\left[X_M\vec{1}_{Mx1} - 1\right]\right)\tilde{c}_{M,t}$$

which yields

$$\tilde{c}_{M,t} = \frac{1}{R_3}(B_2 - E_2)\, c_t - \frac{1}{R_3}(D_2 - Q_3 W_M)\, B_t + \frac{1}{R_3} Q_3 V_M z_t \tag{28}$$

where R_3 is a scalar.

$$R_3 \equiv F_2\vec{1}_{Mx1} - Q_3\left[X_M\vec{1}_{Mx1} - 1\right]$$

Combining (28) and (27) yields

$$\tilde{w}_t = S_3 z_t + T_3 B_t + U_3 c_t \tag{29}$$

where

$$S_3 \equiv \frac{1}{A_{2,M}\vec{1}_{Mx1}}\left(V_M + \left[X_M\vec{1}_{Mx1} - 1\right]\frac{1}{R_3} Q_3 V_M\right)$$

$$T_3 \equiv \frac{1}{A_{2,M}\vec{1}_{Mx1}}\left(W_M - \left[X_M\vec{1}_{Mx1} - 1\right]\frac{1}{R_3}(D_2 - Q_3 W_M)\right)$$

$$U_3 \equiv \frac{1}{A_{2,M}\vec{1}_{Mx1}}\left(\left[X_M\vec{1}_{Mx1} - 1\right]\frac{1}{R_3}(B_2 - E_2)\right)$$

Now, plugging (28) and (29) into (22) yields

$$\left(I_M - X\vec{1}_{Mx1}\left[\frac{1}{R_3}(B_2 - E_2)\right]\right) c_t = \left(V + X\vec{1}_{Mx1}\frac{1}{R_3} Q_3 V_M\right) z_t + \left(W - X\vec{1}_{Mx1}\frac{1}{R_3}(D_2 - Q_3 W_M)\right) B_t - A_2\vec{1}_{Mx1}\tilde{w}_t$$

$$\left(I_M - X\vec{1}_{Mx1}\left[\frac{1}{R_3}(B_2 - E_2)\right]\right)c_t = \left(V + X\vec{1}_{Mx1}\frac{1}{R_3}Q_3V_M\right)z_t + \left(W - X\vec{1}_{Mx1}\frac{1}{R_3}(D_2 - Q_3W_M)\right)B_t - A_2\vec{1}_{Mx1}(S_3z_t + T_3B_t + U_3c_t)$$

$$V_3c_t = W_3z_t + A_4B_t$$

where

$$V_3 \equiv I_M - X\vec{1}_{Mx1}\left[\frac{1}{R_3}(B_2 - E_2)\right] + A_2\vec{1}_{Mx1}U_3$$

$$W_3 \equiv V + X\vec{1}_{Mx1}\frac{1}{R_3}Q_3V_M - A_2\vec{1}_{Mx1}S_3$$

$$A_4 \equiv W - X\vec{1}_{Mx1}\frac{1}{R_3}(D_2 - Q_3W_M) - A_2\vec{1}_{Mx1}T_3$$

Thus, the closed-form expression for c_t is

$$c_t = cCoeffzz_t + cCoeffBB_t$$

where

$$cCoeffzz_t \equiv V_3^{-1}W_3 \qquad cCoeffBB_t \equiv V_3^{-1}A_4$$

Given this closed-form expression, we can now similarly find closed-form expressions for all of the other variables in the system. Then (29) implies a closed-form expression for $\tilde{w}_t$.

$$\tilde{w}_t = wCoeffzz_t + wCoeffBB_t \tag{30}$$

where

$$wCoeffz \equiv S_3 + U_3cCoeffz$$

$$wCoeffB \equiv T_3 + U_3cCoeffB$$

And similarly for (28).

$$\tilde{c}_{M,t} = \left(\frac{1}{R_3}(B_2 - E_2)\,cCoeffz + \frac{1}{R_3}Q_3V_M\right)z_t + \left(\frac{1}{R_3}(B_2 - E_2)\,cCoeffB - \frac{1}{R_3}(D_2 - Q_3W_M)\right)B_t \tag{31}$$

Thus,

$$\tilde{c}_{M,t} = c^n Coeffz z_t + c^n CoeffB B_t \tag{32}$$

where

$$c^n Coeffz \equiv \frac{1}{R_3}\left(B_2 - E_2\right) cCoeffz + \frac{1}{R_3} Q_3 V_M$$

$$c^n CoeffB \equiv \frac{1}{R_3}\left(B_2 - E_2\right) cCoeffB - \frac{1}{R_3}\left(D_2 - Q_3 W_M\right)$$

Using (18) I obtain

$$y_t = L\left(cCoeffB B_t + cCoeffz z_t\right) + O B_t + \left(P \vec{1}_{Mx1}\right)\left(c^n CoeffB B_t + c^n Coeffz z_t\right)$$

which implies

$$y_t = yCoeffB B_t + yCoeffz z_t \tag{33}$$

where

$$yCoeffB \equiv O + LcCoeffB + \left(P \vec{1}_{Mx1}\right) c^n CoeffB$$

$$yCoeffz \equiv LcCoeffz + \left(P \vec{1}_{Mx1}\right) c^n Coeffz$$

Using (23) I obtain

$$x_t = \left(A + DO\right) B_t + \left(DL + F\right)\left(cCoeffB B_t + cCoeffz z_t\right) + \left[\left(DP + E\right)\vec{1}_{Mx1}\right]\left(c^n CoeffB B_t + c^n Coeffz z_t\right)$$

which implies

$$x_t = xCoeffB B_t + xCoeffz z_t \tag{34}$$

where

$$xCoeffB \equiv \left(A + DO\right) + \left(DL + F\right) cCoeffB + \left[\left(DP + E\right)\vec{1}_{Mx1}\right] c^n CoeffB$$

$$xCoeffz \equiv \left(DL + F\right) cCoeffz + \left[\left(DP + E\right)\vec{1}_{Mx1}\right] c^n Coeffz$$

Using (20) I obtain

$$n_t = \left(\Omega_{\phi B} - GA\right) B_t + (H - GD)\,(yCoeffBB_t + yCoeffzz_t) + \left[(J - GE)\,\vec{1}_{Mx1}\right](c^n CoeffBB_t + c^n Coeffzz_t) + ...$$

$$... + (K - GF)\,(cCoeffBB_t + cCoeffzz_t) - \vec{1}_{Mx1}\,(wCoeffBB_t + wCoeffzz_t)$$

which implies

$$n_t = nCoeffBB_t + nCoeffzz_t \tag{35}$$

where

$$nCoeffB \equiv (\Omega_{\phi B} - GA) + (H - GD)\,yCoeffB + \left[(J - GE)\,\vec{1}_{Mx1}\right] c^n CoeffB + (K - GF)\,cCoeffB - \vec{1}_{Mx1} wCoeffB$$

$$nCoeffz \equiv (H - GD)\,yCoeffz + \left[(J - GE)\,\vec{1}_{Mx1}\right] c^n Coeffz + (K - GF)\,cCoeffz - \vec{1}_{Mx1} wCoeffz$$

Using (10) I obtain

$$\tilde{N}_t = \Omega_{Nn}\,(nCoeffBB_t + nCoeffzz_t)$$

which implies

$$\tilde{N}_t = NCoeffBB_t + NCoeffzz_t \tag{36}$$

where

$$NCoeffB \equiv \Omega_{Nn} nCoeffB \quad NCoeffz \equiv \Omega_{Nn} nCoeffz$$

Using (3) I obtain

$$p_t = \vec{1}_{Mx1}\,(c^n CoeffBB_t + c^n Coeffzz_t) - cCoeffBB_t - cCoeffzz_t$$

which implies

$$p_t = pCoeffBB_t + pCoeffzz_t \tag{37}$$

where

$$pCoeffB \equiv \vec{1}_{Mx1} c^n CoeffB - cCoeffB \quad pCoeffz \equiv \vec{1}_{Mx1} c^n Coeffz - cCoeffz$$

Using (4) I obtain

$$\ell_t = \Omega_{\ell p}\left(pCoeffBB_t + pCoeffzz_t\right) + \Omega_{\ell y}\left(yCoeffBB_t + yCoeffzz_t\right) - xCoeffBB_t - xCoeffzz_t$$

which implies

$$\ell_t = \ell CoeffBB_t + \ell Coeffzz_t \tag{38}$$

where

$$\ell CoeffB \equiv \Omega_{\ell p} pCoeffB + \Omega_{\ell y} yCoeffB - xCoeffB$$

$$\ell Coeffz \equiv \Omega_{\ell p} pCoeffz + \Omega_{\ell y} yCoeffz - xCoeffz$$

Using (5) I obtain

$$\nu_t = xCoeffBB_t + xCoeffzz_t - \Omega_{\nu y}\left(yCoeffBB_t + yCoeffzz_t\right)$$

which implies

$$\nu_t = \nu CoeffBB_t + \nu Coeffzz_t \tag{39}$$

where

$$\nu CoeffB \equiv xCoeffB - \Omega_{\nu y} yCoeffB \quad \nu Coeffz \equiv xCoeffz - \Omega_{\nu y} yCoeffz$$

Finally, using (6) I obtain

$$\phi_t = \Omega_{\phi B} B_t - \Omega_{\phi \nu}\left(\nu CoeffBB_t + \nu Coeffzz_t \nu_t\right) - \Omega_{\phi \ell}\left(\ell CoeffBB_t + \ell Coeffzz_t\right)$$

which implies

$$\phi_t = \phi CoeffBB_t + \phi Coeffzz_t \tag{40}$$

where

$$\phi CoeffB \equiv \Omega_{\phi B} - \Omega_{\phi \nu} \nu CoeffB - \Omega_{\phi \ell} \ell CoeffB$$

$$\phi Coeffz \equiv -\Omega_{\phi\nu}\nu Coeffz - \Omega_{\phi\ell}\ell Coeffz$$

Thus, equations (30)-(40) yield the closed form solution of the behavior of the economy to liquidity and productivity shocks.

Since, in the model, $GDP = wN$, then the first-order approximated percentage change in GDP is given by

$$G\tilde{D}P_t = \tilde{w}_t + \tilde{N}_t$$

$$= wCoeffBB_t + wCoeffzz_t + NCoeffBB_t + NCoeffzz_t$$

$$= (wCoeffB + NCoeffB)\, B_t + (wCoeffz + NCoeffz)\, z_t$$

Constructing Liquidity and Productivity Shocks

Using this solution for y_t and n_t given by (33) and (35), I constructing industry-level liquidity and productivity shocks (B_t and z_t, respectively) from observed output growth data $\hat{y}_t$ and $\hat{n}_t$ as follows. Solving (35) for B_t yields

$$B_t = [nCoeffB]^{-1}\,(\hat{n}_t - nCoeffzz_t) \tag{41}$$

Plugging this into (33) yields

$$\hat{y}_t = yCoeffB\left([nCoeffB]^{-1}\,(\hat{n}_t - nCoeffzz_t)\right) + yCoeffzz_t \tag{42}$$

Solving this for z_t yields

$$z_t = Q_3^{-1}\hat{y}_t - Q_3^{-1}yCoeffB\,[nCoeffB]^{-1}\,\hat{n}_t \tag{43}$$

where

$$Q_3 \equiv yCoeffz - yCoeffB\,[nCoeffB]^{-1}\,nCoeffz$$

Then plugging (43) back into (41) yields

$$B_t = [nCoeffB]^{-1}\,(\hat{n}_t - nCoeffzz_t) \tag{44}$$

Thus, the shocks at time t which hit each industry are observed fluctuations in output and employment, filtered for the effects of credit and input-output linkages in propagating them to other industries, and are given by (43) and (44).