mbreaks: R Package for Estimating and Testing Multiple Structural Changes in Linear Regression Models

Linh Nguyen*
Purdue University

Pierre Perron†
Boston University

Yohei Yamamoto‡
Hitotsubashi University

January 25, 2023

Abstract

This article provides a hands-on guide for an R package to implement a comprehensive analysis of issues related to multiple structural changes in the coefficients of linear regression models proposed by Bai and Perron (1998, 2003). The original theoretical framework and computational algorithms are valid for models with non-trending and regime-wise stationary regressors, although some results remain useful when the regressors have deterministic or stochastic time trends, and even endogenous regressors, provided appropriate modifications are made. The package provides methods of constructing the confidence intervals of the break dates, testing for the presence of structural changes and selecting the number of structural changes. It can also plot the conditional mean functions with and without structural changes. Two empirical examples illustrate how the results are presented and how they can be used in subsequent analyses.

Keywords: multiple structural changes, global minimizers, confidence intervals, hypothesis tests

*Krannert School of Management, Purdue University, 403 West State Street, West Lafayette, IN, 47907 (nguye535@purdue.edu).

†Department of Economics, Boston University, 270 Bay State Rd., Boston, MA, 02215 (perron@bu.edu).

‡Department of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo Japan, 186-8601 (yohei.yamamoto@r.hit-u.ac.jp).

1 Introduction

Estimating and testing for multiple structural changes in the linear regression model has been an active research area in theoretical econometrics and empirical economics over the past decades. See Perron (2006) and Casini and Perron (2019) for comprehensive surveys. In particular, Bai and Perron (1998, 2003) provided a theoretical framework and computational algorithms to estimate and test for multiple structural changes at unknown dates in linear regression models, where the regressors are non-trending or regime-wise stationary [1]. General forms of heteroskedasticity and serial correlations are permitted in the error term with possible structural changes in variance provided they occur at the same time as those in the coefficients. A number of useful theoretical developments extended these methods to wide range of popular empirical settings; e.g., Perron and Zhu (2005) analyzed models with a linear time trend as a regressor, Kejriwal and Perron (2008, 2010a) considered cointegrated regressions with integrated or $I(1)$ regressors as well as stationary ones, Perron and Yamamoto (2014, 2015) considered models with endogenous regressors.

Here, we provide a hands-on guide for methods in the R (R Core Team 2023) package `mbreaks` to implement a comprehensive analysis of issues related to multiple structural changes in the coefficients of linear regression models proposed by Bai and Perron (1998, 2003). They are based on an efficient method to estimate multiple structural changes by minimizing the overall sum of squared residuals (SSR) using the so-called dynamic programming algorithm. We also cover methods for constructing the confidence intervals of the break dates, testing for the presence of structural changes and selecting the number of structural changes. Although the theoretical framework and computation algorithms are built for models with non-trending and regime-wise stationary regressors, some components are valid even when the regressors have deterministic and stochastic trends. Appropriate modifications can be made when the regressor is a linear time trend or the regressors include $I(1)$ variables. We also give some guide for these cases, although the readers should refer to the exact results obtained in the aforementioned papers for details. Note that the original program of the `mbreaks` package was developed by Pierre Perron in the Gauss programming language. Later, the program was translated into a version of the MATLAB language. These are available on the author's website for non-profit academic purposes.[2] In addition, the `mbreaks`

[1] Throughout, we use the terminology "stationary" in a rather loose way. More precisely what is required is short-memory stationarity, or mixing-type conditions so that the partial sums of the variables scaled by $T^{-1/2}$ (with T the sample size) converge to a scaled Wiener process, also called Brownian motion. We will stick with the label "stationary" since it makes the exposition less cumbersome.

[2] The existing R package `strucchange` (Zeileis et al., 2002) can deal with some structural change tests

package enables plotting the conditional mean functions with and without accounting for structural changes.

This article has the following structure. Section 2 explains the models and the efficient dynamic programming algorithm to estimate multiple break dates when the number of breaks is known. Methods for constructing the confidence intervals of the break dates, testing for the presence of structural changes and selecting the number of structural changes are also outlined. Section 3 explains the main functions of the `mbreaks` package to implement these methods in R and provides the entire list of options specific to each function. The plotting function is also explained. Section 4 presents two empirical examples to illustrate how the results are presented and how these can be used in subsequent more general analyses with the proper R program presented in the Appendix. Section 5 concludes.

2 Econometric framework

This section discusses the main model of interest. We then discuss methods related to the efficient dynamic programming algorithm to estimate multiple break dates when the number of breaks is known, the construction of the confidence intervals for the break dates, testing for the presence of structural changes and selecting the number of structural changes.

2.1 Model and estimation

We suppose that we have a sample of T observations for the variables $\{y_t, x_t', z_t'\}$. These are related by the following multiple linear regression model that specifies a partial structural change structure with m break dates (or $m+1$ regimes):

$$y_t = x_t'\beta + z_t'\delta_j + u_t, \quad t = T_{j-1}+1, ..., T_j,$$

for $j = 1, ..., m+1$. The m break dates are denoted by $\{T_1, \cdots, T_m\}$ with the convention that $T_0 = 0$ and $T_{m+1} = T$. The scalar y_t is the dependent variable at time t, x_t and z_t are $p \times 1$ and $q \times 1$ vectors of regressors at time t, β and δ_j, $j = 1, ..., m+1$, are the corresponding vectors of coefficients and u_t is the error term. In the `mbreaks` package, y_t, x_t and z_t are specified as `y`, `x` and `z`. In the following, we may label the entire set of regressors

such as CUSUM tests of Brown et al. (1975) and tests for single break at some unknown date discussed in Andrews (1993) and Andrews and Ploberger (1994). Zeileis et al. (2003) discussed methods of estimating multiple break points suggested by Bai and Perron (2003) in the same package. However, it does not cover comprehensive tools to deal with multiple structural change models such as confidence intervals and tests for multiple structural changes.

by $w_t = [x_t', z_t']'$. We assume $E(w_t u_t) = 0$. The case with this last assumption violated, namely the presence of endogenous regressors, was investigated in Perron and Yamamoto (2014, 2015) and will be discussed in Section 4.2.

There must be an interval with minimal length h between any two adjacent break dates including the beginning and end of the sample, T_0 and T_{m+1}. In standard applications, h is specified as a small fraction of the total sample $h = \epsilon T$ with a trimming parameter $\epsilon \in (0, 1)$, so that the two break dates are asymptotically distant as $T \to \infty$. Typical choices of the trimming parameter are $\epsilon = 0.05$, 0.10 or 0.15 and this can be specified by the option `eps1` in the `mbreaks` package. Note that the maximum number of structural changes is constrained by this choice. For example, if $\epsilon = 0.15$, a maximum of five structural changes can be considered. If $\epsilon = 0.10$, at most eight, no matter how large T is. This trimming in relation to the sample size is needed when conducting hypothesis testing for structural change or when estimating the confidence intervals since each segments must increase with the sample size to obtain consistent estimates of some relevant quantities. If one is solely interested in estimating the break dates, h can be directly specified as a fixed integer via the option `h`, where $h \geq q$. For example, if we set $h = 15$, at most 5 structural changes are considered when $T = 100$, but as many as 65 structural changes can be considered when $T = 1,000$. For example, Lu and Perron (2010) develop a model that predicts a given number of mean changes in the variance of some asset returns; e.g., 15 for a sample of about 10,000 daily observations on the log absolute returns of the S&P 500 index. Since breaks can occur consecutively, they set $h = 1$.

The `mbreaks` package provides as estimates of the unknown break dates, the values $\{\hat{T}_1, \cdots, \hat{T}_m\}$ that minimize the overall sum of squared residuals. once these are obtained, one can recover the regression coefficients. If $p = 0$, x_t is null and all the coefficients related to z_t are subject to change. This is called a pure structural change model. If $p \geq 1$, we have a partial structural change model in which the estimate of the parameter vector β depends on all segments. Adopting a partial structural change model can be beneficial in terms of increased efficiency. However, care must be applied not to constrain parameters that can change.

In the standard setting, the regressors $w_t = [x_t', z_t']'$ are assumed to be non-trending and regime-wise stationary. The usual heteroskedasticity and serial correlation corrections can be applied to construct the relevant confidence intervals. This can be done using only adjacent regimes when the researcher wants to allow the variance or autocovariance of u_t to change at the same date as the parameters. If the researcher is confident that the distribution of u_t

is stable, the full sample can be used.

2.2 An efficient algorithm to estimate the break dates

The estimation of the multiple break dates is based on the least squares principle. Suppose we know the number of structural changes m. Then, the least squares estimates of β and δ_j are obtained by minimizing the sum of m-partitioned squared residuals with δ_j being specific to the segment between $t = T_{j-1} + 1$ and T_j so that the SSR over the entire sample is

$$\sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} (y_t - x_t'\beta - z_t'\delta_j)^2.$$

Let $\hat{\beta}(\{T_j\}_{j=1}^m)$ and $\hat{\delta}(\{T_j\}_{j=1}^m)$ denote the estimates accounting for the m-partition at $\{T_1, \cdots, T_m\}$. If we plug these in the above SSR, the objective function becomes

$$S_T(T_1, \cdots, T_m) \equiv \sum_{j=1}^{m+1} \sum_{t=T_{j-1}+1}^{T_j} [y_t - x_t'\hat{\beta}(\{T_j\}_{j=1}^m) - z_t'\hat{\delta}(\{T_j\}_{j=1}^m)]^2,$$

while the estimated break dates $\{\hat{T}_1, \cdots, \hat{T}_m\}$ are such that

$$\{\hat{T}_1, \cdots, \hat{T}_m\} = \arg \min_{T_1, \cdots, T_m} S_T(T_1, \cdots, T_m).$$

A straightforward algorithm of this optimization is based on a grid search procedure. However, it requires least squares operations of order $O(T^m)$ and may incur prohibitive computational burden. To avoid this problem, the `mbreaks` package uses an algorithm with least squares operations of order $O(T^2)$ at most. In fact only $O(T)$ matrix inversions are needed. The key idea is that, with a sample of size T, the total number of possible segments is at most $T(T+1)/2$. If we consider a matrix with the vertical axis representing the initial date of the segment and the horizontal axis being the ending date of the segment, then a upper triangular matrix can be constructed with each entry representing the estimated SSR for any one of the possible segments.[3] Then, for any value of m, the following algorithm compares possible combinations to achieve a global minimum of the SSR.

Let us first consider the case of $p = 0$. Once the SSRs of the relevant segments are computed and stored in the upper triangular matrix, the algorithm based on the principle of dynamic programming proceeds via a sequential estimation of optimal one break partition that allows a possible break from observations h to $T - mh$. Then, the next step searches for optimal two break partitions. Such partitions have ending dates ranging from $3h$ to

[3]In practice, less than $T(T+1)/2$ segments are permissible as some minimum distance between each break is usually imposed.

$T-(m-2)h$. For each of these possible ending dates, one break partition can be inserted to achieve the minimal SSR. The method continues sequentially until a set of optimal $m-1$ break partitions are obtained. The final step is to see which of these optimal $m-1$ break partitions yields an overall minimal SSR with an additional segment.

If $p \geq 1$, since the estimate of β depends on the m break partitions, an iterative procedure is required. We start with some initial values of β, say β^*, and apply the above mentioned algorithm for $p=0$ with $y_t - x_t'\beta^*$ as the dependent variable. After obtaining the m break partitions $\{T_1^*, \cdots, T_m^*\}$, we update estimates of β and δ_j via the OLS regression $y_t = x_t'\beta + z_t'\delta_j + u_t$ where the regimes are defined by $\{T_1^*, \cdots, T_m^*\}$. Then, using the updated estimate for β, we iterate the dynamic programming algorithm for the $p=0$ case to update the m break partitions. This procedure is continued until the change in the objective function $S_T(T_1, \cdots, T_m)$ becomes smaller than some arbitrary ε. To avoid local minima, an appropriate choice of the initial value of β is required to start the iteration. To this end, we first apply the dynamic programming algorithm with treating all coefficients as being subject to change and obtain the m break partitions $\{T_1^a, \cdots, T_m^a\}$ and the break coefficients $\delta_1^a, \cdots, \delta_{m+1}^a$ (we also obtain estimates for β in each segment). Then, we estimate β using the OLS regression $y_t - z_t\delta_j^a = x_t\beta + u_t$ over the entire sample and use it as an initial value for β. In the `mbreaks` package, the initial value for β is specified by the options `fixb` and `betaini`. The convergence criteria ε is specified by the option `eps` and the maximum number of iteration is set by the option `maxi`.

Here, the assumption that w_t be non-trending or regime-wise stationary is not needed. Indeed, the break fraction estimator $(\hat{T}_j/T)$ is shown to be consistent to the true break fraction as $T \rightarrow \infty$, irrespective of the nature of w_t (subject to technical conditions). Furthermore, the asymptotic distributions of the OLS coefficient estimates in each segment are not affected by the uncertainty of the break dates. Hence, standard inference for the subsample coefficients in each segment can be implemented as if the estimated break dates were known.

There is another method of estimating multiple break dates using a sequential procedure. It first estimates a single break date using the entire sample, then estimate a single break date in each of the subsamples before and after the estimated break date. The significance of a newly found structural changes can be verified using the $\sup F_T(l+1|l)$ test outlined in Section 2.5.1. This can be continued until m structural changes are found. This method reduces the computational burden as it only requires the least squares operations at rate $O(T)$. Bai (1997b) showed that when using this sequential method, the break date estimates are consistent even when m is underspecified if the regressors are non-trending. The limit distribution is, however, different. To remedy this problem, Bai (1997b) suggested a procedure called 'repartition'. This amounts to re-estimating each break date conditional on the adjacent break dates. For example, let the initial estimates of the break dates be denoted by $(\hat{T}_1^a, ..., \hat{T}_m^a)$. The second round estimate for the i^{th} break date is obtained by fitting a one break model to the segment starting at date $\hat{T}_{i-1}^a + 1$ and ending at date $\hat{T}_{i+1}^a$ (with the convention that $\hat{T}_0^a = 0$ and $\hat{T}_{m+1}^a = T$). The estimates obtained from this repartition procedure have the same limit distributions as those obtained simultaneously.

2.3 Confidence intervals for the break dates

The `mbreaks` package offers a method to construct confidence intervals for the break dates. This is based on the following limiting distribution of $\hat{T}_j$ obtained by adopting a theoretical framework in which the magnitudes of the structural changes converge to zero as $T \to \infty$. For notational simplicity, we focus on the case with $p = 0$. It is then given by:

$$\frac{(\Delta_j' Q_j \Delta_j)^2}{(\Delta_j' \Omega_j \Delta_j)} (\hat{T}_j - T_j^0) \Rightarrow \arg\max_s V^{(j)}(s),$$

where $\Rightarrow$ denotes convergence in distribution, $V^{(j)}(s) = W_1^{(j)}(-s) - |s|/2$ if $s \leq 0$, $V^{(j)}(s) = \sqrt{\xi_j}(\phi_{j,2}/\phi_{j,1})W_2^{(j)}(s) - \xi_j |s|/2$ if $s > 0$, $\xi_j = \Delta_j' Q_{j+1} \Delta_j / \Delta_j' Q_j \Delta_j$, $\phi_{j,1}^2 = \Delta_j' \Omega_j \Delta_j / \Delta_j' Q_j \Delta_j$, $\phi_{j,2}^2 = \Delta_j' \Omega_{j+1} \Delta_j / \Delta_j' Q_j \Delta_j$. Also, $W_1^{(j)}(s)$ and $W_2^{(j)}(s)$ are independent standard Wiener processes defined on $[0, \infty)$, starting at the origin when $s = 0$. The cumulative distribution function of $\arg\max_s V^{(j)}(s)$ is derived in Bai (1997a). To make use of this result, all we need are estimates of $\Delta_j = \delta_{j+1} - \delta_j$, $Q_j = (T_j - T_{j-1})^{-1} \sum_{t=T_{j-1}+1}^{T_j} z_t z_t'$, and Ω_j being the long-run covariance matrix estimator of $z_t u_t$ using data over the jth segment.

For the long-run covariance matrix estimator of $z_t u_t$, the `mbreaks` package adopts the heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator proposed by Andrews (1991) using the Quadratic Spectral kernel with the bandwidth selected by an AR(1) approximation. One can also use the prewhitening procedure proposed by Andrews and Monahan (1992), i.e., a VAR(1) filter applied to $z_t \hat{u}_t$ where $\hat{u}_t$ is the regression residuals. The HAC covariance matrix estimator is then constructed based on the filtered series and the VAR(1) coefficient estimates are parametrically accounted for.

Here, z_t and u_t have to be regime-wise stationary. In addition, if either or both are stationary not only in the segments but also over the entire sample, the following options for z_t and u_t can be used to enhance finite sample efficiency.

- If no serial correlation is present in u_t, use `robust=0`. Then, the long-run covariance matrix of $z_t u_t$ is constructed as if u_t is a martingale difference sequence. If `robust=1`, the HAC covariance matrix estimator is used.

- When `robust=1` and the errors are not highly serially correlated, use `prewhit=0`. If `prewhit=1`, the prewhitening procedure proposed by Andrews and Monahan (1992) is applied.

- If the moments of z_t are identical across all the segments, use `hetq=0`. Then, $Q_j = Q$ for all j, which is estimated by $\hat{Q} = T^{-1}\sum_{t=1}^{T} z_t z_t'$. If `hetq=1`, $\hat{Q}_j = (T_j - T_{j-1})^{-1}\sum_{t=T_{j-1}+1}^{T_j} z_t z_t'$ for each j.

- If the long-run covariance matrices of $z_t u_t$ are identical across all the segments, use `hetvar=0`. Then $\Omega_j = \Omega$ for all j and is estimated using a long-run covariance matrix estimator of $z_t u_t$ over the entire sample. If `hetvar=1`, the long-run covariance matrix is estimated in each segment. This last option should be applied only when all segments contain a sufficient number of observations.

When z_t is a linear time trend, the limit distributions of the break date estimate are provided in Perron and Zhu (2005) for a variety of cases. When z_t includes $I(1)$ variables so that we are dealing with a cointegrating regression, the limiting distributions were derived by Kejriwal and Perron (2008, 2010a).

2.4 Tests for the presence of structural changes

The `mbreaks` package provides the following two hypothesis testing procedures to investigate the presence of structural changes. One is suitable to a specific situation where the number of potential structural changes is known and the second is applicable when the number of potential structural changes is unknown. In most empirical studies, the number of structural changes is unknown, thus the latter test has a higher power and is more useful. Both types of tests are based on the supremum F type tests (the supF_T tests) and their asymptotic critical values are also shown in the output of the `mbreaks` package. Since the null hypothesis is that no break occurs, z_t and $z_t u_t$ must be stationary over of the whole sample to ensure that the provided critical values are valid. When z_t includes $I(1)$ variables and y_t and z_t are cointegrated, one can use the produced test statistics in the `mbreaks` package but the critical values to be used are those derived in Kejriwal and Perron (2008). In what follows, we cover the case with $p = 0$ for simplicity of illustration.

2.4.1 Tests for zero versus a fixed number of structural changes

The first type of hypothesis testing procedure is the supF_T test for the null hypothesis of no structural change against an alternative hypothesis of a fixed number (m) of structural changes. Let R be the conventional matrix such that $(R\delta)' = (\delta_1' - \delta_2', \cdots, \delta_m' - \delta_{m+1}')$ and define the following F test allowing serial correlation in the errors:

$$F_T(T_1, \cdots, T_m) = \frac{1}{T}\left(\frac{T-(k+1)q-p}{kq}\right)\hat{\delta}' R'(R\hat{V}(\hat{\delta})R')^{-1}R\hat{\delta},$$

where $\hat{V}(\hat{\delta})$ is an estimate of the long-run covariance matrix of $\hat{\delta}$. Since the break fractions are T-consistent even with correlated errors, an asymptotically equivalent version is to first take the supremum of the original F_T test that assume i.i.d. errors to obtain the break points. The robust version of the test is constructed as $\sup F_T(m) = F_T(\hat{T}_1, \cdots, \hat{T}_m)$ where $\{\hat{T}_1, \cdots, \hat{T}_m\}$ globally minimize the SSR. They are equivalent to taking the maximum value of $F_T(T_1, \cdots, T_m)$ over the set of $\{T_1, \cdots, T_m\}$ when the F test is constructed under the spherical errors assumption.

Similar to the confidence intervals of the break dates, some prior knowledge on restrictions pertaining to serial correlations in u_t may improve the finite sample properties of the tests. In particular, the following option is available:

- Options `robust` and `prewhit` can be applied if the errors are serially correlated (and heteroskedastic).

- If z_t is imposed to have the same second moments across all the segments let `hetdat=0`. Then, the long-run covariance matrix estimator is constructed accordingly.

- If the long-run covariance matrices of $z_t u_t$ are imposed to be identical across all segments let `hetvar=0`. Then, $\hat{V}(\hat{\delta})$ is constructed using $z_t u_t$ over the entire sample. If `hetvar=1`, $\hat{V}(\hat{\delta})$ is constructed in each segment.

Setting `hetdat=0 and hetvar=0` amounts to estimating the variance imposing the null hypothesis, while `hetdat=1 and hetvar=1` implies estimating it under the alternative hypothesis. If the sample size is large enough it is recommended to estimate under the alternative hypothesis to avoid low or even non-monotonic power problems.

2.4.2 Double maximum tests

One usually has no prior information regarding the true number of structural changes. Bai and Perron (1998) introduced tests for the null hypothesis of no structural change against an unknown number of structural changes given some upper bound M. These are called the double maximum tests. In particular, the `mbreaks` package offers the $UD\max_T$ test:

$$UD\max{}_T(M) = \max_{1\leq m\leq M} F_T(\hat{T}_1, \cdots, \hat{T}_m),$$

where the largest number of structural changes (M) is specified by option `m`. In the `mbreaks` package, the asymptotic critical values for $M = 5$ are available for $\epsilon = 0.05$, 0.10 and 0.15, those for $M = 3$ are available for $\epsilon = 0.20$ and those for $M = 2$ for $\epsilon = 0.25$. Note that if the model contains m structural changes, the $UD\max_T$ test may be slightly less powerful than the correctly specified $\sup F_T(m)$ test. However, the $UD\max_T$ test has a higher power than the $\sup F_T(m)$ test if the number is misspecified. Also, the $UD\max_T$ test is particularly useful when two structural changes exist in opposite directions. Bai and Perron (2006) showed that in such a case the $\sup F_T(1)$ test does not have sufficient power but the $UD\max_T$ test does. Also, the critical values do not change much as M is increased beyond 5, so that the test can also be applied with larger values using the critical values for $M = 5$. Finally, simulations in Bai and Perron (2006) showed that the $UD\max{}_T(M)$ test has power almost as high as the $\sup F_T(m)$ test that uses the correct value of m. Hence, it is arguably the most useful and should be used in all cases.

2.5 Selecting the number of structural changes

In this section, we discuss two avenues to select the number of structural changes. One is based on a sequential testing procedure, the other on the use of information criteria.

2.5.1 Tests for l versus $l + 1$ structural changes

The `mbreaks` package offers tests for the null hypothesis of l structural changes against an alternative hypothesis of $l + 1$ structural changes. This is called the $\sup F_T(l + 1|l)$ test. The method amounts to the application of $l + 1$ tests of the null hypothesis of no structural change versus an alternative hypothesis of single structural change applied to each segment containing the observations from $T_{j-1} + 1$ to T_j for $j = 1, ..., l + 1$. It concludes for a rejection in favour of a model with $l + 1$ structural changes if the overall minimal value of the SSR over all segments where an additional structural change is included is sufficiently smaller than the SSR from the model with l structural changes. The asymptotic critical values of the $\sup F_T(l+1|l)$ tests are provided in the `mbreaks` package for $\epsilon = 0.05$, 0.10, 0.15, 0.20 and 0.25 and for q ranging from 1 to 10. The level of significance is chosen by the option `signif`. The method of sequential testing to select the number of structural changes goes as follows. Use the $UD\max_T$ or $\sup F_T(1)$ test to establish whether at least one break is present. If so, continue with the $\sup F_T(2|1)$ test. If the test fails to reject the null hypothesis, the number of structural change is considered to be one. If the test rejects, we sequentially proceed to the $\sup F_T(3|2)$ test and so on. If the $\sup F_T(l+1|l)$ test does not reject, then one concludes that l breaks are present. Note that, in the presence multiple structural changes, the $\sup F_T(1|0)$ test may suffer from low power, which is why it is advisable to start using the $UD\max_T$ test to establish whether at least one break is present.

When z_t is integrated so that we have a cointegrated system, Kejriwal and Perron (2010b) derived the limiting distributions of the $\sup F_T(l+1|l)$ tests. The output from the `mbreaks` package for these tests can be used but their significance should be assessed using the critical values provided by Kejriwal and Perron (2010b). However, there are a few caveats regarding this procedure if z_t has time trends. Yang (2017) showed that in the presence of multiple structural changes, the first estimated break date needs not converge to one of the true break dates, which invalidates the sequential testing and estimation procedure as well as the repartition method.

2.5.2 Information criteria

Another popular class of procedures to select the number of structural changes is information criteria. The `mbreaks` package provides the Bayesian Information Criterion (BIC) suggested by Yao (1988), the modified Schwarz Information Criterion (LWZ) proposed by Liu et al. (1997) and the modified BIC of Kurozumi and Tuvaandorj (2011). These are defined as follows:

$$\begin{aligned}
BIC(m) &= \log[S_T(T_1,\cdots,T_m)/T] + q(m+1)\times(\ln T)/T, \\
LWZ(m) &= \log[S_T(T_1,\cdots,T_m)/T] + q(m+1)\times c_0(\ln T)^{2+\eta}/T, \\
KT(m) &= \textstyle\sum_{j=1}^{m+1}(\hat{T}_j-\hat{T}_{j-1})\log\hat{\sigma}_j^2 + \sum_{j=1}^{m+1} q\log(\hat{T}_j-\hat{T}_{j-1}) + 2m\log T,
\end{aligned}$$

with $c_0 = 0.299$ and $\eta = 0.1$ in $LWZ(m)$ and $\hat{\sigma}_j^2 = (\sum_{t=\hat{T}_{j-1}+1}^{\hat{T}_j}\hat{u}_t^2)/T$ with $\hat{u}_t$ being the OLS residuals which account for the estimated break dates in $KT(m)$. For all criteria, the value m which minimizes it is considered as the number of structural changes present.

3 Main functions and options in `mbreaks` package

In the `mbreaks` package, all the procedures discussed are implemented via the comprehensive main functions `mdl()` as well as the specific functions `dotest()`, `doseqtests()`, `dosequa()`, `doorder()`, `dorepart()` and `dofix()`. For first-time users, we recommend using the `mdl()` function, as it yields a set of outputs for the standard analysis. In this section, we explain these functions and a plotting function `plot_model()`. Table I summarizes the purposes of these functions.

Table I. Summary of main and plotting functions

comprehensive function	`mdl()`	(`sbtests,seqtests`) $\sup F_T(m)$, UDmax_T, $\sup F_T(l+1\|l)$
		(`model`) SEQ, KT, BIC, LWZ
specific functions	`dotest()`	(`sbtests`) $\sup F_T(m)$, UDmax_T
	`doseqtests()`	(`seqtests`) $\sup F_T(l+1\|l)$
	`dosequa()`	(`model`) SEQ
	`doorder()`	(`model`) KT, BIC, LWZ
	`dorepart()`	(`model`) REPART
	`dofix()`	(`model`) FIX
plotting function	`plot_model()`	Plot any `model` above

`sbtests, eqtests` and `model` are S3 class objects. For class of `model`,

SEQ: model selected by the sequential testing

BIC: models selected by BIC proposed by Yao (1988)

LWZ: model selected by the modified SIC proposed by Liu et al. (1997)

KT: model selected by the modified BIC proposed by Kurozumi and Tuvaandorj (2011)

REPART: model selected by the sequential method proposed by Bai (1997a)

FIX: model selected by a prespecified or known number of structural changes

In what follows, we show results which appear in the console by using a mean shift model with ready-to-use data `real` included in the `mbreaks` package. The details of this example are presented in Section 4.1, but because `x_name` is empty and `z_name` includes only a constant, one can simply produce the results by applying the main functions, for example, `mdl('rate',data=real)`.

3.1 Comprehensive function `mdl()`

The generic format of `mdl()` is:

```
> result_mdl = mdl(y_name,z_name,x_name,data,options)
```

In the output `result_mdl`, there are two objects of hypothesis testing results `$sbtests` and `$seqtests`. `$sbtests` includes a summary of the $\sup F_T$ and $UD\max_T$ tests as well as the critical values assuming stationary regressors. To view the summary tables:

```
>print(result_mdl$sbtests)
```

```
a) SupF tests against a fixed number of breaks

        1 break 2 breaks 3 breaks 4 breaks 5 breaks
Sup F    57.906   43.014   33.323   24.771   18.326
10% CV    7.040    6.280    5.210    4.410    3.470
5% CV     8.580    7.220    5.960    4.990    3.910
2.5% CV  10.180    8.140    6.720    5.510    4.340
1% CV    12.290    9.360    7.600    6.190    4.910

b) UDmax tests against an unknown number of breaks

   UDMax 10% CV  5% CV 2.5% CV  1% CV
1 57.906  7.460  8.880  10.390 12.370
```

Also, `$seqtests` includes a summary of the $\sup F_T(l+1|l)$ tests and their critical values assuming stationary regressors. To view the summary table:

```
>print(result_mdl$seqtests)
```

```
supF(l+1|l) tests using global optimizers under the null

         supF(1|0) supF(2|1) supF(3|2) supF(4|3) supF(5|4)
Seq supF    57.906    33.927    14.725     0.033     0.000
10% CV       7.040     8.510     9.410    10.040    10.580
5% CV        8.580    10.130    11.140    11.830    12.250
2.5% CV     10.180    11.860    12.660    13.400    13.890
1% CV       12.290    13.890    14.800    15.280    15.760
```

Importantly, the outputs from the `mdl()` function include the results of model estimation as `model` class in R, where the number of structural changes is selected by the sequential testing (`$SEQ`) and the information criteria (`$BIC`, `$LWZ` and `$KT`). For the former, the level of significance is chosen by the option `signif`. For the latter, the results of all the three criteria are included. For each `model` object the selected number of structural changes is contained in `$nbreak`, the estimated break dates are in `$date` and their confidence intervals are in `$CI`.

Furthermore, it includes the subsample OLS coefficient estimates for δ_j in the top rows of `$beta` and their standard errors in the same position of `$SE`. If applicable, the full sample OLS coefficient estimates for β is in the bottom rows of `$beta` and their standard errors in the same positions of `$SE`. The minimized SSR, the residuals and the fitted values of the final model are stored in `$SSR`, `$resid` and `$fitted.values`, respectively. One often wants to use some results of `mdl()` function as intermediate inputs for the subsequent analyses. For example, the estimated break dates when the number of structural changes is selected by the sequential testing will be stored in `break_date` object and can be retrieved via:

```
>break_date = result_mdl$SEQ$date
```

```
> break_date
     [,1]
[1,]   24
[2,]   47
[3,]   79
```

3.2 Specific main functions

The outputs from the `mdl()` function can also be produced by using the following specific main functions. First, the results of the tests for the presence of structural changes are obtained from the `dotest()` function.

```
> result_dotest = dotest(y_name,z_name,x_name,data,options)
```

In `result_dotest`, we have four objects which summarize the results. `$ftest` contains the $\sup F_T(m)$ tests, `$cv_supF` includes the 10%, 5%, 2.5% and 1% critical values. `$UDmax` contains the $UD\max_T$ tests and `$cv_Dmax` contains their critical values. The summary tables, which present the same as `result_mdl$sbtests` from the `mdl()` function, are obtained using:

```
>print(result_dotest)
```

Similarly, the results of the sequential tests are obtained by specifically using the `doseqtests()` function. It contains the $\sup F_T(l+1|l)$ test statistics in `$supfl` and their critical values in `$cv_supFl`.

Second, the model estimation results from the `mdl()` function can also be produced using the following specific functions. When the model is selected by the sequential testing, the results are obtained using the `dosequa()` function.

```
> result_dosequa = dosequa(y_name,z_name,x_name,data,options)
```

Similarly, when the number of structural changes is selected by the information criteria, the results are obtained using the `doorder()` function, whereas the criterion (BIC, LWZ or KT) may be specified from the option `ic` in the `doorder()` function. If unspecified, it returns the results using KT.

```
> result_doorder = doorder(y_name,z_name,x_name,data,options)
```

For any of the model estimation results, useful results are stored as explained in Section 3.1. The summary tables, which present the same as `result_mdl$SEQ` from the `mdl()` function, are obtained by:

```
>print(result_dosequa)
```

3.3 Specific main functions of other suboptimal methods

There may be an interest in the following two methods of estimating the break dates. The first method is the repartition method explained in Section 2.2. The `dorepart()` function implements this method and estimates multiple structural changes one at a time, while the number of breaks are selected by the sequential testing. To do this:

```
> result_dorepart = dorepart(y_name,z_name,x_name,data,options)
```

The second method is when the number of structural changes is known or prespecified. One can use the main function `dofix()` to estimate the model where the number of structural changes is specified via the option `fixn`.

```
> result_dofix = dofix(y_name,z_name,x_name,data,fixn, options)
```

Note that if `fixn` is not specified in `dofix()` function, a model with $m = 5$ is chosen.

3.4 Plotting function

The `mbreaks` package offers a plotting function using the `ggplot2` package. The `plot_model()` function produces a figure of the following items obtained from an estimated structural change model (`result_dosequa`, `result_doorder`, `result_dorepart` or `result_dofix`). For example,

```
> plot_model(result_dosequa)
```

- The observed y_t, fitted values $\hat{y}_t^{(m^*)}$ from a model with m^* structural changes (estimated or pre-specified) and fitted values $y_t^{(0)}$ from a model with no structural changes.
- The estimated break dates with labels in chronological order and the confidence intervals of the break dates with confidence level 0.95 as the default value. The confidence level can be changed to 0.90 via the option `CI`.
- The confidence interval of the conditional mean of y_t when the model has m^* structural changes. The confidence level can be changed to 0.90 via the option `CI`.

3.5 Options for the functions

For the main functions `mdl()` as well as `dotest()`, `doseqtests()`, `dosequa()`, `doorder()`, `dorepart()`, `dofix()` and `plot_model()` the following options are available. If these are not specified, they are set at some stated default value.

- `const`: allows to include a constant term in z_t. If `const=0`, z_t does not include a constant. The default value is `const=1`.

- `eps1` : specifies the trimming value ϵ from $\{0, 0.05, 0.10, 0.15, 0.20, 0.25\}$. The default value is `eps1=0.15`. If the input value is not one of the six values or $(m+1)\lfloor \epsilon T \rfloor > T$, it will be set to the default value. If `eps1=0` is chosen, `h` must directly be specified and `dotest()`, `doseqtests()`and `dosequa()` functions are invalidated as explained in Section 2.1.

- `h`: specifies the minimum interval between the two adjacent structural changes. This is an option specific to the occasion of $\epsilon = 0$. When $\epsilon > 0$, $h = \lfloor \epsilon T \rfloor$ is automatically set. If $\epsilon = 0$ and the input value is invalid, i.e. $(m+1)h > T$ or $h < p + q$, it will be set to $h = \lfloor 0.15T \rfloor$.

- `m` : specifies the maximum number of structural changes considered in the model. The default value is `m=5`. If the input value is not an integer or $(m+1)h > T$, it will be set to the default value.

- `signif` : specifies the level of significance in the sequential testing procedures to select the number of structural changes. The default value is `signif=2` corresponding to the 5% significance level. Other values are `signif=1` for the 10%, `signif=3` for the 2.5%, and `signif=4` for the 1% significance levels, respectively.

The following options pertain to the structure of the long-run covariance matrix of $w_t u_t$.

- `robust` : allows heteroskedasticity and autocorrelation in u_t by using the HAC covariance matrix estimator in which the Quadratic Spectral kernel is used with the bandwidth selected via the AR(1) approximation proposed by Andrews (1991). With `robust=0`, the errors are assumed to be a martingale difference sequence. The default value is `robust=1`.

- `prewhit` : prewhitening procedure proposed by Andrews and Monahan (1992), that is, a VAR(1) filter applied to $w_t\hat{u}_t$ where $\hat{u}_t$ is the regression residuals and the HAC covariance matrix estimator is constructed based on the filtered series and the AR1 coefficient estimate are parametrically accounted for. The default value is `prewhit=1`.

The following options pertain to the variance of u_t and the second moment matrices of the regressors w_t:

- `hetvar` : allows for the variance of errors u_t to be different across the segments determined by the estimated breaks dates when constructing the F test statistics. If `hetvar=0`, the errors are assumed to have the same variance across the segments. The default value is `hetvar=1`. Note that `hetvar=0` is not allowed when `robust=1`.

- `hetdat` : allows for the second moment matrices of w_t to be different across the segments when constructing the F test. If `hetdat=0`, w_t is assumed to have the same second moment matrix across the segments. The default value is `hetdat=1`.

- `hetq` : allows for the second moment matrices of w_t to be different across the segments when constructing the confidence intervals of the break dates. If `hetq=0`, w_t is assumed to have the same second moment matrix across the segments. The default value is `hetq=1`.

- `hetomega` : allows for the long-run covariance matrices of $w_t u_t$ to be different across the segments when constructing the confidence intervals of the break dates. If `hetomega=0`, the long-run covariance matrix is assumed to be the same across the segments. The default value is `hetomega=1`.

The following options are specific to estimating partial structural change models explained in Section 2.2:

- `maxi` : specifies the maximum number of iterations if no convergence is attained when running the iterative procedure. The default value is `maxi=20`.

- `eps` : specifies the criterion for convergence of the iterative procedure. The default value is `eps=0.0001`.

- `fixb` : allows specific initial values for β. If `fixb=1`, the values must be supplied as `betaini` of size $p \times 1$. If `betaini` is an invalid value, it will set `fixb=0` and use an OLS estimate for the initial values for β. The default value is `fixb=0`.

The other options:

- `ic` : option specific to the main function `doorder()`. If `ic="BIC"`, the BIC will be used to select the number of structural changes. If `ic="LWZ"`, the LWZ will be used to select the number of structural changes. The default value is `ic="KT"` and KT is used.

- `fixn`: option specific to the main function `dofix()`. The number of structural changes prespecified by the user. The default value is `fixn=5`.

- `printd` : allows to print intermediate outputs of the estimation procedures in the console, if you set `printd=1`. The default value is `printd=0` and the intermediate outputs are suppressed.

- `CI`: option specific to the plotting function `plot_model()`. This specifies the confidence level for the confidence intervals of the break dates and the conditional means in the plot. The options are 0.90 or 0.95. The default value is `CI=0.95`.

4 Empirical examples

The section presents two empirical examples investigated in the previous literature: a) level shifts in the U.S. real interest rate; see Garcia and Perron (1996) and Bai and Perron (2003), b) structural changes in the New Keynesian Phillips curve; see Perron and Yamamoto (2015).

4.1 Level shifts in the US real interest rate

We investigate structural changes in the mean parameter μ_j of the mean shift model:

$$y_t = \mu_j + u_t, \quad \text{for } t = 1, ..., T,$$

and $j = 1, ..., m+1$ with y_t being the U.S. real interest rate series from 1961Q1 to 1986Q3. The `mbreaks` package includes a ready-to-use data set `real` in which the real interest rate data is labelled as `rate`. We allow serial correlation in the errors u_t by using the HAC covariance matrix estimator with prewhitening. These can be specified using the options `robust=1` and `prewhit=1`, which are default settings. Here, we use the `dosequa()` function to select the number of structural changes by the sequential testing.

```
> result_rate = dosequa('rate', data = real)
> print(result_rate)
```

The following output shows that, consistent with Bai and Perron (2003), a model with three structural changes in level is selected and the estimated break dates are 24 (1966Q4), 47 (1972Q3) and 79 (1980Q3). Their 95% confidence intervals are [1965Q2, 1970Q2], [1969Q1, 1973Q1] and [1980Q1, 1981Q2], respectively. The output also presents the coefficient estimate $\hat{\mu}$ in each regime: 1.824% for the first regime, 0.866% for the second regime, -1.796% for the third regime and 5.643% for the fourth regime. Their standard errors computed using the HAC covariance matrix estimator with prewhitening are presented in parentheses.

```
The number of breaks is estimated by sequential procedure
Pure change model with 3 estimated breaks.
Minimum SSR = 445.182

Estimated date:
        Break1  Break2  Break3
Date        24      47      79
95% CI (18,38) (33,49) (77,82)
90% CI (20,35) (37,49) (78,81)

Estimated regime-specific coefficients:
                Regime 1      Regime 2       Regime 3
Const (SE) 1.824 (0.190) 0.866 (0.153) -1.796 (0.511)
                Regime 4
Const (SE) 5.643 (0.603)

No full sample regressors
```

As `result_rate` contains the estimated model with three structural changes, the conditional means with and without structural changes are plotted by

> `plot_model(result_rate)`

In Figure I, the black line shows the data y_t, the blue line corresponds to the conditional mean of y_t when the three structural changes are accounted for. The red long line is the conditional mean of y_t when no structural changes are considered. The estimated three break dates are indicated by the vertical dotted lines in purple with their confidence intervals by short red lines in the bottom of the figure.

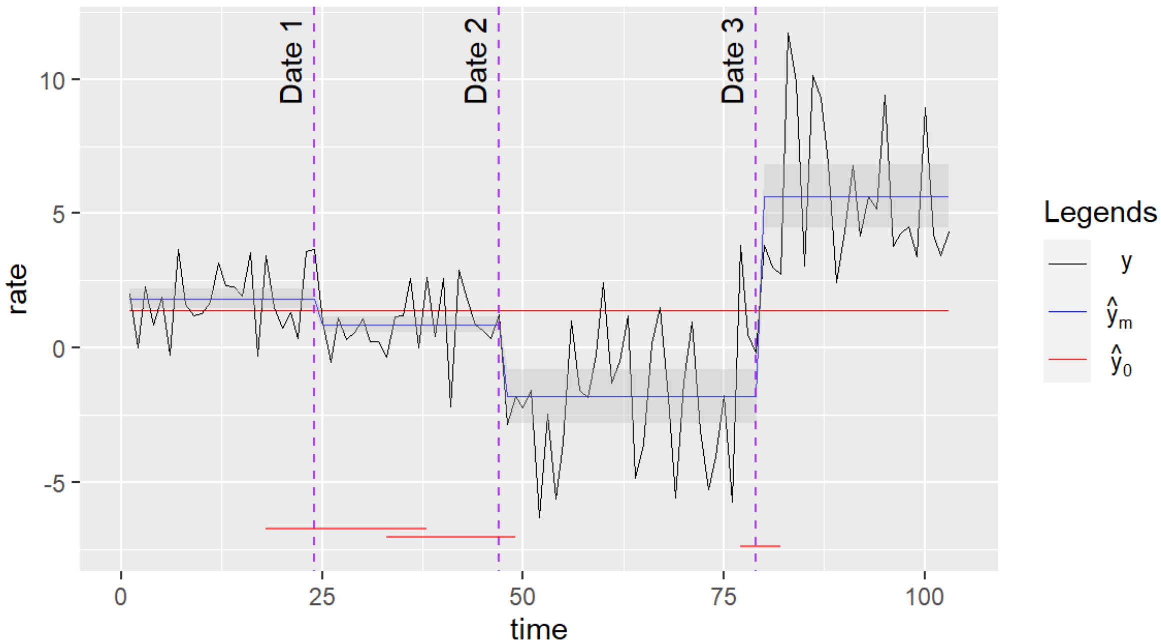

Figure I. Plot of the conditional mean of the US. real interest rates from the `plot_model()` function

4.2 Structural changes in the New Keynesian Phillips curve model

Perron and Yamamoto (2015) studied structural changes in the New Keynesian Phillips curve model proposed by Gali and Gertler (1999). They considered the following linear model:

$$\pi_t = \mu + \gamma\pi_{t-1} + \kappa mc_t + \beta E_t\pi_{t+1} + u_t \quad \text{for } t = 1, ..., T,$$

where π_t is the inflation rate and mc_t is a marginal cost measure, which here is the labor income share, at time t. E_t is the expectation operator conditional on information available up to time t; hence $E_t\pi_{t+1}$ is the expected inflation rate for the next period. The `mbreaks` package includes a ready-to-use data set `nkpc` from Kurmann (2007) and we use `inf` and `inflag` for π_t, π_{t-1}, respectively, and `lbs` for mc_t. The sample period is 1960Q1-1997Q4. An alternative choice for mc_t is the GDP gap (`ygap`) and this can be implemented by replacing `lbs` with `ygap` in the following procedures.

As the expectation term $E_t\pi_{t+1}$ is unobservable, we take the strategy of using π_{t+1} (`inffut`) to proxy $E_t\pi_{t+1}$ in the regression model. However, such a regression model suffers from endogeneity problem as the expectation errors $\pi_{t+1} - E_t\pi_{t+1}$ absorbed in the error term are correlated with the proxy variable π_{t+1}. In assessing structural changes, if some regressors are endogenous, i.e., correlated with the errors, one can simply consider the projection of the regressors on the space spanned by the instrumental variables (IVs) to estimate and test for multiple structural changes (Perron and Yamamoto, 2014). In addition, when the reduced-form has structural changes, the generated regressors may not be regime-wise stationary. Thus, the confidence intervals of the break dates by the IV method need to account for this fact. Perron and Yamamoto (2015) showed that it is preferable to simply estimate the break dates and test for structural changes using the usual OLS framework, as it delivers estimates of the break dates with higher precision and tests with higher power compared to those obtained using the IV method. In this empirical example, we adopt the strategy of Perron and Yamamoto (2015) and estimate and test for structural changes by the OLS method even if endogeneity is an issue. The main reason why OLS is preferable is that even if the parameter estimates of the coefficients are inconsistent the changes are, in general, consistent. Also, it is more efficient since it avoids the use of generated regressors which have less variations especially when the instruments are weak. This is done only to get better estimates of the break dates and more powerful tests for structural changes. Once the structural changes are identified, the coefficients are estimated by using the IV method applied to each segment.

Let us consider the full structural change model and now use the comprehensive function `mdl()`. Following Perron and Yamamoto (2015), we specify the options `prewhit=0`, `eps1=0.1` and `m=5` but these specific choices do not qualitatively affect the main empirical results.

```
> result_nkpc=mdl('inf',c('inffut', 'inflag', 'lbs'),
                                    data=nkpc,prewhit=0,eps1=0.1,m=5)
```

The results of the tests for the presence of structural changes and those of the sequential tests are stored in `$sbtests` and `$seqtests`, respectively. They are of course the same as the original results of Perron and Yamamoto (2015). In `$sbtests`, the $UD\max_T$ test is 69.63 and exceeds the 1% critical value (20.75), suggesting the presence of structural changes in the New Keynesian Phillips curve model.

```
> result_nkpc$sbtests
```

```
a) SupF tests against a fixed number of breaks

         1 break 2 breaks 3 breaks 4 breaks 5 breaks
Sup F     30.592   69.630   37.894   32.765   30.607
10% CV    14.810   13.560   12.360   11.430   10.610
5% CV     16.760   14.720   13.300   12.250   11.290
2.5% CV   18.620   15.880   14.220   12.960   11.940
1% CV     20.750   17.240   15.300   13.930   12.780

b) UDmax tests against an unknown number of breaks

   UDMax 10% CV  5% CV 2.5% CV  1% CV
1 69.630 15.230 17.000  18.750 20.750
```

In `$seqtests`, the sup$F_T(2|1)$ test is 11.408 and is smaller than the 10% critical value (16.70). Thus, there is only one structural change in the model.

```
> result_nkpc$seqtests
```

```
supF(l+1|l) tests using global optimizers under the null

          supF(1|0) supF(2|1) supF(3|2) supF(4|3) supF(5|4)
Seq supF     30.592    11.408    11.595    12.564    26.418
10% CV       14.810    16.700    17.840    18.510    19.130
5% CV        16.760    18.560    19.530    20.240    20.720
2.5% CV      18.620    20.300    21.180    21.860    22.400
1% CV        20.750    22.400    23.550    24.130    24.540
```

In `result_nkpc$SEQ`, we see that the estimated break date is 125 which corresponds to 1991Q1. The OLS coefficient estimates and their standard errors are also produced, although these suffer from endogeneity bias and are invalid.

```
> result_nkpc$SEQ
```

```
The number of breaks is estimated by sequential procedure
Pure change model with 1 estimated breaks.
Minimum SSR = 0.001

Estimated date:
          Break1
Date         125
95% CI (125,127)
90% CI (125,126)

Estimated regime-specific coefficients:
                  Regime 1       Regime 2
Const (SE)   0.001 (0.001)  0.486 (0.040)
inflag (SE) -0.003 (0.005)  0.467 (0.039)
lbs (SE)    -0.003 (0.005) -0.019 (0.134)
inffut (SE)  0.060 (0.041) -0.039 (0.149)

No full sample regressors
```

The next step is to split the entire sample into the two subsamples ([1960Q1, 1991Q1] and [1991Q2, 1997Q4]) and conduct the IV estimation for each segment. Following the literature, we consider the set of instruments available at period $t-1$. We use the first lagged variables of inflation (`inflag`), labor income share (`lbslag`), GDP gap (`ygaplag`), interest spread (`spreadlag`), wage inflation (`dwlag`) and commodity price (`dcplag`) as the set of IVs.

Our analysis illustrates how the results from the `mbreaks` package can be incorporated in subsequent analyses. In the first stage regression, structural changes in the reduced-form model are possible. Using the sequential testing, we found two structural changes in 1973Q1 and 1980Q4. Hence, the fitted values of the endogenous regressor (`inffut`) on the IVs by accounting for those two structural changes are created (`Xpred`). In the second stage regression, we regress the dependent variable (`inf`) on `Xpred` and the exogenous regressors (`inflag` and `lbs`) segmented into the two subsamples [1960Q1, 1991Q1] and [1991Q2, 1997Q4]. Table II reports the coefficient estimates and standard errors equivalent to Perron and Yamamoto (2015) in each segment. See Appendix for the R program.

Table II. IV coefficient estimates in the New Keynesian Phillips Curve model with structural changes

	π_{t-1}	mc_t	$E_t\pi_{t+1}$	const.
1960Q1-1991Q1	0.302	0.001	0.683	0.000
	(0.070)	(0.007)	(0.067)	(0.001)
1991Q2-1997Q4	0.130	0.042	-0.328	0.001
	(0.157)	(0.036)	(0.385)	(0.005)

Note: Standard errors are in parentheses.

5 Conclusion

This article provided a hands-on guide for an R package `mbreaks` to implement a comprehensive analysis of issues related to multiple structural changes in the coefficients of linear regression models proposed by Bai and Perron (1998, 2003). The proposed R package is available from CRAN (R Core Team, 2023). It offers methods for constructing the confidence intervals of the break dates, testing for the presence of structural changes and selecting the number of structural changes. Although the theoretical framework and computation algorithms are built for models with non-trending or regime-wise stationary regressors, some results are still useful even when the regressors have a linear time trend or $I(1)$ variables, or even when some regressors are endogenous. The `mbreaks` package provides methods of constructing the confidence intervals of the break dates, testing for the presence of structural changes and selecting the number of structural changes by using the main functions `mdl()` as well as `dotest()`, `doseqtests()`, `dosequa()`, `doorder()`, `dorepart()`, `dofix()` and to plot the conditional mean functions with and without structural changes. A list of the entire set of options to specify in these functions is provided. Two empirical examples illustrate how to use these functions in practice, how the results are presented as well as how the results can be used in subsequent more general analyses. We hope that this will provide valuable tools to implement state-of-the-art methods to deal with multiple structural changes in a wide range of empirical analyses.

References

Andrews DWK (1991). "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica,* 59, 817-858.

Andrews DWK (1993). "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica*, 61, 821–856.

Andrews DWK, Monahan JC (1992). "An Improved Heteroscedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica,* 60, 953–966.

Andrews DWK, Ploberger W (1994). "Optimal Tests when a Nuisance Parameter is Present only under the Alternative," *Econometrica*, 62, 1383–1414.

Brown RL, Durbin J, Evans JM (1975). "Techniques for Testing the Constancy of Regression Relationships over Time," *Journal of the Royal Statistical Society B*, 37, 149–163.

Bai J (1997a). "Estimation of a Change Point in Multiple Regression Models," *Review of Economics and Statistics,* 79, 551-563.

Bai J (1997b). "Estimating Multiple Breaks One at a Time," *Econometric Theory,* 13, 315-352.

Bai J, Perron P (1998). "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica,* 66, 47-78.

Bai J, Perron P (2003). "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics*, 18, 1-22.

Bai J, Perron P (2006). "Multiple Structural Change Models: A Simulation Analysis," In *Econometric Theory and Practice: Frontiers of Analysis and Applied Research,* eds. D. Corbae, S. Durlauf and B.E. Hansen, New York: Cambridge University Press, pp. 212-237.

Casini A, Perron P (2019). "Structural Breaks in Time Series," In *Oxford Research Encyclopedia of Economics and Finance,* Oxford University Press.

Gali J, Gertler M (1999). "Inflation Dynamics: a Structural Econometric Analysis," *Journal of Monetary Economics*, 44, 195-222.

Garcia R, Perron P (1996). "An Analysis of the Real Interest Rate under Regime Shifts," *Review of Economics and Statistics,* 78, 111-125.

Kejriwal M, Perron P (2008). "The Limit Distribution of the Estimates in Cointegrated Regression Models with Multiple Structural Changes," *Journal of Econometrics,* 146, 59-73.

Kejriwal M, Perron P (2010a). "Testing for Multiple Structural Changes in Cointegrated Regression Models," *Journal of Business and Economic Statistics,* 28, 503-522.

Kejriwal M, Perron P (2010b). "A Sequential Procedure to Determine the Number of Breaks in Trend with an Integrated or Stationary Noise Component," *Journal of Time Series Analysis*, 31, 305-328.

Kurozumi E, Tuvaandorj P (2011). "Model Selection Criteria in Multivariate Models with Multiple Structural Changes," *Journal of Econometrics*, 164, 218-238.

Kurmann A (2007). "VAR-based Estimation of Euler Equations with an Application to New Keynesian Pricing," *Journal of Economic Dynamics and Control*, 31, 767-796.

Liu J, Wu S, Zidek JV (1997). "On Segmented Multivariate Regressions," *Statistica Sinica,* 7, 497-525.

Lu, YK, Perron P (2010). "Modeling and Forecasting Stock Return Volatility Using a Random Level Shift Model," *Journal of Empirical Finance,* 17, 138-156.

Perron P (2006). "Dealing with Structural Breaks," In *Palgrave Handbook of Econometrics*, Patterson, K. and Mills, T.C. (eds.), Vol.1: Econometric Theory, Palgrave Macmillan, 278-352.

Perron P, Yamamoto Y (2014). "A Note on Estimating and Testing for Multiple Structural Changes in Models with Endogenous Regressors via 2SLS," *Econometric Theory,* 30(2), 491-507.

Perron P, Yamamoto Y (2015). "Using OLS to Estimate and Test for Structural Changes in Models with Endogenous Regressors," *Journal of Applied Econometrics,* 30, 119-144.

Perron P, Zhu X (2005). "Structural Breaks with Deterministic and Stochastic Trends," *Journal of Econometrics,* 129, 65-119.

R Core Team (2023). `R`: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria. URL `https://cran.r-project.org/web/packages/mbreaks/index.html/`.

Yang J (2017). "Consistency of Trend Break Point Estimator with Underspecified Break Number," *Econometrics,* 5(4), 1-19.

Yao YC (1988). "Estimating the Number of Change-points via Schwartz' Criterion," *Statistics and Probability Letters,* 6, 181-189.

Zeileis A, Leisch F, Hornik K, Kleiber C (2002). "`strucchange`: An R Package for Testing for Structural Change in Linear Regression Models," *Journal of Statistical Software,* 7(2), 1-38.

Zeileis A, Kleiber C, Krämer W, Hornik K (2003). "Testing and Dating of Structural Changes in Practice," *Computational Statistics & Data Analysis*, 44(1-2), 109-123.

Appendix: R program to obtain the second-stage coefficient estimates in Section 4.2

```
data(nkpc)
T = dim(nkpc)[1]

##### First stage regression #####
#endogenous variable
X_e = as.matrix(nkpc$inffut,drop=FALSE)

#instrumental variables
instruments = c('inflag','lbslag','ygaplag','spreadlag','dwlag','dcplag')
Z = as.matrix(nkpc[,instruments])
Z = cbind(rep(1,T),Z)

#estimate the break dates in the first stage
mdl1 = dofix("inffut",instruments,data=nkpc,fixn=2,prewhit=0,eps1=0.1,m=5)
Tr1 = mdl1$date[1]
Tr2 = mdl1$date[2]


# generate the second-stage regressors (entire sample)
tind1r = seq(1, mdl1$date[1],1)
tind2r = seq(mdl1$date[1]+1,mdl1$date[2],1)
tind3r = seq(mdl1$date[2]+1,T,1)
Z1 = Z[tind1r,]
Z2 = Z[tind2r,]
Z3 = Z[tind3r,]
X_e1 = X_e[tind1r,]
X_e2 = X_e[tind2r,]
X_e3 = X_e[tind3r,]
Xh_e1 = Z1%*%solve(t(Z1)%*%Z1)%*%t(Z1)%*%X_e1
Xh_e2 = Z2%*%solve(t(Z2)%*%Z2)%*%t(Z2)%*%X_e2
Xh_e3 = Z3%*%solve(t(Z3)%*%Z3)%*%t(Z3)%*%X_e3
Xpred = rbind(Xh_e1,Xh_e2,Xh_e3)

##### Second stage regression #####
#independent variables
Y = as.matrix(nkpc[,'inf',drop=FALSE])
#second-stage regressors
Xh = as.matrix(nkpc[,c('inflag','lbs')])
Xh = cbind(rep(1,151),Xh,Xpred)

#sructural change tests and break date estimate by OLS
regressors = c('inffut', 'inflag', 'lbs')
```

```
mdl2 = mdl("inf",regressors,data=nkpc,m=5,eps1=0.1,prewhit=0)
T1 = mdl2$SEQ$date

# partition the second-stage regressors
tind1 = seq(1,T1,1)
tind2 = seq(T1+1,T,1)
Xh1 = Xh[tind1,]
Xh2 = Xh[tind2,]
Y1 = Y[tind1,1,drop=FALSE]
Y2 = Y[tind2,1,drop=FALSE]

#subsample coefficients and standard errors
beta1 = solve(t(Xh1)%*%Xh1)%*%t(Xh1)%*%Y1
beta2 = solve(t(Xh2)%*%Xh2)%*%t(Xh2)%*%Y2
k = dim(Xh)[2]
res1 = Y1-Xh1%*%beta1
res2 = Y2-Xh2%*%beta2
hac1 = correct(Xh1,res1,0)
hac2 = correct(Xh2,res2,0)
vhac1 = solve(t(Xh1)%*%Xh1)%*%hac1%*%solve(t(Xh1)%*%Xh1)
vhac1 = vhac1*(T1-k)
vhac2 = solve(t(Xh2)%*%Xh2)%*%hac2%*%solve(t(Xh2)%*%Xh2)
vhac2 = vhac2*(T-T1-k)
stdhac1 = sqrt(diag(vhac1))
stdhac2 = sqrt(diag(vhac2))
```