On the persistence of near surface temperature dynamics in a warming world

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Supplementary Information

Appendix A. Persistence estimate using daily observations.

In this Appendix, we extend our main analysis of the persistence parameter estimates using daily observations. To this end, we use geographically gridded daily mean air temperature provided by CHELSA (Climatologies at high resolution for the earth's land surface areas; see Karger et al., 2017, for more details). The data set has much higher resolutions (1800 arc sec) than the HadCRUT data set used in our main analysis and covers all land but no ocean area. More importantly, the data has a limited time span from January 1, 1979 to December 31, 2016. To make comparisons with our main analysis easier, we created a monthly data set by taking averages of the original daily observations each month from January 1979 to December 2016. We also produced anomalies out of the daily and monthly data, respectively, by subtracting the average of the same day (or month) over 1979 to 2016. We estimated α in (3) by using ordinary least squares.

Figure A1 shows persistence parameter estimates using monthly anomaly data on the left and those using daily anomaly data on the right. We use linearly detrended data by the ordinary least squares method. To be consistent with our main analysis, we consider a possible structural change in the linear trend in January 1992. This is a simpler method than our state-space model employed in the main analysis. However, it is inevitable due to high computational burden of using daily data and can be justified as the variance estimate of the trend component in the main analyses (σ_{ω}^{2}) is close to zero in most areas. The noise component is not removed so that the downward bias due to the noise may still exist.

We particularly emphasize two features in Figure A1, for the sake of our study. First, the persistence parameter estimate using monthly data, though using a different data set from the main analysis, is roughly consistent with the results presented in Figure 6 for land area. More importantly, the persistence parameter estimate using daily observations is much higher than those using monthly data. The former actually is close to one in the majority of geographical grids. Therefore, as discussed in Section 1, it is harder to detect potential structural changes in the persistence parameter if daily data are used. Second, the trend component does not seem to affect the persistence estimate, because the sample period is short. Also, the noise is larger in the CHELSA data due to their higher resolutions. For these reasons, we stick to HadCRUT data in this study.

Appendix B. A proof that the OLS estimate $\widehat{\alpha}$ in Equation (3) is biased when a trend is present.

In this Appendix, we show that the persistence parameter estimate in (3) using OLS has a bias when linear trend exists but is not accounted for in the estimation. In fact, it is inconsistent and converges to 1 as T increases. Consider model (1) with a linear trend ($\tau_t = bt$) with no measurement error.

$$y_t = bt + x_t,$$

for $t = 1, \dots, T$, where x_t follows (2). We consider the sample autocovariance of y_t of order h, by letting $\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$ and $\tilde{x}_t = x_t - \frac{1}{T}\sum_{s=1}^{T} x_s$,

$$\begin{aligned}
R(h) &= T^{-1}\sum_{t=1}^{T-h}(y_t - \bar{y})(y_{t+h} - \bar{y}),\\
&= T^{-1}\sum_{t=1}^{T-h}\left(bt - b\frac{T+1}{2} + \tilde{x}_t\right)\left\{b(t+h) - b\frac{T+1}{2} + \tilde{x}_{t+h}\right\},\\
&= \frac{b^2}{T}\sum_{t=1}^{T-h}\left\{t^2 - (T+1-h)t + \frac{(T+1)^2 - 2h(T+1)}{4}\right\} + \varphi_h,\\
&= \frac{b^2}{T}\left\{\frac{(T-h)(T-h+1)(2T-2h+1)}{6} - \frac{(T+1-h)(T-h)(T-h+1)}{2},\right.\\
&\qquad \left. + \frac{(T+1)(T+1-2h)(T-h)}{4}\right\} + \varphi_h,\\
&= \frac{b^2(T-h)}{12T}(T^2 - 2hT - 2h^2 - 1) + \varphi_h,
\end{aligned}$$

where $\varphi_h = \frac{b}{T}\sum_{t=1}^{T-h}\left\{\left(t - \frac{T+1}{2}\right)\tilde{x}_{t+h} + \left(t + h - \frac{T+1}{2}\right)\tilde{x}_t\right\} + \frac{1}{T}\sum_{t=1}^{T-h}\tilde{x}_t\tilde{x}_{t+h} = O_p(T^{3/2})$. When $h = 0$, we obtain

$$R(0) = T^{-1}\sum_{t=1}^{T}\left(bt - b\frac{T+1}{2} + \tilde{x}_t\right)^2 = \frac{b^2}{12}(T+1)(T-1) + \varphi_0.$$

Hence, the sample autocorrelation is

$$\begin{aligned}
\rho(h) = \frac{R(h)}{R(0)} &= \frac{(T-h)\left(T^2 - 2hT - 2h^2 - 1\right) + \frac{12}{b^2}T\varphi_h}{T(T+1)(T-1) + \frac{12}{b^2}T\varphi_0},\\
&= \frac{\left(1 - \frac{h}{T}\right)\left(1 - \frac{2h}{T} - \frac{2h^2+1}{T^2}\right) + \frac{12}{b^2T^2}\varphi_h}{\left(1 + \frac{1}{T}\right)\left(1 - \frac{1}{T}\right) + \frac{12}{b^2T^2}\varphi_0}.
\end{aligned}$$

From this, it is straightforward to show that $\lim_{T\to\infty}\rho(h) = 1$ for any fixed h. Our specific case corresponds to $h = 1$ and the same result follows.

Appendix C. The Warm Spell Duration Index (WSDI) and the Cold Spell Duration Index (CSDI) from 1901 to 2018.

The WSDI and CSDI indices shown in Figures 1 and 3 are continuously presented from 1901 to 2018 at the following link.

https://doi.org/10.6084/m9.figshare.24718107.v1

Appendix D. AR(1) coefficient estimate when the data includes measurement errors.

This Appendix shows that y_t follows an ARMA(1,1) process if it is generated by (1) and (2) with $\tau_t = 0$. Also, as the variance of the noise increases, the roots cancel and y_t is uncorrelated.

First, plugging (2) in (1) gives

$$\begin{aligned} y_t &= \alpha(x_{t-1} + e_t) + \omega_t \\ &= \alpha(x_{t-1} + \omega_{t-1}) + e_t + \omega_t - \alpha\omega_{t-1} \\ &= \alpha y_{t-1} + e_t + \omega_t - \alpha\omega_{t-1} \\ &= \alpha y_{t-1} + u_t \end{aligned}$$

where $u_t = e_t + \omega_t - \alpha\omega_{t-1}$. Since

$$\begin{aligned} E(u_t^2) &= \sigma_e^2 + (1+\alpha^2)\sigma_\omega^2, \\ E(u_t u_{t-1}) &= -\alpha\sigma_\omega^2, \\ E(u_t u_{t-\tau}) &= 0, \text{ for } |\tau| > 1, \end{aligned}$$

u_t has an MA(1) representation. Then, we let $u_t = v_t + \theta v_{t-1}$ where $v_t \sim i.i.d.(0, \sigma_v^2)$ and obtain the following two equations

$$E(u_t^2) = \sigma_e^2 + (1+\alpha^2)\sigma_\omega^2 = (1+\theta^2)\sigma_v^2, \qquad \text{(A.1)}$$

$$E(u_t u_{t-1}) = -\alpha\sigma_\omega^2 = \theta\sigma_v^2. \qquad \text{(A.2)}$$

By plugging $\sigma_v^2 = -\alpha\sigma_\omega^2/\theta$ from (A.2) into (A.1), we have the following equation for θ

$$\alpha\sigma_\omega^2\theta^2 + [\sigma_e^2 + (1+\alpha^2)\sigma_\omega^2]\theta + \alpha\sigma_\omega^2 = 0.$$

The condition for the roots to be real is

$$\begin{aligned} \Delta &= [\sigma_e^2 + (1+\alpha^2)\sigma_\omega^2]^2 - 4(\alpha\sigma_\omega^2)^2 \\ &= [\sigma_e^2 + (1+\alpha^2)\sigma_\omega^2 + 2\alpha\sigma_\omega^2][\sigma_e^2 + (1-\alpha)^2\sigma_\omega^2] > 0. \end{aligned}$$

Assuming that the process u_t is invertible, we also need to choose the root $|\theta| < 1$. Thus,

$$\theta = \frac{-[\sigma_e^2 + (1+\alpha^2)\sigma_\omega^2] + \sqrt{\Delta}}{2\alpha\sigma_\omega^2}.$$

In conclusion, y_t is an ARMA(1,1) process of the form

$$y_t - \alpha y_{t-1} = v_t + \theta v_{t-1}.$$

It is easy to show as σ_ω^2 gets large, θ approaches $-\alpha$; hence, the roots cancel and y_t becomes uncorrelated. Hence, an increase in σ_ω^2 imparts a downward bias on the first-order correlation coefficient and the measure of persistence.

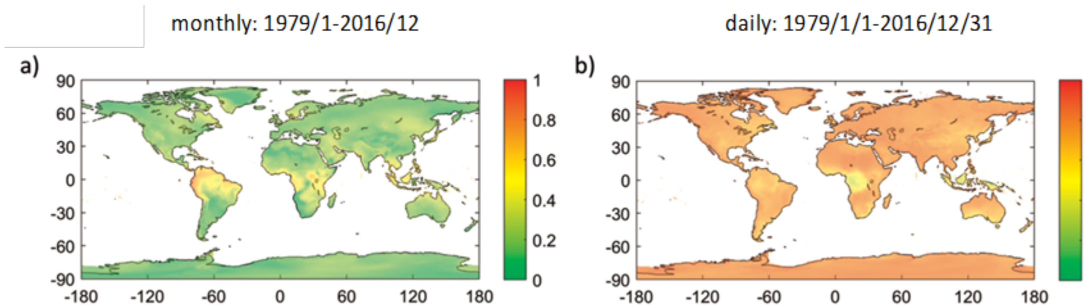

Figure A1. Persistence parameter estimate using monthly (a) and daily (b) land surface temperature data obtained from CHELSA (Karger et al., 2017). The ordinary least squares method is used to estimate α in (3), where y_t is monthly or daily surface temperature linearly detrended with a possible structural change in January 1992.