How Do Firms Advertise When Customer Reviews are Available?*

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Abstract

Online consumer product reviews have become very popular and influential in consumers' purchase decisions. I study how competing firms choose advertising and prices when customer reviews are available and when firms may build up loyal customer bases. The model predicts that higher-rated firms are more likely to be dominant in advertising. In other words, online reviews are a complement to firms' advertising. I also analyze an extreme case of the model: an entry game in which an entrant and an incumbent interact. I find that the availability of customer reviews undoes the "fat-cat" effect of a big incumbent with a lot of loyal customers. An incumbent with a high enough ratio of good reviews can successfully deter entry and maintain a high profit. Comparative statics of the theory model can explain the pattern of advertising response to Yelp rating found in the empirical RDD paper.[1] Intuitively, when the capacity limit of a local business becomes binding, a jump in the display rating will reduce the complemental effect of online reviews on advertising.

Keywords: advertising, online customer reviews, brand loyalty, entry deterrence

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[1]See the empirical findings in "Advertising Response to A Better Online Rating: A Regression Discontinuity Design on Local Restaurants" which can be found on http://blogs.bu.edu/ylei/research/.

1 Introduction

In recent years, customer reviews have become an important part of consumers' shopping experience. The percentage of consumers who read reviews (occasionally or regularly) before purchase to determine the quality of a business has been steadily increasing, and in 2015 this number reached 92%.[2] When consumers can get information about firms' qualities from online customer reviews, how do firms compete when they have different reviews? In particular, my paper studies competing firms' advertising strategies when customer reviews are available, and combining with data, offers new insights on the interaction between online customer reviews and informative advertising: Are they substitutes or complements?

Previous research finds that reviews, specifically good reviews, are a substitute for advertising, and in particular, that firms with better reviews advertise less. However, I introduce two realistic features that complicate this story. The first is loyal customers. In particular, consumers that are satisfied with a local business not only leave good reviews, and they will also return and become loyal customers. Second, in many cases, advertising and reviews offer different types of information. Whereas advertising informs consumers about the existence of a product or raises consumer awareness, reviews offer a more credible source of information about quality. I show these features interact to generate surprising results about the relationship between reviews and advertising.

I consider a non-durable experience good. For such a good, consumers' individual values are initially unknown and can be learned only after purchase, and each consumer may purchase more than once. Advertising for an experience good, as Nelson (1970) points out, cannot convey direct information of product quality because consumers will not trust such information that is not verifiable before purchase. Indirect information that advertising may carry includes a product's existence and price.

By advertising, a firm informs new consumers about the existence of its product, and these informed consumers will have access to the firm's price and reviews before purchase. A familiar example is consumers see various types of advertisements of a local restaurant, and then go to Yelp.com to check this restaurant's reviews and menu (including prices). There are also some other less obvious forms of advertising. Consider the example of Amazon.com: firms need to pay a fee ($39.99) per month to be listed and sell on Amazon. Shoppers visiting Amazon can see all listed firms' prices and reviews. Therefore, advertising in my paper can be interpreted more generally as a marketing tool that raises awareness of consumers about a product, and consequently makes the price

[2] Data source: The Local Consumer Review Survey by BrightLocal. This survey was conducted over 2 weeks in July each year, starting from 2010. The 2015 survey has 2,354 entries with 90% of respondents coming from the US and 10% from Canada.

and reviews of this product observable to those consumers.

Customer reviews in my model are reports of individual satisfaction from previous buyers. Consumers have heterogeneous preferences, and each consumer may be satisfied or dissatisfied with a product. Before purchase, individual satisfactions are unknown and all consumers are *ex ante* homogeneous. Consumers who see reviews of a product, see how previous buyers were satisfied with it, and will use these reviews to update their belief about the likelihood of themselves being satisfied with the product. I assume that consumers report truthfully, but this assumption can be relaxed and does not affect the conclusions.

I define a product's quality to be its *ex ante* satisfaction likelihood for a new consumer. That is, a higher quality product has a higher probability, *ex ante*, to make a random new consumer satisfied. A new firm's product quality is unknown, and consumers and firms hold a common prior belief about it. As a firm receives reviews from its buyers, the belief about its product quality will be updated according to Bayes' rule.

In this model, two firms with unknown qualities compete with each other. Firms first choose whether or not to advertise, and after observing each other's advertising decision, firms choose prices. There are a finite number (n) of consumers randomly drawn from the population to become "shoppers". (Justifications can be the group of consumers who consider having lunch at a local restaurant today in a certain neighborhood, or the group of consumers who visit Amazon.com to search for a specific product in a month.) Only firms that choose to advertise are known to the n shoppers. If only one firm advertises, it can charge a monopoly price. But if both firms advertise, they need to compete for the shoppers in price.

I consider the competition between two firms that have operated for some periods, and hence have already received some reviews. The good reviews that a firm receives come from its previous buyers who were satisfied with their purchase. I assume satisfied previous buyers will repeat purchase and become "loyal customers" of this firm.[3] Firms know about their own and each other's loyal customer base (because they are automatically aware of each other and can see each other's reviews). If a firm advertises, the new shoppers will be able to see this firm's reviews and use them to update the belief about this firm's product quality. The shoppers make purchase decisions based on both prices and reviews of the advertised firm(s).

For a firm with some reviews and a nonzero loyal customer base, in choosing a price, this firm trades off between charging a high price to sell only to loyal customers, and lowering price to attract new shoppers. In the price competition between two advertised firms, there is no pure strategy equilibrium if at least one firm owns nonzero loyal

[3]Repeat purchase from loyal customers is a very important part of local restaurants' business.

customers. However, there is a unique mixed strategy equilibrium, in which both firms randomize pricing, and the firm with better reviews and more loyal customers randomizes over a higher range of prices.

In the subgame perfect Nash equilibrium of the two-stage advertising-pricing game, firms' advertising strategies will depend on how differently the two firms are rated in their reviews. If firms are similarly rated, there will be multiple equilibria and one firm advertises only if the rival does not. In other words, advertising is a strategic substitute for similarly rated firms. However, if one firm is rated much better than the rival, the better-rated firm is the only one that advertises. Therefore, having a relatively higher rating helps a firm to be dominant in advertising.

In an extension, I show that these conclusions still hold if I allow a group of consumers to search for firms. The "Searchers" do not need to receive advertisements to be informed about firms, and they can see all firms' prices and reviews. This robustness check makes the implications of my model applicable to a more general case, where consumers may search but firms can advertise to reach more consumers.

In the main body of this paper, I consider the case where two firms have the same number of previous buyers. But the conclusions also hold if two firms have different numbers of previous buyers, i.e. asymmetric firm histories. In particular, I extend the main model to an entry deterrence problem which can be interpreted as an extreme case of asymmetric firm histories. An incumbent firm interacts with a potential entrant, and entry happens if the entrant chooses to advertise. The incumbent already has some reviews from previous buyers, and owns a loyal customer base. In this extension, I find that entry can be deterred if the incumbent was successful enough in the past, i.e. has a high ratio of good reviews. There is an interesting interpretation of my model vis-à-vis the "fat-cat" effect as in Fudenberg and Tirole (1984). When the incumbent has a big group of loyal customers (a "fat cat"), in Fugenberg and Tirole's model, it should be weak in the competition with potential entrants. However, with the availability of customer reviews, if the incumbent has a large enough percentage of good reviews, it will successfully deter entry and is therefore a "fat-but-strong cat".

Finally, I use data on local restaurants' advertising spending and Yelp reviews to test the model prediction and find supporting evidence on the positive effect of a higher average rating on restaurants' advertising spending. In addition, by using a Regression Discontinuity Design, I successfully separate the effect of display rating from the effect of average rating.[4] In the discussion section, I explain why we observe these two effects having opposite directions in their relationship with advertising.

[4]See Figure 14 in Appendix 2 for an example Yelp web page showing the display rating and the average rating of a restaurant.

2 Literature Review

My paper first contributes to the literature of advertising (for a review, see Bagwell (2007)), and in particular relates to the papers on informative advertising for experience goods. Nelson's (1970) seminal paper differentiates between search goods and experience goods, and starts a discussion about indirect informative advertising for experience goods (Nelson, 1974). An important benefit of such advertising is creating repeat purchases. Following Nelson's conjecture, Schmalensee (1978), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986) and Hertzendorf (1993) have formally studied this repeat-purchase effect of advertising. Because of the various restrictions these papers put on firms' or consumers' dynamic decisions, the repeat-purchase effect has not been fully investigated. Although creating repeat purchases in the future is a long-term benefit of advertising (the repeat-purchase effect), the current repeated purchases from previous buyers reduces a firm's incentive to advertise in the current period (the loyal-customer effect). In my paper, I take the loyal-customer effect into account to study firms' advertising strategies.

There have been several papers investigating the Bertrand price competition between firms with loyal customers, and showing that such Bertrand games have no pure strategy equilibrium. Varian (1980) and Narasimhan (1988) study the competition between firms when each firm has an exogenous group of loyal customers ("uninformed consumers" in Varian 1980), and analyze the mixed strategy equilibrium of the Bertrand game. In McGahan and Ghemawat (1994) and Chioveanu (2008), firms are allowed to invest first in building their loyal customer bases. They both assume that the size of a firm's loyal customer base is determined with certainty by firms' decisions (service in McGahan and Ghemawat 1994 or persuasive advertising in Chioveanu 2008). In this paper, I study the two-stage advertising-pricing game, and the mixed strategy equilibrium only happens in a subgame and actually serves as a threat to the firm without advantage in the price competition.

Another literature that my paper contribute to is the one on customer reviews. There have been many great papers showing the importance of online customer reviews. Using data on Amazon.com and BN.com, Chevalier and Mayzlin (2006) show that online reviews are very influential to consumers' purchase decisions about books. Luca (2011) uses data from Yelp.com to study how reviews affect firms' sales and how consumers learn from reviews. Sun (2012) demonstrates how the variance and the average of ratings interact in affecting firms' sales.

Within this literature, my paper is closely related to a specific group of papers that studies the interaction between firms' marketing strategies and product reviews. Chen and Xie (2005) study two competing firms' advertising and pricing strategies in response to third-party product reviews when a lot of consumers have strong preference on horizontal attributes. Mayzlin (2006) talks about when firms post fake reviews in response to different ratings from customer reviews. Another paper by Chen and Xie (2008) studies how a monopoly firm 1) chooses how much product attribute information to reveal, 2) decides whether to make previous customer reviews available to future consumers, and 3) proactively control the informativeness of customer reviews, for different types of products. In Chen and Xie's (2008) paper, all customer reviews are the same and give one signal, match or mismatch. My paper takes into account the heterogeneity in customer reviews and uses Bayesian learning to model new consumers' belief updating using these reviews. Therefore, when firms are rated differently, their advertising strategies in competition will be different.

The rest of this paper proceeds as follows: Section 3 gives the setting of the main model, and Section 4 contains the equilibrium analysis, Section 5 contains two extensions: entry game and Searchers model. In Section 6 I test the model prediction with data, Section 7 discusses the empirical findings and Section 8 concludes.

3 Main Model

3.1 Players and Information

There is a continuum of consumers in the population, and a finite number ($n \in \mathbb{N}$) of consumers are randomly drawn to be shoppers. Each would like to buy at most one unit of a good. The good is an *experience* good. After purchase, a consumer derives either a value of 0 or 1 from consuming the good, but this individual match value is unknown before purchase, and consumers are *ex ante* homogeneous. I also assume that consumers are initially unaware about the availability of the good, but they can be informed by firms through advertising.

Firm A and Firm B sell the good, and compete for only one period. Marginal cost of production is assumed to be the same for both firms, and normalized to 0. An advertisement by a firm informs shoppers about the availability of the firm's product. If both Firms A and B advertise, the n shoppers are informed of both firms' goods. Advertising is a discrete-choice variable. A firm can choose either to advertise ($M = 1$) or not to ($M = 0$). The cost of advertising is fixed and denoted by $c \in \mathbb{R}_+$.

Let θ^A and θ^B denote, respectively, the probabilities that products of Firms A and B will yield a value 1 to a shopper. Neither firms nor consumers know these probabilities *ex ante*. However, it is common knowledge that θ^A and θ^B are drawn independently and identically from a uniform distribution on $[0, 1]$. We may call θ^A and θ^B firms' product qualities, or their types.

There is a special group of consumers, called "loyal customers". If a previous buyer derives a value of 1 from a firm's product, this consumer will repeat purchase and become a "loyal customer" of this firm thereafter. Assume that, at the beginning of this model, each firm has already built up a loyal customer base, denoted by $L^k, k = A, B$. Let T^A and T^B denote the total number of previous buyers of Firm A and Firm B respectively.[5] Assume T^A and T^B are both finite numbers, i.e. Firms A and B have not been operated for infinite periods, so that the ratio L^k/T^k cannot predict precisely the value of θ^k.

Customer reviews are defined to be consumers' truthful reports of their satisfaction with a product after consuming it. Reviews can be good (taking value 1), or bad (taking value 0). Assume that all previous buyers wrote reviews.[6] Firm A's good reviews come from those previous buyers of Firm A who derived value 1, and these consumers now constitute Firm A's loyal customer base (L^A). Firm A's bad reviews come from the dissatisfied previous buyers of Firm A, $T^A - L^A$. Similarly, for Firm B, the good reviews come from its satisfied previous buyers who now constitute Firm B's loyal customer base (L^B), and bad reviews come from its dissatisfied previous buyers, $T^B - L^B$.

Once a consumer is aware of a firm's existence, she will have free access to all of the firm's previous customer reviews, L^k good and $T^k - L^k$ bad, and the firm's price p^k. Firms can always see each other's previous customer reviews.

The firms and consumers interact over two stages in the single period.

Stage 1 (Advertising) Each firm chooses whether to advertise. The new consumers (n) are informed by firms' advertisements, and become aware of the advertised firm(s).

Stage 2 (Pricing) Observing each other's advertising decision, firms now choose prices simultaneously. The new consumers have access to the advertised firms' customer reviews and prices, and then make purchase decisions. Each firm's loyal customers repeat purchase.

If both firms advertise, the n shoppers choose between two firms based on their reviews and prices. Loyal customers do not consider switching because they are already enjoying the highest possible value – 1. I assume that a firm cannot discriminate between its loyal customers and new shoppers. The same price is charged to all buyers.

[5]For this customer review model to be meaningful, assume at least one of T^A and T^B is nonzero. In other words, assume at least one firm has some reviews.

[6]This assumption is not crucial and can be relaxed, because new consumers learn about a product's quality only through the available reviews. The shopping experience, good or bad, of those previous buyers who have not left reviews failed to be conveyed to new consumers. What matters is the available reviews.

If Firm A's quality θ^A is known, after purchasing from Firm A, a consumer gets value 1 with probability θ^A; analogously with probability θ^B if the purchase is from Firm B. However, two firms' qualities are unknown, to consumers and to firms themselves.[7] The probability of being satisfied (i.e., getting value 1) with a product is unknown, and everyone learns from firms' previous customer reviews.

3.2 Belief Updating

I assume consumers are rational. First, before knowing the individual value of a product, a rational consumer updates her belief of the product quality according to Bayes' rule. Second, a consumer values a product by the expected quality in the initial purchase, and by her individual value of this product in repeated purchases, and a consumer is rational in that she will not pay for prices above her value of a product.

Recall that a firm's product quality is defined to be the probability ($\theta \in [0, 1]$) that its product will yield a value 1 to a randomly chosen shopper. A belief about a firm's quality is therefore a probability distribution on $[0, 1]$. I use the Beta distribution to model beliefs.[8] Good and bad reviews of a firm can be viewed as successes and failures of Bernoulli trials, and all serve as signals to update belief. For a prior belief that is described by a Beta distribution, after updating with the Bernoulli trials, the posterior belief again follows Beta distribution, only with updated parameters.[9] Specifically, the belief-updating process is as follows.

For a new firm with no reviews, the common prior belief of its quality is the Beta distribution with parameters 1 and 1, $Beta(1, 1)$, which is equivalent to the uniform distribution on $[0, 1]$. Let $\tilde{\theta}_0$ denote the expected quality of a new firm, then $\tilde{\theta}_0 = 1/2$. Therefore, all new firms share the same prior belief and expected quality, 1/2, even though they may have different true qualities.

As firms start receiving reviews, the beliefs about their qualities will be updated. If a firm has received a total number T of reviews, among which there are L good reviews, using Bayes' rule, the updated expected quality will be $\tilde{\theta} = \frac{1+L}{2+T}$.

Specifically, in this model, Firm A has, in total, T^A reviews, and L^A out of T^A are good reviews. Update the common prior belief, $Beta(1, 1)$, with these good and bad reviews, and the posterior belief will be distributed as $Beta(1+L^A, 1+T^A-L^A)$. Therefore, when

[7] This assumption can be interpreted as the uncertainty about whether consumers will like the food of a restaurant. If restaurant owners know exactly what consumers like, they will all provide the most favorable food, and we won't see bad reviews at all. However, obviously this is not true.

[8] The Beta distribution, $Beta(a, b)$ (a and b are parameters), is a continuous distribution on $[0, 1]$, and the expectation is $\frac{a}{a+b}$.

[9] For more details about the Beta distribution and its property of being a conjugate prior distribution, please refer to DeGroot, M.H. & M.J. Schervish (2011), *Probability and Statistics*, 4th Ed (specifically p. 327-333 and Theorem 7.3.1 on p. 394).

consumers see Firm A's reviews, their belief is updated such that the expected quality of Firm A becomes

$$\tilde{\theta}^A = \frac{1 + L^A}{2 + T^A} \tag{1}$$

Similarly, Firm B has L^B good reviews, and $T^B - L^B$ bad reviews. Therefore, updated with these reviews, Firm B's expected quality is

$$\tilde{\theta}^B = \frac{1 + L^B}{2 + T^B} \tag{2}$$

Only new consumers need the reviews to update their beliefs. Loyal customers of each firm have already learned their personal match value, which is 1, with the product they are buying.

4 Equilibrium Analysis

In this model, firms' actions in Stage 1 are publicly observable in Stage 2, and once a firm has advertised, new consumers and firms will have symmetric information about the advertised firm's (expected) product quality. Therefore, I solve by backward induction for the subgame perfect Nash equilibrium of this two-stage game.

At the beginning of Stage 1, the state of the game is described by two firms' total reviews, T^A and T^B respectively, and good reviews, L^A and L^B respectively. For the following analysis, I use a special case where $T^A = T^B = T > 0$. The analyses for the other two cases, $T^A > T^B \geq 0$ and $T^B > T^A \geq 0$, will be essentially the same, and are briefly explained in the Appendix.[10]

Each firm owns a loyal customer base, the size of which equals to the number of the firm's good reviews, $L^k, k = A, B$. These loyal customers are willing to pay price 1 for the firm's product. If the firm advertises, new consumers read its reviews and are willing to pay $\tilde{\theta}^k = \frac{1+L^k}{2+T^k} < 1$ for the firm's product.

By the equilibrium analysis of this two-stage game, we want to see firms' advertising strategies (M^k) in competition when they have different ratios of good reviews (L^k/T^k).[11] Advertising is costly (fixed cost c), and the benefit it brings is *"expansion"*: to expand a firm's customer base and sell to more consumers.

For a firm with nonzero loyal customers, if it does not expand (by advertising), it always has a *"secured profit"* because this firm can charge price 1 to its loyal customers. So Firm A's secured profit is L^A, and Firm B's is L^B. The existence of secured profit

[10] An extreme case of the asymmetric previous-buyers setting is an entry game with an established incumbent and a new entrant, $T^A > T^B = 0$, which is analyzed in Section 5 as an extension.

[11] The ratio of good reviews can be roughly corresponded to a firm's rating.

Table 1: The Normal Form of The Game

		Firm A	
		$M^A = 1$	$M^A = 0$
Firm B	$M^B = 1$	Pricing Subgame	$L^A, (L^B + n)\tilde{\theta}^B - c$
	$M^B = 0$	$(L^A + n)\tilde{\theta}^A - c, L^B$	L^A, L^B

reduces a firm's incentive to advertise. When Firm A owns a group of loyal customers $L^A > 0$, if the highest profit that Firm A can obtain from expansion is lower than its secured profit (L^A), Firm A will never choose to expand (by advertising). The highest profit Firm A can get from expansion is when the opponent does not advertise, and Firm A sells to all new consumers (n) and its loyal customers (L^A) at the monopoly price $\tilde{\theta}^A$: $(L^A + n)\tilde{\theta}^A - c$. In other words, it is profitable for Firm A to expand through advertising only if

$$(L^A + n)\tilde{\theta}^A - c \geq L^A \tag{3}$$

We say Firm A satisfies the *"Profitable Expansion" (PE)* condition if (3) is satisfied. Analogously, Firm B satisfies the PE condition if

$$(L^B + n)\tilde{\theta}^B - c \geq L^B \tag{4}$$

If no firms satisfy the PE condition, the equilibrium will be trivial: no firm advertises and each firm sells to its loyal customer base at price 1. If there is only one firm, say Firm A, that satisfies the PE condition, the equilibrium will be that only Firm A advertises. And in this equilibrium, Firm A charges a price $p^A = \tilde{\theta}^A$ to both new and loyal customers, while Firm B charges 1 and earns its secured profit L^B from loyal customers.

Suppose the state variables are such that both firms satisfy PE, i.e., both (3) and (4) hold, then the normal form of the game is as shown in Table 1. For subgames where at most one firm advertises, the payoffs are straightforward. If both firms advertise, the n shoppers are aware of both firms and can see all customer reviews. A pricing subgame follows, and I show next how two established firms compete for new shoppers by price.

The Pricing Subgame

Firms have different expected qualities unless they have exactly the same number of good reviews.[12] The pricing subgame is therefore a Bertrand competition between firms producing goods of different expected qualities (at the same cost 0). New shoppers choose

[12] Recall that I assume two firms have the same total number of reviews for now.

the product from Firm A if and only if

$$\tilde{\theta}^A - p^A > \tilde{\theta}^B - p^B$$

Let $d = \tilde{\theta}^A - \tilde{\theta}^B$. Therefore, Firm A tends to undercut Firm B's price p^B by charging just below $p^B + d$, and similarly, Firm B tends to undercut Firm A by charging just below $p^A - d$.

Unlike in common Bertrand games, firms here are unwilling to undercut each other all the way down to the marginal cost of production (here it is 0). Because of loyal customers (and hence the secured profit), there is a lowest price that a firm is willing to charge in the pricing subgame, which I call the firm's *"reservation price"* in the price competition. Firm A does not want to charge any price that yields a lower profit from the price competition than Firm A's secured profit L_2^A, and the lowest price that Firm A is willing to charge in the price competition satisfies $(L^A + n)p^A = L^A$. So I call Firm A's reservation price

$$\gamma^A = \frac{L^A}{L^A + n}$$

Similarly, Firm B's reservation price in the price competition is

$$\gamma^B = \frac{L^B}{L^B + n}$$

The existence of loyal customers creates jump discontinuities in firms' best response functions in the pricing subgame. For each firm, the jump happens at the point of the firm's reservation price. If undercutting the rival requires Firm A to charge a price below its reservation price γ^A, Firm A would rather charge price 1 to its loyal customers and do not sell to new shoppers. The discontinuity at firms' reservation prices, caused by the existence of loyal customers, leads to the following lemma (see all proofs in the Appendix).

Lemma 1. *When both firms advertise and compete in price for new shoppers, if at least one firm has a nonzero loyal customer base, there will be no pure strategy equilibrium in this pricing subgame.*

The existence of loyal customers creates asymmetric information among consumers. Like papers in the literature of price dispersion (Varian 1980; Chioveanu 2008), this asymmetry in information among consumers leads to a mixed strategy equilibrium. How firms act in the mixed strategy equilibrium depends on the relationship between γ^A and $\gamma^B + d$ (recall that $d = \tilde{\theta}^A - \tilde{\theta}^B$). Intuitively, it depends on which firm can undercut the opponent further in the price competition. In particular, if γ^A and γ^B satisfy

$$\gamma^A < \gamma^B + d, \tag{5}$$

when Firm B charges its reservation price $p^B = \gamma^B$, Firm A can undercut it by charging just below $\gamma^B + d$, which is still above Firm A's reservation price γ^A. I say Firm A satisfies the *"Advantage in Price Competition" (APC)* condition if (5) is satisfied. Similarly, Firm B satisfies the APC condition if

$$\gamma^A > \gamma^B + d. \tag{6}$$

How the mixed strategy equilibrium depends on this inequality is shown in the next proposition.

Proposition 1. *Consider the pricing subgame when Firm A and Firm B both advertise. Suppose that at least one of them has a nonzero loyal customer base. In particular, suppose Firm A has a loyal customer base L^A and expected quality $\tilde{\theta}^A$, and analogously Firm B has L^B and $\tilde{\theta}^B$. This pricing subgame has a unique mixed strategy equilibrium.*

Let $d = \tilde{\theta}^A - \tilde{\theta}^B, \gamma^A = \frac{L^A}{L^A+n}, \gamma^B = \frac{L^B}{L^B+n}$.

(1) If Firm A has advantage in competition, i.e., (5) holds, in the mixed strategy equilibrium $(F_A(p), F_B(p))$, Firm A gets an expected profit of $(L^A + n)(\gamma^B + d)$, Firm B gets an expected profit of L^B, and $F_A(p)$ first order stochastically dominates $F_B(p)$. The distribution functions in the equilibrium are

$$F_A(p) = \begin{cases} 0 & p \leq \gamma^B + d \\ 1 - \frac{L^B}{n(p-d)} + \frac{L^B}{n} & \gamma^B + d \leq p \leq \tilde{\theta}^A \\ 1 & p \geq \tilde{\theta}^A \end{cases}$$

and

$$F_B(p) = \begin{cases} 0 & p \leq \gamma^B \\ 1 - \frac{(L^A+n)(\gamma^B+d)}{n(p+d)} + \frac{L^A}{n} & \gamma^B \leq p \leq \tilde{\theta}^B \\ 1 & p \geq \tilde{\theta}^B \end{cases}$$

(2) If Firm B has advantage in price competition, i.e., (6) holds, in the mixed strategy equilibrium $(F_A(p), F_B(p))$, Firm A gets an expected profit of L^A, Firm B gets expected profit $(L^B + n)(\gamma^A - d)$, and $F_B(p)$ first order stochastically dominates $F_A(p)$. The distribution functions in the equilibrium are

$$F_A(p) = \begin{cases} 0 & p \leq \gamma^A \\ 1 - \frac{(L^B+n)(\gamma^A-d)}{n(p-d)} + \frac{L^B}{n} & \gamma^A \leq p \leq \tilde{\theta}^A \\ 1 & p \geq \tilde{\theta}^A \end{cases}$$

Table 2: Payoffs When Condition APC-A Holds

		Firm A: $M^A = 1$	Firm A: $M^A = 0$
Firm B	$M^B = 1$	$(L^A + n)(\gamma^B + d) - c, L^B - c$	$L^A, (L^B + n)\tilde{\theta}^B - c$
	$M^B = 0$	$(L^A + n)\tilde{\theta}^A - c, L^B$	L^A, L^B

and

$$F_B(p) = \begin{cases} 0 & p \leq \gamma^A - d \\ 1 - \frac{L^A}{n(p+d)} + \frac{L^A}{n} & \gamma^A - d \leq p \leq \tilde{\theta}^B \\ 1 & p \geq \tilde{\theta}^B \end{cases}$$

If a firm has advantage in price competition, it gets an expected profit from the pricing subgame higher than its secured profit. The firm without advantage in price competition only earns the same (in expectation) as its secured profit. In general, a firm satisfies the APC condition if it has a relatively higher ratio of good reviews. See the green area in Figure 1.

The APC condition only predicts the winner (in terms of expected profit) in the pricing subgame if both firms have advertised. However, it does not give any information about whether the expected winner might want to enter this pricing subgame.

Advertising Game

Without loss of generality, in the following analysis, I suppose Firm A satisfied the APC condition, i.e., condition (5) holds.

From Proposition 1 and the fact that (5) is true (APC-A), in the pricing subgame after both firms advertise, Firm A gets an expected payoff of $(L^A + n)(\gamma^B + d) - c$, and Firm B gets an expected payoff of $L^B - c$. We now have payoffs in the pricing subgame, and the advertising game with payoffs when APC-A holds is shown in Table 2.

Equilibria in cases where at least one firm do not satisfy the PE condition, i.e., (3) or (4) does not hold, have been analyzed earlier, and here I study the equilibrium for the case where it is profitable for both firms to expand through advertising. That is, (3) and (4) hold simultaneously.

From Table 2, we can see that Firm B (the firm that does not satisfy the APC condition) advertises only if Firm A does not advertise. Firm A also advertises if Firm B does not, but it may still choose to advertise even if Firm B advertises. Therefore, the relationship between Firm A's expected payoff from the pricing subgame, $(L^A + n)(\gamma^B + d) - c$, and Firm A's secured profit, L^A, determines how firms advertise in the equilibrium.

Proposition 2. *Consider the competition between two firms, Firm A with a loyal customer base L^A and expected quality $\tilde{\theta}^A$, Firm B with loyal customer base L^B and expected* *quality $\tilde{\theta}^B$. Suppose Firm A satisfies the APC condition, i.e.,* (5) *holds, and suppose both* (3) *and* (4) *are satisfied, then,*

1) If

$$(L^A+n)(\gamma^B+d)-c \geq L^A, \tag{7}$$

there is a unique equilibrium in which only Firm A advertises $(M^A=1, p^A=\tilde{\theta}^A; M^B=0, p^B=1)$.

2) If

$$(L^A+n)(\gamma^B+d)-c < L^A, \tag{8}$$

there are multiple equilibria: i) only Firm A advertises $(M^A=1, p^A=\tilde{\theta}^A; M^B=0, p^B=1)$*; ii) only Firm B advertises* $(M^A=0, p^A=1; M^B=1, p^B=\tilde{\theta}^B)$*; and iii) both firms randomize advertising with probabilities respectively* $\lambda^A=\frac{L^B-(L^B+n)\tilde{\theta}^B+c}{L^B-(L^B+n)\tilde{\theta}^B}$ *and* $\lambda^B=\frac{L^A-(L^A+n)\tilde{\theta}^A+c}{(L^A+n)(\gamma^B-\tilde{\theta}^B)}$.

To summarize the equilibrium analysis, the advertising equilibrium is characterized by three conditions on state variables $(L^A, L^B, T^A=T, T^B=T)$, or equivalently $(L^A, L^B, \tilde{\theta}^A, \tilde{\theta}^B)$. The three conditions are Profitable Expansion (PE), Advantage in Price Competition (APC), and Uniqueness (U), and are summarized below.

Profitable Expansion (PE)

- (PE – Firm A) $(L^A+n)\tilde{\theta}^A-c>L^A$
- (PE – Firm B) $(L^B+n)\tilde{\theta}^B-c>L^B$

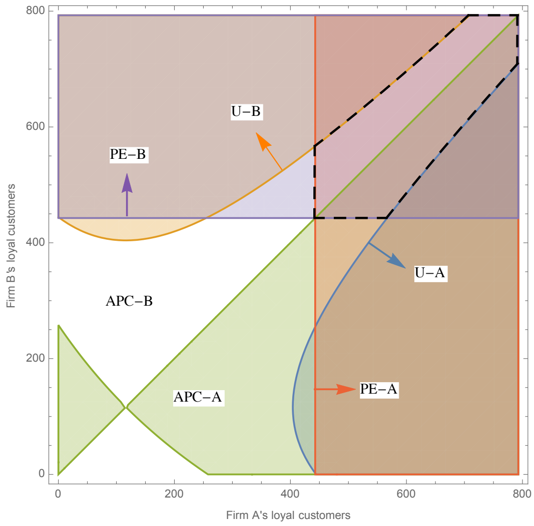
Advantage in Price Competition (APC)

- (APC – Firm A) $\gamma^A<\gamma^B+d$
- (APC – Firm B) $\gamma^A>\gamma^B+d$

Uniqueness (U)

- (U – Firm A) $(L^A+n)(\gamma^B+d)-c\geq L^A$
- (U – Firm B) $(L^B+n)(\gamma^A-d)-c\geq L^B$

As Proposition 2 shows, the condition of equilibrium uniqueness for Firm A ($U-A$) is relevant only when both firms satisfy the PE condition ($PE-A$, and $PE-B$), and Firm A satisfies the APC condition ($APC-A$). Similarly, $U-B$ is relevant only when $PE-A$, $PE-B$ and $APC-B$ are satisfied at the same time.



Horizontal axis: L^A; Vertical axis: L^B
$T^A = T^B = T$

Figure 1: Three conditions to characterize equilibria

The interaction of the three conditions is shown in Figure 1. When two firms share the same number of previous buyers, $T^A = T^B = T$, the state variables can be summarized by L^A and L^B, which are the axes in Figure 1. Note that expected qualities $(\tilde{\theta}^A, \tilde{\theta}^B)$ are increasing linear functions of the corresponding loyal customer bases (L^A, L^B): The bigger a firm's loyal customer base is, fixing T, the higher is the firm's expected quality (or rating).

Figure 1 shows that, the condition *Uniqueness* is satisfied only when the firm with advantage in price competition (APC) has a much bigger loyal customer base than the rival. In other words, if Firm A and Firm B have similar numbers of loyal customers (i.e. $|L^A - L^B|$ is small), advertising is a strategic substitute between two firms. In this case, there are multiple equilibria, and one firm chooses to advertise only when the rival does not. However, if firms differ a lot in their loyal customer bases (i.e., $|L^A - L^B|$ is big), advertising is a dominant strategy for the firm with more loyal customers, and the firm with less will never advertise.

We can see from Figure 1 that the area of each *Uniqueness* condition is a strict subset of the area of the corresponding APC condition. In particular, $U - B$ is a strict subset of $APC - B$, and $U - A$ is a strict subset of $APC - A$. This observation leads to the following proposition.

Proposition 3. *If advertising cost is positive, $c > 0$, there exists a set of state variables (L^A, L^B, T^A, T^B) such that multiple equilibria exist in the competition between two established firms.*

The area of multiple equilibria is marked by black dashed lines in Figure 1. Proposition 3 is saying that this area is nonempty as long as advertising cost is positive ($c > 0$). Therefore, if firms are rated similarly, multiple equilibria always exist, and one case we might see is that only the worse-rated firm advertises.

If one firm does not satisfy the PE condition, i.e. expansion through advertising is not profitable for this firm, whereas the other firm satisfies the PE condition, there is also a unique equilibrium in which only the firm that satisfied PE advertises. If neither firm satisfies the PE condition, the unique equilibrium is that no firm advertises. We can see from Figure 1 that a firm finds it profitable to expand through advertising if its loyal customer base exceeds a certain level (fixing T).

Combining all cases in the entire set of state variables ($0 \leq L^A \leq T, 0 \leq L^B \leq T$), I use Figure 2 to illustrate the areas of different competition results. In the lower-left white block, expansion is not profitable for either firm, so neither firm advertises. In the lower-right blue area, Firm A has a much higher ratio of loyal customers (or good reviews) than Firm B, there is a unique equilibrium and only Firm A advertises. Correspondingly, in the upper-left red area, there is a unique equilibrium, and only Firm B advertises. In the upper-right green area, firms both have a high ratio of good reviews, and their ratings differ a little, there are multiple equilibria, and either firm might be the one that is advertising. We can see that a firm will be dominant in advertising if it has a much higher ratio of good reviews.

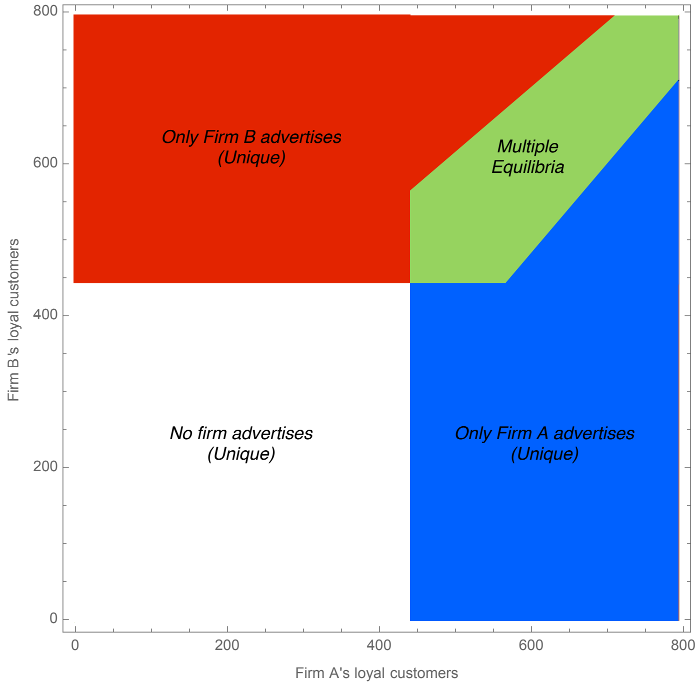
5 Extensions

In this section, I give two extensions of the main model. First, I consider an extreme case of the main model in an entry setting. In this entry game, the incumbent is an established firm facing potential entry, the incumbent and the entrant sequentially make advertising decisions, and if entry takes place, they compete in price. Customer reviews interact with advertising in the entry deterrence problem.

In the second part, I consider an extension where a group of consumers are allowed to search for firms and can see the reviews of each firm. Such consumers are called "Searchers", and they do not need to see advertisements to be informed about the existence of products. I check robustness of the conclusions from the main model to this extension.

5.1 An Entry Game

The settings on the product and on consumers remain the same as the main model.



Horizontal axis: L^A; Vertical axis: L^B

$T^A = T^B = T$

Figure 2: Multiple Equilibria or Unique Equilibrium?

On the firm side there are two firms, an incumbent, Firm A, and a potential entrant, Firm B. Firms' true qualities, respectively θ^A and θ^B, are unknown.

Firm A has operated for several periods, had $T \in \mathbb{N}$ previous buyers and $L^A \leq T$ loyal customers. Assume that $T \geq n$. Firm A has a secured profit equal to L^A. All previous buyers have written reviews, so Firm A has T reviews in total, and L^A out of them are good reviews (1's).

Firm B, the entrant, has no previous buyers and thus no loyal customers and no reviews. As a result, Firm B has no secured profit.

The incumbent and the entrant interact in three stages. In the first (pre-entry) stage, the incumbent, Firm A, chooses whether to advertise ($M^A = 1, 0$). In the second (entry) stage, after observing the incumbent's advertising decision, the entrant, Firm B, chooses whether to enter and advertise ($M^B = 1, 0$). In the third (post-entry) stage, n shoppers are randomly drawn to receive advertisements and see customer reviews (if exist) of the advertised firms, and firms simultaneously choose prices.

Customer reviews of Firm A are observable to both firms and all informed shoppers. It is also common knowledge that Firm B is a new firm and has no customer reviews nor loyal customers. Therefore, firms and informed shoppers share the same belief that Firm A's expected quality is $\tilde{\theta}^A = \frac{1+L^A}{2+T}$ and Firm B's expected quality is $\tilde{\theta}^B = \frac{1}{2}$. Let d

denote the difference between firms' expected qualities, $d = \tilde{\theta}^A - \tilde{\theta}^B = \tilde{\theta}^A - \frac{1}{2}$.

Using backward induction, I solve for the subgame perfect Nash equilibrium of the three-stage game. As in the main model, the equilibrium here is characterized by three conditions, Profitable Expansion (PE), Advantage in Price Competition (APC), and Uniqueness (U).

Before analyzing the equilibrium of the entry game, we need first to look at the PE condition. If it is not profitable for the incumbent to expand, entry deterrence will never happen. If the entrant does not satisfy the PE condition, entry will never happen.

The PE condition is defined in the same way as in the main model. Firm A satisfies the PE condition if its loyal customer base L^A and expected quality $\tilde{\theta}^A$ satisfy (3). Firm B has $L^B = 0$ and $\tilde{\theta}^B = \frac{1}{2}$, and satisfies PE if $\frac{1}{2}n \geq c$. I assume that advertising is not too costly, specifically $c < \frac{1}{2}n$, so that an entrant is willing to enter the market and advertise if there is no competition. In other words, the entrant, Firm B, always satisfies the PE condition.

If expansion is not profitable for at least one firm, there will be no entry game. In the following study of entry deterrence, I consider only the case where both firms find it profitable to expand. That is, suppose (3) is satisfied.

Third (post-entry) stage:

In the third stage, firms carry out pricing decisions. If entry did not happen in the second stage, i.e., Firm B did not advertise, the n shoppers will be aware only of Firm A. In this case, Firm A selects its monopoly price, $\tilde{\theta}^A$, and sells to both loyal customers (L^A) and new shoppers (n). Firm B does not move in this stage because it did not enter the market.

If entry occurred (i.e., the entrant – Firm B– advertised), and Firm A also advertised, the n shoppers will be aware of both firms, and Firm A and Firm B need to compete in price for these new shoppers. Lemma 1 implies that as long as Firm A has a nonzero loyal customer base, there is no pure strategy equilibrium of the price competition, and firms' expected payoffs in the mixed strategy equilibrium is determined by the condition "Advantage in Price Competition" (APC). In the competition between an established firm and a new firm, we have the following lemma about the APC condition.

Lemma 2. *If the incumbent (Firm A) and the entrant (Firm B) have both advertised, then in the pricing subgame of the third stage, Firm B has advantage in price competition (APC-B), i.e.,* $\gamma^B < \gamma^A - d$*, for all values of* L^A *and the corresponding* $\tilde{\theta}^A$.

Lemma 2 and Proposition 1 together provide us with the firms' expected payoffs in the pricing subgame after entry occurred. Firm A gets L^A in expectation, and Firm B

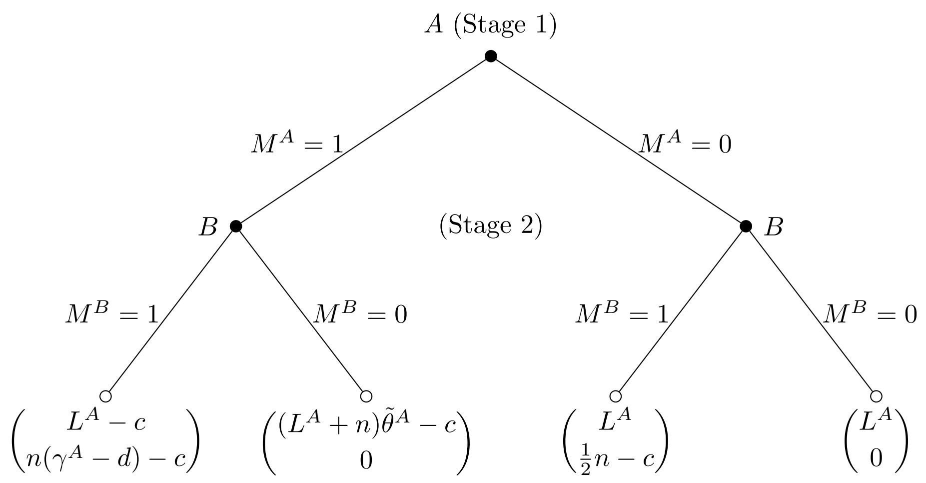


Figure 3: Stage 1 and Stage 2 of the entry game

gets $n(\gamma^A - d)$ in expectation.

In an extreme case that the incumbent never satisfied consumers: if Firm A has no loyal customers ($L^A = 0$ and $\tilde{\theta}^A = \frac{1}{2+T}$), there is a pure strategy equilibrium of the pricing competition, and firms get the same payoffs equal to the expected payoff in the mixed strategy equilibrium, with certainty. That is, Firm A does not sell and gets zero profit, and Firm B wins all new shoppers and gets $n(\tilde{\theta}^B - \tilde{\theta}^A) = n(\gamma^A - d)$.

First and second (pre-entry and entry) stage:

Given the (expected) payoffs in the third stage, I analyze how the incumbent and the entrant make advertising decisions sequentially in Stage 1 and Stage 2. The game tree is shown in Figure 3.

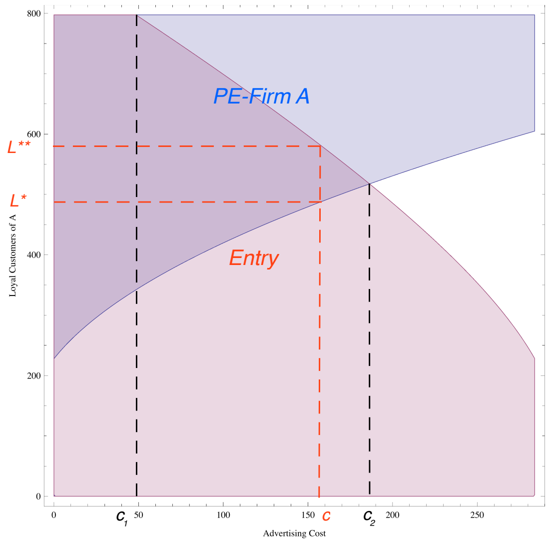
If Firm A did not advertise in Stage 1, entry is accommodated and Firm B will advertise in Stage 2. If Firm A has advertised in Stage 1, Firm B may still enter and advertise if $n(\gamma^A - d) - c \geq 0$. If Firm B enters, Firm A eventually gets a payoff $(L^A - c)$ less than its secured profit L^A.

Therefore, entering is a dominant strategy for Firm B if $n(\gamma^A - d) - c \geq 0$ is satisfied. This is exactly the "Uniqueness" condition in the main model. I call it the "Entry" condition here. If the "Entry" condition is satisfied, Firm A will not advertise in Stage 1, and Firm B will enter and sell to all new shoppers in the equilibrium. If the "Entry" condition is not satisfied, Firm A, having the first-move advantage, will advertise, and Firm B will not enter. In this case, entry is successfully deterred.

To sum up, there are two conditions relevant in this entry game:

(PE − Firm A): $(L^A + n)\tilde{\theta}^A - c \geq L^A$

(Entry): $n(\gamma^A - d) - c \geq 0$



Horizontal axis: advertising cost c
Vertical axis: Firm A's loyal customer base L^A

Figure 4: Conditions PE and Entry in the entry game

The entry game is defined by two variables, Firm A's loyal customer base L^A and expected quality $\tilde{\theta}^A = \frac{1+L^A}{2+T}$. These two variables can be summarized by L^A alone. There are three parameters, previous buyers of Firm A (T), new shoppers (n), and advertising cost (c). Firm A has operated alone for more than one periods, therefore, T is assumed to be greater than n. Figure 4 shows how the two conditions interact for different values of L^A and the parameter c, fixing the values of T and n.

We can see from Figure 4 that, 1) if advertising cost c is too small ($c \leq c_1$), entry always occurs for all values of L^A; 2) if advertising is too costly ($c \geq c_2$), whenever the incumbent (Firm A) is willing to expand through advertising (i.e., satisfies the PE condition), entry will not occur; and 3) for moderate advertising cost ($c_1 < c < c_2$), entry may still occur even when Firm A satisfies the PE condition ($L^A \geq L^*$), and only if Firm A has a big enough loyal customer base (L^A) relative to its total previous buyers (b), then entry will be successfully deterred.

From this entry extension, I show that for an entry game with a moderate advertising cost (case 3), entry can be deterred only if the incumbent has a high enough ratio of loyal customers, or in other words, only if the incumbent has a big percentage of good reviews.

Comparing this to what Fudenberg and Tirole (1984) show: when there are no customer reviews, a big incumbent is weak in the competition with entrant and cannot deter entry. If customer reviews do not exist, even if the incumbent advertises, it will lose the price competition with the entrant. Therefore, in a world where customer reviews do not exist, the incumbent will not advertise and cannot deter entry.

We see that the availability of customer reviews undoes the "fat-cat" effect of big incumbents, and strengthens incumbents with a high ratio of good reviews.

5.2 Extension: Searchers

Now I consider an extension of the main model where "Searchers" are allowed. Searchers are a group of consumers who are used to searching for firms instead of watching advertisements, like the tech-savvy consumers. Searchers are not affected by firms' advertisements, and can always search for all firms and see each firm's reviews and price.

Assume that there are two types of new consumers, the traditional new consumers and the Searchers. Traditional consumers are the same as the consumers in the main model: they need to see a firm's advertisements to be informed about the firm's existence. Searchers are as defined above. Besides searching, there is no other difference between traditional new consumers and Searchers. In other words, if a product has true quality θ, a traditional new consumer and a searcher have the same probability (θ) of being satisfied in consuming this product.

In this model, there are $n \in \mathbb{N}$ traditional shoppers and $s \in \mathbb{N}$ Searchers. Each consumer chooses between Firm A and Firm B, and purchase, at most, one unit of the good.

Firm A has L^A loyal customers and an expected quality $\tilde{\theta}^A = \frac{1+L^A}{2+T^A}$, and Firm B has L^B loyal customers, and an expected quality $\tilde{\theta}^B = \frac{1+L^B}{2+T^B}$.

The timing of the model is:

The s Searchers are aware of both firms, and have access to their customer reviews. Firms A and B first make advertising decisions simultaneously. The n traditional shoppers are informed by firms' advertisements, and have access to the advertised firm's customer reviews. After observing each other's advertising decision, firms choose prices. Traditional shoppers and Searchers then make purchase decisions based on firms' reviews and prices, and loyal customers of each firm repeat purchases.

The key difference of having Searchers is the option to compete for the group of Searchers even if a firm does not advertise. Therefore, in this extension model, there is a pricing subgame for every combination of firms' advertising decisions, whereas in the main model (without Searchers), firms compete in price only when both firms advertise. The four combinations are $(M^A = 1, M^B = 1)$, $(M^A = 1, M^B = 0)$, $(M^A = 0, M^B = 1)$ and $(M^A = 0, M^B = 0)$.

Recall that in the pricing subgame, the condition "Advantage in Price Competition" (APC) determines which firm wins the pricing subgame (in terms of expected profit). Here in the extension with Searchers, we will have four APC conditions, one for each pricing subgame.

Pricing Subgame 1: $M^A = 1, M^B = 1$

This case is very close to the price competition in the main model. When both firms choose to advertise, the traditional consumers are informed about the existence of both firms. Therefore, now the traditional consumers and Searchers have exactly the same information, and can be viewed as one group in this pricing subgame. Firm A and Firm B compete in price for these $n + s$ new consumers.

Given Firm A's loyal customer base L^A, the reservation price, γ_{11}^A, of Firm A in this subgame ($M^A = 1, M^B = 1$) is the lowest price that it is willing to charge in the price competition:

$$\gamma_{11}^A = \frac{L^A}{L^A + n + s}$$

Similarly, Firm B has L^B loyal customers, and the reservation price γ_{11}^B of Firm B in this pricing subgame is

$$\gamma_{11}^B = \frac{L^B}{L^B + n + s}$$

The APC condition in this subgame is denoted as APC_{11}. The condition of Firm A having advantage in price competition is $APC_{11} - A$:

$$\gamma_{11}^A < \gamma_{11}^B + d \tag{9}$$

where d is the difference in ratings of two firms, $d = \tilde{\theta}^A - \tilde{\theta}^B$. Firm B has APC if (9) does not hold.

If Firm A has advantage in this pricing subgame, i.e. $APC_{11} - A$ holds, Firm A wins this pricing subgame (in terms of expected profit), and two firms expected profit will be:

$$\begin{aligned} \pi_{11}^A &= (L^A + n + s)(\gamma_{11}^B + d) - c \\ \pi_{11}^B &= L^B - c \end{aligned}$$

Pricing Subgame 2: $M^A = 1, M^B = 0$

If only Firm A advertises, those n traditional consumers will not be aware of Firm B, and they only consider buying from Firm A. However, two firms may compete for Searchers (s). Reservation prices will be different because of the existence of Searchers.

For Firm A, it now has two reservation options in pricing. It may charge price $p^A = 1$, and sell only to its loyal customers L^A. Or, it may charge price $p^A = \tilde{\theta}^A$, and at least loyal customers L^A and traditional shoppers n will buy from Firm A for certain. Therefore, for any price p that Firm A charges in pricing competition, it has to satisfy

$$p(L^A + n + s) \geq \max\{L^A, (L^A + n)\tilde{\theta}^A\}$$

Then Firm A's reservation price in this pricing subgame is

$$\gamma_{10}^A = \frac{\max\{L^A, (L^A+n)\tilde{\theta}^A\}}{L^A+n+s}$$

Firm B's reservation option in pricing is still only one: charging $p^B = 1$ and sell only to its loyal customers L^B. Therefore, Firm B's reservation price in this pricing subgame is

$$\gamma_{10}^B = \frac{L^B}{L^B+s}$$

Note that if s is small, γ_{10}^B will be close to 1, and higher than $\tilde{\theta}^B$, which is the highest price that Searchers would accept for product B. Therefore, if only Firm A advertises, there will be competition for Searchers only when

$$\gamma_{10}^B \leq \tilde{\theta}^B \tag{10}$$

The APC condition in this pricing subgame is denoted as APC_{10}. And $APC_{10} - A$ is satisfied if

$$\gamma_{10}^A < \gamma_{10}^B + d \tag{11}$$

And again, $APC_{10} - B$ is satisfied if (11) does not hold.

If Firm B is willing to compete for Searchers (i.e., (10) holds), and Firm A satisfies the APC condition (i.e., (11) holds), two firms' expected profits will be

$$\begin{aligned}\pi_{10}^A &= (L^A+n+s)(\gamma_{10}^B + d) - c \\ \pi_{10}^B &= L^B\end{aligned}$$

Pricing Subgame 3: $M^A = 0, M^B = 1$

This subgame is similar to subgame 2, only with firms switching roles, so I will just list the conditions for this subgame.

Firm A's reservation price in this subgame is $\gamma_{01}^A = \frac{L^A}{L^A+s}$, and Firm B's reservation price in this subgame is $\gamma_{01}^B = \frac{\max\{L^B,(L^B+n)\tilde{\theta}^B\}}{L^B+n+s}$.

Firm A is willing to compete for Searchers only if

$$\gamma_{01}^A \leq \tilde{\theta}^A \tag{12}$$

The APC condition for this subgame is denoted as APC_{01}, and $APC_{01} - A$ is satisfied if

$$\gamma_{01}^A < \gamma_{01}^B + d \tag{13}$$

If both (12) and (13) are satisfied, two firms' expected profits will be

$$\begin{aligned}\pi_{01}^A &= (L^A + s)(\gamma_{01}^B + d) \\ \pi_{01}^B &= \max\{L^B, (L^B + n)\tilde{\theta}^B\} - c\end{aligned}$$

Pricing Subgame 4: $M^A = 0, M^B = 0$

If neither firm advertises, this pricing subgame will again be similar to the pricing subgame in the main model, except that firms are now competing for Searchers, not traditional consumers.

The two firms' reservation prices are $\gamma_{00}^A = \frac{L^A}{L^A+s}, \gamma_{00}^B = \frac{L^B}{L^B+s}$. And they will compete for Searchers only if

$$\gamma_{00}^A \leq \tilde{\theta}^A \quad (14)$$

$$\gamma_{00}^B \leq \tilde{\theta}^B \quad (15)$$

are both satisfied.

In this subgame, Firm A satisfies the APC condition, i.e. $APC_{00} - A$ holds, if

$$\gamma_{00}^A < \gamma_{00}^B + d \quad (16)$$

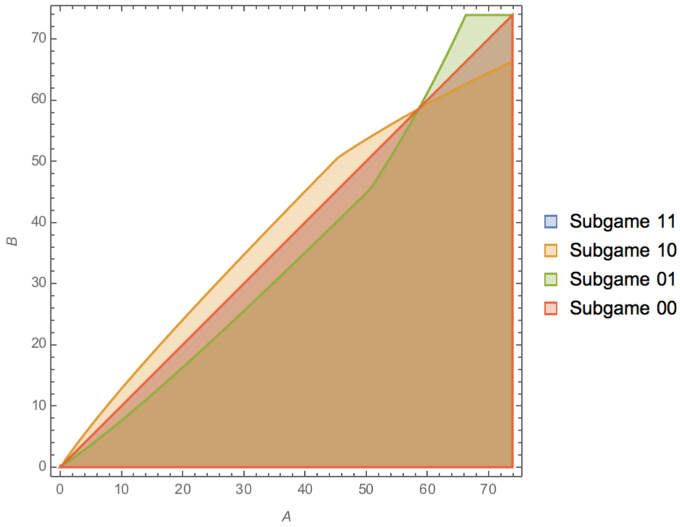
If (14), (15) and (16) all hold, the two firms' expected profits will be

$$\begin{aligned}\pi_{00}^A &= (L^A + s)(\gamma_{00}^B + d) \\ \pi_{00}^B &= L^B\end{aligned}$$

Next, we will see how the four $APC - A$ conditions interact with each other. For simplicity of analysis, here I focus on the case where two firms have the same number of previous buyers: $T^A = T^B = T > 0$.

As shown in Figure 5, all four $APC - A$ conditions are satisfied when Firm A has a higher ratio of good reviews than Firm B. That is, if Firm A has a much higher ratio of good reviews than Firm B, Firm A will have advantage in price competition (APC) in all four pricing subgames. And as the value of s increases, the four conditions converge to be the same area. Therefore, if the group of Searchers (s) is very big, a firm either has advantage in price competition (APC) in all four pricing subgames, or has APC in no pricing subgame.

Suppose the group of Searchers (s) is very big, and Firm A is the one with a higher ratio of good reviews, and hence satisfies the APC conditions of all four pricing subgames. The profit functions for each combination of advertising strategies by Firms A and B are



Horizontal axis: Firm A's loyal customers L^A
Vertical axis: Firm B's loyal customers L^B

Figure 5: Condition $APC - A$ in four pricing subgames

Table 3: Firms' Profits When s is Big, and $L^A > L^B$

	$M^B = 1$	$M^B = 0$
$M^A = 1$	$(L^A + n + s)(\gamma_{11}^B + d) - c$ $L^B - c$	$(L^A + n + s)(\gamma_{10}^B + d) - c$ L^B
$M^A = 0$	$(L^A + s)(\gamma_{01}^B + d)$ $\max\{L^B, (L^B + n)\tilde{\theta}^B\} - c$	$(L^A + s)(\gamma_{00}^B + d)$ L^B

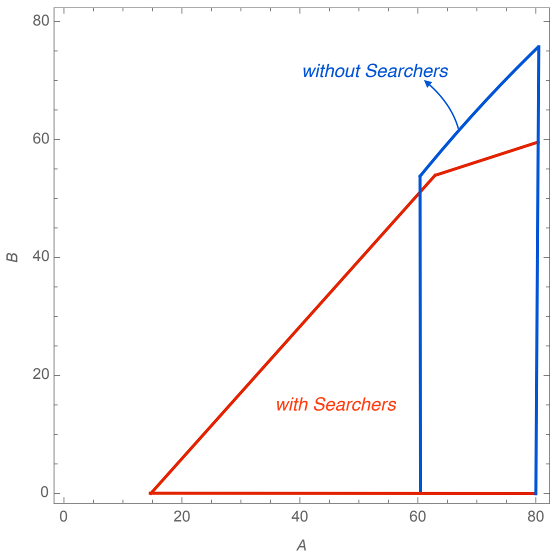
provided in Table 3. We can see from the table that when Firm A has advantage in price competition, if Firm A advertises, Firm B never wants to advertise; but if Firm A does not advertise, Firm B would want to advertise. This is the same as in the main model.

Next, let us see how Firm A's advertising strategy here differs with the main model where there are no Searchers. Recall that in the main model, the Profitable Expansion (PE) condition is defined such that a firm is willing to advertise when the opponent does not advertise. Here, the PE condition for Firm A (PE-A) in the extension model with Searchers is

$$(L^A + n + s)(\gamma_{10}^B + d) - c > (L^A + s)(\gamma_{00}^B + d) \tag{17}$$

which can be simplified to: $n(\frac{L^B}{L^B+s} + d) > c$. If Firm A wants to advertise even when Firm B chooses to advertise, we say the condition Uniqueness (U-A) is satisfied, that is, there will be a unique equilibrium where only Firm A advertises. The condition (U-A) is

$$(L^A + n + s)(\gamma_{11}^B + d) - c > (L^A + s)(\gamma_{01}^B + d) \tag{18}$$



Horizontal axis: Firm A's loyal customer base L^A
Vertical axis: Firm B's loyal customer base L^B
Assuming $T^A = T^B = T$.

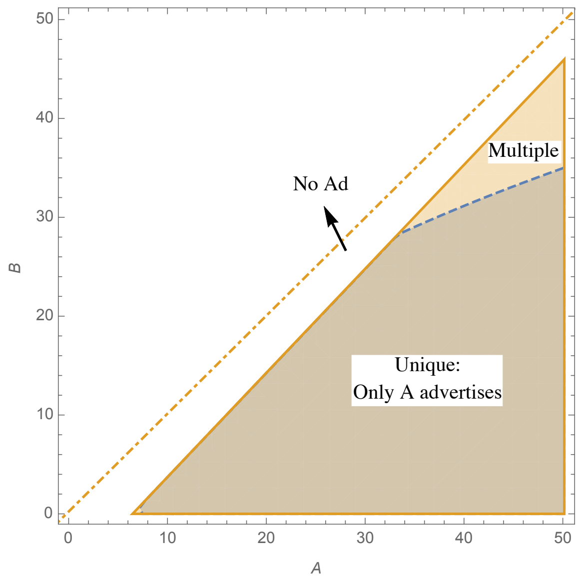
Figure 6: With and without Searchers: when does Firm A always advertise?

When (17) and (18) are simultaneously satisfied, Firm A always advertises, regardless of Firm B's advertising decision, and the unique equilibrium is that only Firm A advertises.[13] The intersection of these two conditions (PE-A and U-A) is illustrated by the red-contoured area in Figure 6. In this area, Firm A advertises no matter what.

Compare the extension model with Searchers to the main model. The area of Firm A always advertising in the main model is contoured by blue curves. We can see that Firm A (the one with higher ratio of good reviews) has a bigger chance to be dominant in advertising when there are Searchers. However, if the opponent (Firm B) has a very high ratio of good reviews already, it is harder for Firm A to be dominant in advertising in the Searchers model than in the main model.

By this comparison, we see that the spirit of the main model still holds: having a high percentage of good reviews is important for a firm to win the competition, and the areas of unique equilibrium and multiple equilibria all exist when there are Searchers. In particular, the equilibrium partition in the Searchers extension model is given in Figure 7.

[13]Note that conditions $APC_{11} - A$, $APC_{10} - A$, $APC_{01} - A$ and $APC_{00} - A$ are already satisfied, as we are considering the case when s is big and $L^A > L^B$.



Horizontal axis: Firm A's loyal customer base L^A
Vertical axis: Firm B's loyal customer base L^B
Assuming $T^A = T^B = T$.

Figure 7: Equilibrium Partition in the Searchers Model: Areas of Multiple Equilibria or Unique Equilibrium

6 Data Evidence

In this section I use advertising data and reviews data of local restaurants to test the main prediction of my theoretical model: a firm with a relatively higher average rating, in general, advertises more.

I combine two datasets. The first dataset is obtained from the advertising spending database of Kantar Media; it contains local restaurants' advertising spending in the year 2014. The advertising spending amount in this dataset is the total amount of ad spending in all channels: TV, magazines, Internet, newspapers, radio, outdoor, etc.

The second dataset is scripted from Yelp.com and contains the corresponding Yelp reviews and location information of those local restaurants in the first (advertising) dataset. I only took those reviews posted before January 1st, 2014. Merging two datasets together, then we have the total advertising spending amount and Yelp reviews of these local restaurants.

In this section, I first use a small dataset to graphically show the distribution of local restaurants' advertising spending levels in the two dimensional space of their total number of reviews and the ratio of good reviews. And in the second subsection, I use a large dataset with Regression Discontinuity Design to find the relationship between advertising spending and average rating on Yelp.

Table 4: Summary Statistics for The Graphical Analysis Dataset

Variable	Mean	Std. Dev.
Good reviews (4/5 stars)	35.84991	85.78242
All reviews	57.7613	122.1084
Advertising spending (000)	5.622604	11.34311

Notes. Observations: 553.

6.1 Graph Illustration

The graphical analysis in this section can be directly linked to the prediction of the model, and in particular corresponds to the red-contoured area (with Searchers) in Figure 6.

The small dataset contains only the local restaurants that advertise in the New York City market, one of the DMA (designated marketing area) regions, in Q1 of 2014. To repeat here, only the Yelp reviews posted by Jan 1 of 2014 are collected for these local restaurants. Consumers in this region share the same access for advertisements in all channels.

Filtering out restaurants that are not listed on Yelp, I have 553 local restaurants left in this dataset. Summary statistics are provided in Table 4.

To match with the simplifying setting of my model, I take the four-star and five-star reviews to be the good reviews (L). Consumers who give four stars or five stars to a local restaurant are highly likely to come back and purchase again.[14] Therefore, I approximate a local restaurant's loyal customer base by the group of consumers who rated this restaurant four or five stars.

I use contour plots to visualize the advertising pattern of restaurants with different levels of good reviews (L) and all reviews (T). I divide the dataset into several subsets and plot the advertising pattern for each subset. Each subset contains restaurants that have the same number of opponents in its neighborhood.[15] That is, restaurants that are the only restaurant in its neighborhood belong to one subset, restaurants that locate in the neighborhoods with only two local restaurants are in another subset, and so on.[16]

Using loyal customer base (i.e., L in the model) as the vertical axis, and total number of reviews (T) as the horizontal axis, Figure 8 shows the advertising spending levels of the 90 restaurants that are in neighborhoods with only two restaurants, and different colors indicate different levels. From this graph, restaurants with a very high ratio of good reviews (or loyal customers) generally have very high level of advertising spending.

[14]See an evidence for this from the word clouds of Yelp reviews analyzed by Max Woolf at http://minimaxir.com/2014/09/one-star-five-stars/.

[15]This neighborhood concept is defined by the "city" information of restaurants on Yelp pages.

[16]Note that, here I am not saying that these neighborhoods really have only one or two local restaurants, but that they only have one or two local restaurants that are both advertising in the New York market and are listed on Yelp.

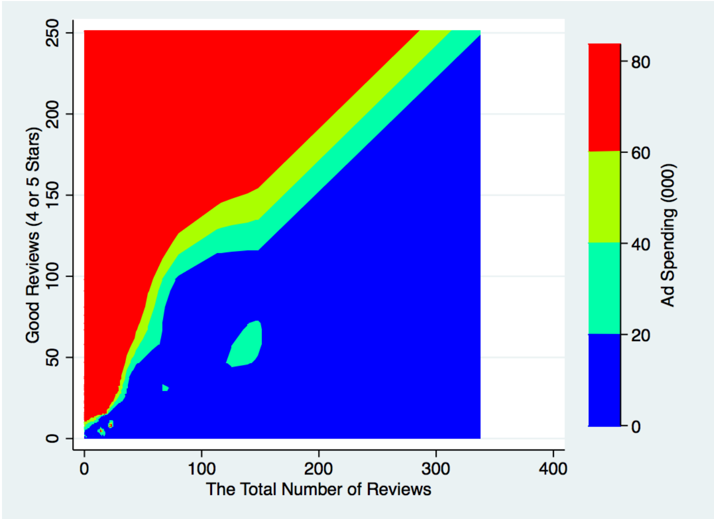


Figure 8: The Advertising Pattern of Local Restaurants in Neighborhoods with Only Two Restaurants (that are both in Kantar Media Database and on Yelp.com)

In the middle range (around the 45 degree line), advertising spending levels are mixed: some restaurants advertise a lot while some others advertise a little. Then for restaurants with a very low ratio of good reviews, their advertising spending level is really low.

A more extreme subset is the one that contains the local restaurants located in the neighborhood of New York City, and there are in total 107 restaurants in this subset. The contour plot is shown in Figure 9. We can again see a similar pattern.

From the contour plots of advertising spending levels for local restaurants, we can see that firms with a high ratio of good reviews are indeed more likely to be dominant in advertising. Next, I will provide supporting evidence from regression analysis as well.

6.2 Regression Analysis

To do regression analysis, I use a larger dataset that contains all US local restaurants that advertised in 2014 and are listed on Yelp. To be specific, this dataset is merged from an advertising dataset with 13,360 local restaurants' total advertising spending in 2014, and a Yelp information dataset with reviews (by Jan 1, 2014) and other information of those local restaurants. Summary statistics of this dataset is given in Table 5.

In the regression analysis, I regress with restaurants' average rating instead of the ratio of 4- and 5-star reviews (i.e., the ratio of good reviews). This is because restaurants do not make decisions based on the simplification concept "the ratio of good reviews" but rather on their average rating, and average rating and the ratio of good reviews are not one-to-one corresponding to each other, so running regression directly with the ratio of good reviews includes too much unnecessary noise. More importantly, the ratio of good

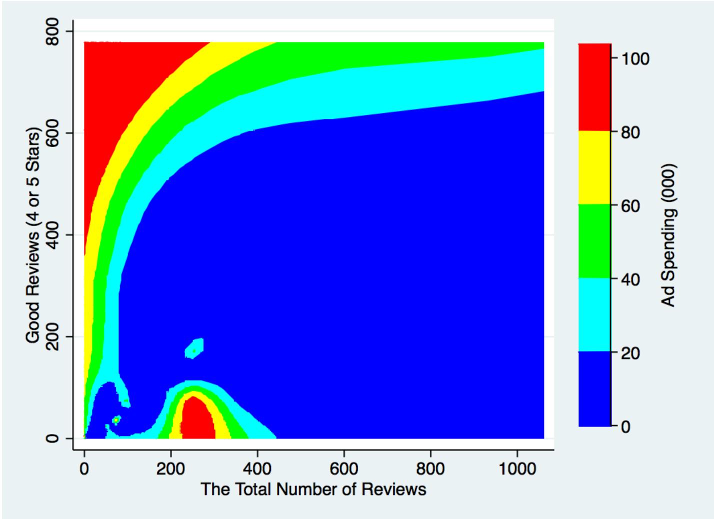


Figure 9: The Advertising Pattern of Local Restaurants in New York City

Table 5: Summary Statistics for The Regression Anslysis Dataset

Variable	Mean	Std. Dev.
Percentage of one-star reviews	0.099	0.113
Percentage of two-star reviews	0.116	0.103
Percentage of three-star reviews	0.172	0.120
Percentage of four-star reviews	0.331	0.150
Percentage of five-star reviews	0.281	0.191
Advertising spending (000)	12.724	44.290

Notes. Observations: 13,360.

reviews exactly corresponds to the true average rating in the theoretical model setting with only 1 and 0 reviews. Therefore, using data from a five-star review system, we should regress with the true average rating.

To analyze the relationship between local restaurants' advertising spending and their average rating, I use the following specification:

$$Ad_i = \alpha + \beta \cdot R_i + \gamma^{DR} \cdot dressy_i + \gamma^{DE} \cdot deliver_i + \gamma^{W} \cdot waitor_i + \Gamma^{P} \cdot P_i + \varepsilon_i \qquad (19)$$

where R_i is the average rating of a restaurant, $dressy_i$ is an indicator variable that equals to 1 if the dressing code is "Dressy", $deliver_i$ is an indicator variable that equals 1 if delivery is available, $waitor_i$ is an indicator variable of the availability of waiter service, and P_i consists of three price dummy variables indicating the price level ($$, $$$, or $$$$).

If we simply run such a regression with all restaurants, we will get a regression result indicating a negative relationship between restaurants' average rating and advertising spending amount. See column (1) in Table 6. The estimated coefficient of variable R_i is significantly negative (-0.807). However, this regression is incorrect since there is a

Figure 10: The Relationship between Ad Spending and True Rating without Recognition of RD Drops

complication caused by the discrepancy between the display rating and the true rating of a restaurant on Yelp. It is out of the scope of the current paper to go into all the details, so I will use some graphs to briefly show the effect on advertising caused by such discrepancy. More detailed discussion can found in another paper of mine that investigates the effect of a higher display rating on restaurants' advertising spending using a Regression Discontinuity design.

According to the empirical findings from the empirical RDD paper, there exist significant drops in advertising spending when the true average rating crosses the thresholds of 3.25, 3.75 and 4.25. That is, for relatively higher-rated restaurants, a higher display rating induces drops in advertising spending of local restaurants. This effect of display rating needs to be separated from the effect of true rating in order for us to learn the real relationship between a local restaurant's (true) average rating and its advertising spending.

Figure 10 is a binned scatterplot of advertising spending for restaurants with different true average ratings. Each dot represents the average level of advertising spending of the restaurants within that bin.[17] We can see that, when the average rating goes above 3, advertising spending decreases with the average rating, and this seems consistent with the traditional conclusions in the literature of product reviews. However, once we look in detail at the plot with the recognition of the effects of display rating (i.e., the drops at RD thresholds), we can see from Figure 11 that the downward trend is entirely caused by

[17] Each bin contains about 130 restaurants.

Figure 11: The Relationship between Ad Spending and True Rating with Recognition of RD Drops

the drops at the RD thresholds (where the display rating jumps by 0.5 star), and in each interval between two thresholds advertising spending in fact goes up with the average rating. This can be seen more clearly from the pooled RD regression results as shown in Table 6.

Column (2) in Table 6 provides the estimates of the relationship between advertising spending and average rating to the left and to the right of the RD thresholds, where the display ratings are constant. I pool all RD thresholds together and assign a dummy for each interval (that contains a threshold and has length 0.5). The average rating of each observation is normalized by its nearest threshold.

We can see directly from the estimates in column (2) that the relationship between advertising spending and average rating is significantly positive (2.811) to the left of the thresholds. I also test for the significance of the slope to the right of the thresholds, and find that it is significantly positive at 87% confidence level.[18]

In summary, controlling for the disturbing effect of Yelp display ratings on advertising, there is in fact a significantly positive relationship between advertising spending and the local restaurants' average rating. Therefore, we have seen supporting empirical evidence for the model prediction: Higher-rated restaurants advertise more.[19]

[18]The two-tail p value is 26%. Because the point estimate is positive (2.811 - 0.954 = 1.957), therefore we can reject against the hypothesis that it is positive at 87% confidence level.

[19]It is out of the scope of the current paper to analyze the effect of Yelp display rating at each threshold, but graphs showing better details about the drops at thresholds above 3 and below 3 are provided in Figure 15 and Figure 16 in Appendix 3.

Table 6: RD: Pooled Regression

	(1)	(2)
Average rating	-0.807***	2.811*
	(0.150)	(1.551)
Above threshold		-1.085***
		(0.351)
Average rating × Above threshold		-0.854
		(2.349)
Rating [1.5, 2)		2.965
		(2.240)
Rating [2, 2.5)		1.299
		(1.930)
Rating [2.5, 3)		1.922
		(1.890)
Rating [3, 3.5)		1.446
		(1.876)
Rating [3.5, 4)		1.201
		(1.875)
Rating [4, 4.5)		0.295
		(1.877)
Rating [4.5, 5]		-0.187
		(1.907)
Dressy	0.819*	0.838*
	(0.454)	(0.454)
Waiter service	-0.920***	-0.931***
	(0.301)	(0.301)
Delivery	-0.505**	-0.511**
	(0.236)	(0.236)
Reservation	0.195	0.176
	(0.213)	(0.213)
Price level $$	0.466*	0.384
	(0.262)	(0.262)
Price level $$$	1.437***	1.337***
	(0.424)	(0.424)
Price level $$$$	0.353	0.311
	(0.898)	(0.898)
Constant	9.287***	6.018***
	(0.631)	(1.909)
Observations	10197	10197

Notes. The dependent variable in both specifications is the total amount of advertising spending (unit: USD Thousands). Average rating is normalized by the nearest threshold. Regress only with observations that advertise in only one market, and outliers are ruled out. All RD jump thresholds of display rating are pooled together, and interval dummies are used. Significance levels: ***1%, **5%, *10%.

7 Discussion: The Capacity Limit

We have seen from the empirical findings that advertising of local restaurants goes up with average ratings away from RD thresholds (where the display rating is constant) and that advertising goes down with display ratings above 3. The positive relationship between advertising and average ratings is consistent with the theory prediction in this paper that better online reviews increase the benefit of advertising and therefore complement advertising. However, the negative response in advertising to display ratings above 3 seems surprising and counterintuitive because display rating is also an information on reviews, only coarser than the information of average rating.

The essential difference between display rating and the average rating is the way of

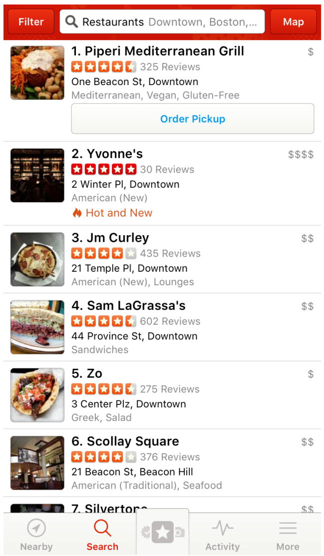


Figure 12: An Example of Yelp Search Results Page

their interaction with firm profits. Display ratings of restaurants are shown on the search results page if a consumer search for nearby restaurants. See Figure 12. An increase (by 0.5 stars) in the display rating of a restaurant will increase the click rates of this restaurant. In other words, the group size of Searchers for a restaurant will increase every time the display rating jumps. On the other hand, when average rating increases between two adjacent jump thresholds of display rating, the display rating is constant, therefore even though the reviews become better (which is the reason of the increase in average rating), the number of Searchers will not change because any change in reviews other than the display rating is unobservable from the search results page. However, for the consumers who have opened the Yelp page of a restaurant and see the reviews, the increase in average rating is observable and will raise consumers' belief about the quality. Apply the prediction of the theory model in this paper, a higher average rating complement advertising by increasing the benefit of advertising.

In short, a higher display rating increases a firm's profit by raising the number of consumers, i.e., this is a "*volume* increase" that leads to a higher profit; a higher average rating increases a firm's profit by raising consumers' willingness to pay, i.e., this is a "*margin* increase" that leads to a higher profit. Both ways work to increase a firm's profit, but they have different interactions with advertising. The benefit of advertising is the part of the profit that comes from the group "traditional new consumers" who are attracted by advertisements. The margin increase will increase the firm's profit margin over every single consumer, including the ones that are attracted by advertisements, i.e., the traditional new consumers. Therefore, the margin increase resulting from a higher average rating will complement the benefit from advertising and make advertising more profitable, but the volume increase resulting from a higher display rating has almost no interaction with advertising and the benefit of advertising basically remains the same.

A key thing to notice is that a volume increase might be bound by the capacity limit of a local business, but a margin increase will never be bound and is always "the more the better".

Most local businesses have the concern of capacity limits. A local restaurant, a hair salon, or a hotel, cannot accommodate an unlimited number of customers. In the rest of this section, I use comparative statics of the Searchers model (presented in Subsection 5.2) to show the volume increase and the margin increase in profit, and the effect of capacity limits on them.

A capacity limit is an upper bound on the total number of consumers that a restaurant can accommodate. Let $\bar{n}_{k=A,B}$ denote the capacity limits of Firm A and Firm B. The sum of loyal customers, traditional new consumers and Searchers of a firm cannot exceed its capacity limit: $L_k + n_k + s_k \leq \bar{n}_k$.

Without Capacity Limit

First, suppose there is no capacity limit, and a firm may accommodate as many customers as there will be. Because Firm A and Firm B are symmetric, here I discuss only Firm A.

If Firm A's average rating increases between two jump thresholds of display rating, namely when the display rating remains constant while average rating increases, there is no change in s: $\triangle s = 0$, but only $\tilde{\theta}_A$ increases. Firm A's profit always increases with $d = \tilde{\theta}_A - \tilde{\theta}_B$, which in turn increases with $\tilde{\theta}_A$. This increase in Firm A's profit caused by a higher average rating is the "margin increase".

If Firm A's average rating increases across a jump threshold of display rating and causes its display rating on Yelp to jump by half a star, then besides the increase in average rating $\tilde{\theta}_A$, there is also an increase in s: $\triangle s > 0$, more Searchers find Firm A. We can see from Table 3 that Firm A's profit always increase with s. This increase in profit caused by the jump in display rating is the "volume increase". At the same time, due to the increase in average rating $\tilde{\theta}_A$, there is also a "margin increase".

In terms of a firm's total profit, we can see that an increase in average rating increases a firm's profit by a larger amount, i.e., margin increase + volume increase, if this increase goes across a jump threshold of display rating and makes the display rating to increase by 0.5 stars. But if an increase in the average rating happens between two jump thresholds of display rating, where the display rating remains constant, the firm's profit will increase by a smaller amount, i.e., margin increase only.

A firm's total profit increases more if its display rating increases together with its average rating. To see the interaction between rating and advertising, however, we need to find the change in the benefit and cost of advertising resulting from a change in rating. An increase in total profit might come with a decrease in the benefit of advertising.

The cost of advertising is always $c > 0$.

When Firm A advertises and has advantage in price competition (i.e., Firm A has a higher ratio of loyal customers), see Table 3, the benefit of advertising (denoted by $\mathcal{R}$) for Firm A is $\mathcal{R}_A(M_B = 1) = (L^A + n + s)(\gamma_{11}^B + d) - (L^A + s)(\gamma_{01}^B + d)$ if Firm B advertises, and is $\mathcal{R}_A(M_B = 0) = (L^A + n + s)(\gamma_{10}^B + d) - (L^A + s)(\gamma_{00}^B + d)$ if Firm B does not advertise.

If Firm A's average rating, i.e., $\tilde{\theta}_A$, increases by $\epsilon > 0$ but its display rating remains the same, and hold all else constant, the change in $\mathcal{R}_A$ will come only from $d = \tilde{\theta}_A - \tilde{\theta}_B$. In particular, d will increase by ϵ. The change in the benefit of advertising resulting from the change in Firm A's average rating is

$$\triangle\mathcal{R}_A(M_B = 1; d + \epsilon) = \triangle\mathcal{R}_A(M_B = 0; d + \epsilon) = n\epsilon \tag{20}$$

Therefore a higher average rating unambiguously increases a firm's benefit from advertising by raising consumers' willingness to pay: the profit margin on each traditional new consumer attracted by advertisement increases by ϵ. The change in advertising benefit is also a margin increase and matches the change in Firm A's total profit when only the average rating goes up.

If Firm A's average rating $\tilde{\theta}_A$ increases by $\epsilon > 0$ and crosses a jump threshold of display rating, i.e., its display rating jumps by 0.5 star, two things will be changing, $d = \tilde{\theta}_A - \tilde{\theta}_B$ and s. The change in d is again ϵ, and the change in s ($\triangle s > 0$) is the number of extra Searchers that are attracted by the jump in Firm A's display rating. Let $s' = s + \triangle s$, and recall that $\gamma_{11}^B = \frac{L^B}{L^B+n+s^B}$, $\gamma_{01}^B = \frac{\max\{L^B,(L^B+n)\tilde{\theta}^B\}}{L^B+n+s^B}$, and $\gamma_{10}^B = \gamma_{00}^B = \frac{L^B}{L^B+s^B}$.[20] The changes in Firm A's benefit of advertising resulting from the change in both average rating and display rating are

$$\begin{aligned} \triangle\mathcal{R}_A(M_B = 1; d + \epsilon, s' = s + \triangle s) &= \mathcal{R}_A(M_B = 1; d + \epsilon, s') - \mathcal{R}_A(M_B = 1; d, s) \\ &= n\epsilon + \triangle s \cdot \frac{L^B - \max\{L^B, (L^B + n)\tilde{\theta}^B\}}{L^B + n + s^B} \end{aligned} \tag{21}$$

and

$$\begin{aligned} \triangle\mathcal{R}_A(M_B = 0; d + \epsilon, s' = s + \triangle s) &= \mathcal{R}_A(M_B = 0; d + \epsilon, s') - \mathcal{R}_A(M_B = 0; d, s) \\ &= n\epsilon \end{aligned} \tag{22}$$

[20] Note that here I use s^B instead of s to denote the group of Searchers for Firm B and s^B is held constant, because the number of Searchers that can find Firm B will not be affected by a change in Firm A's rating.

First, note that $\triangle\mathcal{R}_A(M_B = 0; d+\epsilon, s' = s+\triangle s) = \triangle\mathcal{R}_A(M_B = 0; d+\epsilon)$, that is the increase in s does not change Firm A's benefit of advertising if Firm B does not advertise. If Firm B advertises, the increase in s brings a new part in $\triangle\mathcal{R}_A$: $\triangle s \cdot \frac{L^B - \max\{L^B, (L^B+n)\tilde{\theta}^B\}}{L^B+n+s^B}$, which equals 0 if $L^B \geq (L^B+n)\tilde{\theta}^B$ and is negative otherwise. If $L^B < (L^B+n)\tilde{\theta}^B$ is satisfied, intuitively it means that the competitor of Firm A has a very good rating ($\tilde{\theta}^B$) or the advertising is very effective in reaching and bringing new consumers (n). In this case, $\triangle s \cdot \frac{L^B - \max\{L^B, (L^B+n)\tilde{\theta}^B\}}{L^B+n+s^B} = \triangle s \cdot \frac{L^B - (L^B+n)\tilde{\theta}^B}{L^B+n+s^B} < 0$, i.e., the increase in s reduces the benefit of advertising.

The effect of $\triangle s$ enters $\triangle\mathcal{R}_A$, i.e., $\triangle s \cdot \frac{L^B - (L^B+n)\tilde{\theta}^B}{L^B+n+s^B} < 0$, if either $\tilde{\theta}^B$ or n is large. In this model, the effectiveness of advertising (n) must be large enough for the costly advertising to be ever profitable, and the group of Searchers (s) is assumed to have a nontrivial size to make a difference. And in the real world, take Yelp as an example, a high enough $\tilde{\theta}^B$ comes with a large s^B, a restaurant with a high rating will be found by a large number of Searchers. Therefore the magnitude of $\frac{L^B - (L^B+n)\tilde{\theta}^B}{L^B+n+s^B}$ is very small. Rewrite (21) as $n(\epsilon + \frac{\triangle s}{n} \cdot \frac{L^B - (L^B+n)\tilde{\theta}^B}{L^B+n+s^B})$. Therefore if $\triangle s$ is less than or only slightly larger than n, $\triangle\mathcal{R}_A$ would still be positive and small. In case of $\triangle s$ being very large and exceeding n a lot, $\triangle\mathcal{R}_A$ might become negative, but because both n and s^B are nontrivial, the change in the marginal benefit of advertising will be small.

We can see that, without capacity limit, if a jump in display rating comes with the increase in average rating, the resulting increase in s (Searchers) increases Firm A's total profits unambiguously, but in most cases it has no effect on Firm A's benefit of advertising ($\mathcal{R}_A$). In the case that $\triangle s$ does change $\mathcal{R}_A$, it reduces the margin increase ϵ that comes with the higher average rating $\tilde{\theta}^A$. But the reduction force from the jump in display rating is only of a small magnitude.

With Capacity Limit

Now I consider the case that only Firm A's capacity limit becomes binding, i.e., $L^A + n + s = \bar{n}^A$, when Firm A advertises and wins the Searchers.

Firm A's benefit of advertising when capacity limit is binding is $\mathcal{R}_A^{CL}(M_B = 1; \bar{n}^A) = (L^A+n+s)(\gamma_{11}^B + d) - (L^A+s)(\gamma_{01}^B + d) = \bar{n}^A(\gamma_{11}^B + d) - (L^A+s)(\gamma_{01}^B + d)$ if Firm B advertises, and is $\mathcal{R}_A^{CL}(M_B = 0; \bar{n}^A) = (L^A+n+s)(\gamma_{10}^B + d) - (L^A+s)(\gamma_{00}^B + d) = \bar{n}^A(\gamma_{10}^B + d) - (L^A+s)(\gamma_{00}^B + d)$ if Firm B does not advertise.

If Firm A's average rating $\tilde{\theta}^A$ increases by ϵ between two jump thresholds of display rating, i.e., the display rating remains constant, then only $d = \tilde{\theta}^A - \tilde{\theta}^B$ increases (by ϵ) in $\mathcal{R}_A^{CL}$. In particular, the changes in Firm A's advertising benefit when its average rating increases under a binding capacity limit are

$$\triangle\mathcal{R}_A^{CL}(M_B = 1; d + \epsilon, \bar{n}^A) = \triangle\mathcal{R}_A^{CL}(M_B = 0; d + \epsilon, \bar{n}^A) = n\epsilon \tag{23}$$

Therefore with capacity limit the change in advertising benefit is still a margin increase (ϵ) that raises Firm A's profit margin from each traditional new consumer (n) attracted by Firm A's advertisements. A higher average rating always increases a firm's benefit and also its willingness to advertising.

If Firm A's average rating $\tilde{\theta}^A$ increases by $\epsilon > 0$ and also crosses a jump threshold of display rating, i.e., its display rating jumps by 0.5 star, again both $d = \tilde{\theta}^A - \tilde{\theta}^B$ and s will be increasing. But now with capacity limit, if s increases by $\triangle s > 0$, the number of traditional new consumers that are attracted by advertisements and can be accommodated will change as well, and $\triangle n = -\triangle s$ to make $L^A + n' + s' = \bar{n}^A$ still hold. So if Firm A's display rating changes with its average rating, the changes in Firm A's benefit of advertising when capacity limit is binding are

$$\begin{aligned}\triangle\mathcal{R}_A^{CL}(M_B = 1; d + \epsilon, s' = s + \triangle s, \bar{n}^A) &= \mathcal{R}_A^{CL}(M_B = 1; d + \epsilon, s', \bar{n}^A) - \mathcal{R}_A^{CL}(M_B = 1; d, s, \bar{n}^A) \\ &= n\epsilon - \triangle s(\gamma_{01}^B + d + \epsilon) \end{aligned} \tag{24}$$

and

$$\begin{aligned}\triangle\mathcal{R}_A^{CL}(M_B = 0; d + \epsilon, s' = s + \triangle s, \bar{n}^A) &= \mathcal{R}_A^{CL}(M_B = 0; d + \epsilon, s', \bar{n}^A) - \mathcal{R}_A^{CL}(M_B = 0; d, s, \bar{n}^A) \\ &= n\epsilon - \triangle s(\gamma_{00}^B + d + \epsilon) \end{aligned} \tag{25}$$

We can see that, when capacity limit becomes binding, the jump in display rating (that leads to $\triangle s > 0$) that comes with the increase in average rating ($\triangle\tilde{\theta}^A = \epsilon$) will always reduce the advertising benefit. In particular, if $\triangle s$ is big enough, the jump in display rating will overturn the margin increase caused by the higher average rating, and results in a margin decrease in advertising benefit after the average rating has increased. For example, if $\triangle s$ is close to the size of n, then $\triangle\mathcal{R}_A(M_B = 1) = -\triangle s(\gamma_{01}^B + d)$ and $\triangle\mathcal{R}_A(M_B = 0) = -\triangle s(\gamma_{00}^B + d)$. If $\triangle s$ is larger than n, the margin decrease in advertising benefit will be larger than $\frac{\triangle s}{n}(\gamma^B + d)$.[21]

[21] Take (25) for example. Rewrite it as $n[\epsilon - \frac{\triangle s}{n}\epsilon - \frac{\triangle s}{n}(\gamma_{00}^B + d)]$. If $\triangle s > n$, then $n[\epsilon - \frac{\triangle s}{n}\epsilon - \frac{\triangle s}{n}(\gamma_{00}^B + d)] < 0$ and the marginal benefit of advertising decreases by $|\epsilon - \frac{\triangle s}{n}\epsilon - \frac{\triangle s}{n}(\gamma_{00}^B + d)| = \frac{\triangle s}{n}\epsilon - \epsilon + \frac{\triangle s}{n}(\gamma_{00}^B + d) > \frac{\triangle s}{n}(\gamma_{00}^B + d)$.

8 Conclusion

In this paper, I use game theory and Bayesian learning to study how competing firms with different customer reviews choose their advertising strategies, with and without Searchers. As an extension, I also study how the availability of customer reviews changes the advertising strategies of the incumbent and entrant firms in an entry game.

The key question that my paper answers is "Do online customer reviews complement or substitute firms' advertising?" In the literature of online customer reviews, good reviews and advertising are often thought to be substitutes, since a high rating can improve the effectiveness of advertising and can even directly substitute advertising when people can search for ratings. However, findings from the RDD analysis (see Figure 11) show that local restaurants' advertising spending goes up with their average rating on Yelp, but drops with display ratings above 3. This opposite pattern in advertising implies that online reviews are in fact complements to advertising, and display ratings above 3 work as substitutes for advertising.

The RDD analysis enables us to distinguish between the effect of average rating and the effect of display rating on advertising spending. It also provides an explanation of why we have been seeing a negative correlation between rating and advertising spending in regressions all the time: the display rating (above 3) has a strong negative effect and the downward trend comes entirely from this negative effect of display rating. By controlling for the effect of display rating using RDD, I find a significantly positive relationship between advertising and average rating, i.e., if consumers consider only the average rating in their purchase decision, a higher rated firm will be advertising more.

The reason for display rating (above 3) to have a negative effect on advertising is the capacity limits of local businesses. A higher display rating increases the click rates and, therefore, increases the number of Searchers visiting the restaurant. However, when the display rating is high enough, the capacity limit becomes binding and the increased Searchers will crowd out the benefit from those new consumers that are attracted by advertising, thus the benefit of advertising starts to decrease. Comparative statics of the theory model, presented in Section 7, show that the jump in display rating cannot cause a significant change in advertising benefit if there is no capacity limit.

Applying the findings of this research to other industries with online reviews and capacity limits, if the rounding algorithm is less discrete (for example, Expedia and Priceline use a rounding algorithm to the nearest tenth), we would expect to see smaller drops and more increasing trend as the average rating increases. The effect of jumps in display rating should be smaller if the rounding algorithm rounds to the nearest tenth.

Another interesting prediction comes from the extension to an entry game. An incumbent firm with a long history and a big loyal customer base is not necessarily intimidating. If the incumbent does not have a high enough ratio of good reviews, i.e., the ratio of its loyal customers to all of its previous buyers is not high enough, this big incumbent is weak in the competition with the entrant, and therefore entry is profitable and cannot be deterred. Such incumbent is a "fat cat" as in Fudenberg and Tirole (1984). But the difference caused by the availability of customer reviews is that a big incumbent with a high ratio of good reviews is very strong and is able to deter entry. Such application in entry deterrence problem might be tested in future research.

In summary, the existence of customer reviews provides an effective information channel for consumers to learn about firms' qualities, so firms should evaluate the effect of their marketing strategies in this new environment, and adjust their marketing strategies accordingly. We have seen that the availability of customer reviews brings big changes to the traditional predictions on firms' advertising strategies. A lot of other traditional topics about firms can be revised in the "Age of the Internet", and we might get many interesting new results.

References

Bagwell, K. (2007). *The Economic Analysis of Advertising*, Volume 3 of *Handbook of Industrial Organization*, Chapter 28, pp. 1701–1844. Elsevier.

Chen, Y. and J. Xie (2005). Third-party product review and firm marketing strategy. *Marketing Science 24*(2), pp. 218–240.

Chen, Y. and J. Xie (2008). Online consumer review: Word-of-mouth as a new element of marketing communication mix. *Management Science 54*(3), 477–491.

Chevalier, J. A. and D. Mayzlin (2006). The effect of word of mouth on sales: Online book reviews. *Jornal of Marketing Research 43*(3), 345–354.

Chioveanu, I. (2008). Advertising, brand loyalty and pricing. *Games and Economic Behavior 64*(1), 68–80.

DeGroot, M. and M. Schervish (2011). *Probability and Statistics* (4th ed.). Pearson.

Fudenberg, D. and J. Tirole (1984). The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *American Economic Review 74*(2), 361–66.

Hertzendorf, M. N. (1993). I'm not a high-quality firm – but i play one on tv. *RAND Journal of Economics 24*(2), 236–247.

Kihlstrom, R. E. and M. H. Riordan (1984). Advertising as a signal. *Journal of Political Economy 92*(3), 427–50.

Luca, M. (2011, September). Reviews, reputation, and revenue: The case of yelp.com. Harvard Business School Working Papers 12-016, Harvard Business School.

Mayzlin, D. (2006). Promotional chat on the internet. *Marketing Science 25*(2), 155–163.

McGahan, A. M. and P. Ghemawat (1994). Competition to retain customers. *Marketing Science 13*(2), pp. 165–176.

Milgrom, P. and J. Roberts (1986). Price and advertising signals of product quality. *Journal of Political Economy 94*(4), 796–821.

Narasimhan, C. (1988). Competitive promotional strategies. *The Journal of Business 61*(4), pp. 427–449.

Nelson, P. (1970). Information and consumer behavior. *Journal of Political Economy 78*(2), pp. 311–329.

Nelson, P. (1974). Advertising as information. *Journal of Political Economy 82*(4), pp. 729–754.

Schmalensee, R. (1978). A model of advertising and product quality. *Journal of Political Economy 86*(3), 485–503.

Sun, M. (2012). How does the variance of product ratings matter? *Management Science 58*(4), 696–707.

Varian, H. R. (1980). A model of sales. *American Economic Review 70*(4), 651–59.

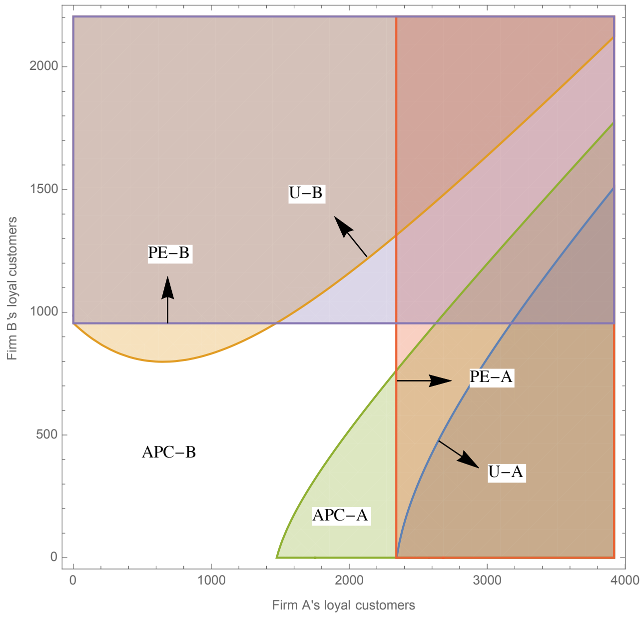


Figure 13: Three Conditions for Case $T^A > T^B > 0$

Appendix

Appendix 1. Equilibrium Analysis for The Asymmetric Previous Buyers Cases

Since Firm A and Firm B are symmetric, here I only study the case $T^A > T^B > 0$. The three conditions characterizing the equilibrium for this case are shown in Figure 13. We can see they are essentially the same as the symmetric case ($T^A = T^B = T$) in the main model, a firm needs to have a high enough ratio of good reviews to be the dominant firm in advertising. What is different here is that, when Firm A has more previous buyers than Firm B, it is harder for Firm A to reach the "high enough ratio" of good reviews.

Appendix 2. The Yelp Web Page of A Restaurant: Average Rating and Display Rating

See Figure 14. When consumers open the web page of a restaurant on Yelp.com, they directly see the display rating: the colored stars (4 stars here) displayed right below the restaurant name. If they click on the button "Details" beside the display star rating, they can see the full distribution of reviews. The display rating rounds the average rating calculated from the distribution to the nearest half star. The distribution of reviews allow consumers to visually approximate and compare between restaurants with the same display star rating.

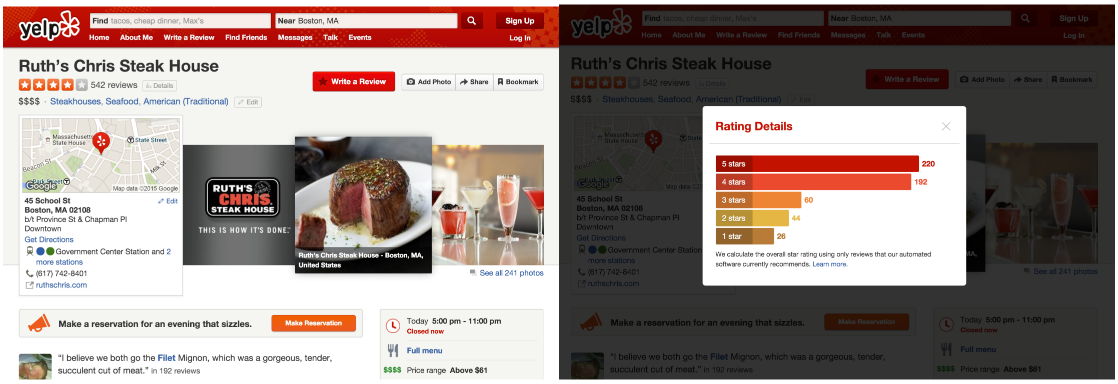


Figure 14: The Display Rating and Distribution of Reviews of a Restaurant

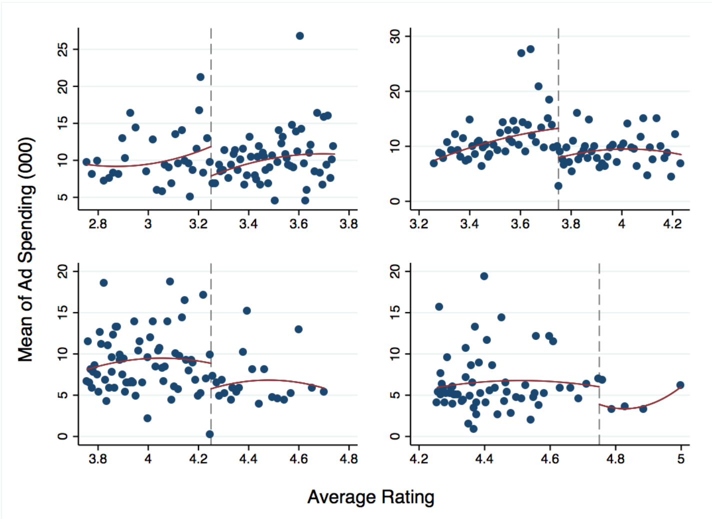


Figure 15: Advertising Spending by Local Restaurants' Average Rating around Threshold 3.75

Appendix 3. RD Graphs Showing The Effects of Yelp Display Rating on The Advertising Spending

Figure 15 shows the drops at the thresholds above 3, including thresholds 3.25, 3.75, 4.25 and 4.75. Figure 16 shows the insignificant drops at the thresholds below 3, including thresholds 1.25, 1.75, 2.25 and 2.75. Empirical estimates show that there are insignificant drops at the thresholds below 3 with precise estimates.

Appendix 4. Proofs

Proof of Lemma 1. (No pure strategy equilibrium)

Proof. The best responses of two firms are shown in Figure 17. There are no intersection of two firms' best response functions, hence no pure strategy equilibrium in the Bertrand

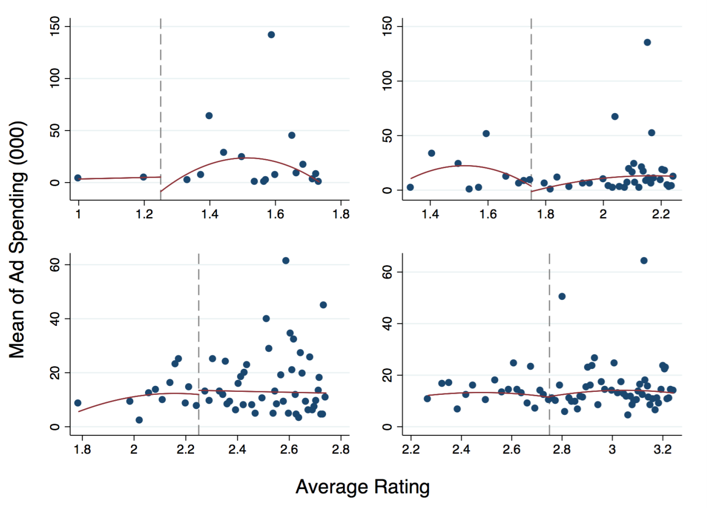


Figure 16: Advertising Spending by Local Restaurants' Average Rating Around Threshold 4.25

game.

□

Proof of Proposition 1. (The mixed strategy equilibrium of the pricing subgame)

Proof. It has been shown that the pricing subgame has no pure strategy equilibrium. Here I show how to construct the mixed strategy equilibrium for the case where firm A has advantage, and the analysis when firm B has advantage is the same.

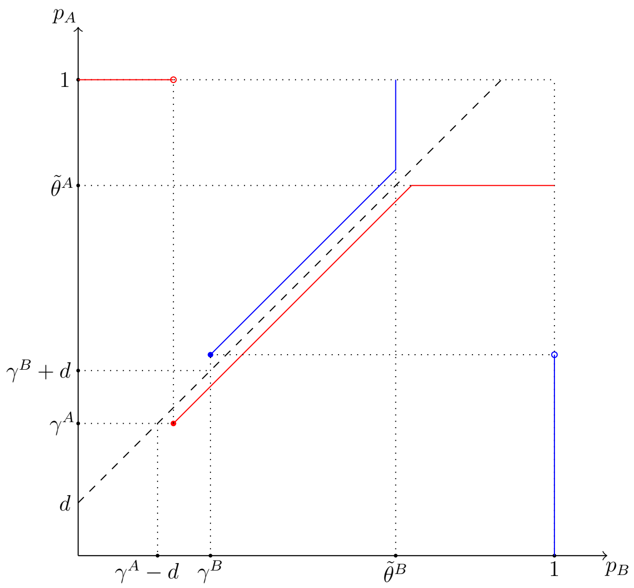
In the mixed strategy equilibrium, both firms mix over a range of prices. We know that a firm would never charge a price below its reservation price γ^k, so the prices that firm k mixes must be above γ^k. Also, it is not optimal for a firm to charge a price higher than 1, since no consumer would purchase at that price.

Here I give the proof only for the case where firm A has advantage, i.e. $\gamma^A < \gamma^B + d$. The proof for the case of firm B having advantage is the same.

If firm A has advantage in price competition, by charging a price of $\gamma^B + d$, firm A can force firm B out of the competition. Therefore, the highest profit that firm A could secure is $(L^A + n)(\gamma^B + d)$, which is higher than L^A since $\gamma^A < \gamma^B + d$; the highest profit that firm B could secure is L^B. In the mixed strategy equilibrium, firms get an expected profit equal to their highest secured profit. Let $F_A(p)$ and $F_B(p)$ denote two firms' equilibrium mixed strategies, then we have

$$L^A p^A + [1 - F_B(p^A - d)]np^A = (L^A + n)(\gamma^B + d) \qquad \gamma^B + d \leq p^A \leq 1 \tag{26}$$

$$L^B p^B + [1 - F_A(p^B + d)]np^B = L^B \qquad \gamma^B \leq p^B \leq 1 \tag{27}$$



Red: Firm A's best response function; Blue: Firm B's best response function.
This case shown in the graph is when Firm A has more loyal customers: $L^A > L^B$, i.e. $d = \tilde{\theta}^A - \tilde{\theta}^B > 0$.

Figure 17: Best Responses in the Pricing Subgame

Note that firm A does not mix below $\gamma^B + d$ $(> \gamma^A)$ as firm B will never choose prices below γ^B, and $p^A = \gamma^B + d$ is already enough to beat the lowest price of firm B.

We can therefore calculate each firm's mixed strategy distribution function from (26) and (27):

Before calculate for the distributions of all mixed prices, let us look at the special interval $(\tilde{\theta}^k, 1]$. For $p^A > \tilde{\theta}^A$, no new consumers purchase product A (even if $p^A < p^B + d$), so we have $1 - F_B(p^A - d) = 0$ for $p^A > \tilde{\theta}^A$ (or $p^A - d > \tilde{\theta}^B$). Therefore, $F_B(p) = 1$ for $p > \tilde{\theta}^B$, which means F_B has no mass point at 1, nor at any price between $\tilde{\theta}^B$ and 1. Similarly, $1 - F_A(p^B + d) = 0$ for $p^B > \tilde{\theta}^B$ (or $p^B + d > \tilde{\theta}^B + d = \tilde{\theta}^A$). So $F_A(p) = 1$ for $p > \tilde{\theta}^A$, F_A has no mass point at 1 (or any other price above $\tilde{\theta}^A$) either.

Then we get the distribution functions (F_A and F_B) in the mixed strategy equilibrium as follows.

$$F_A(p) = \begin{cases} 0 & p \leq \gamma^B + d \\ 1 - \frac{L^B}{n(p-d)} + \frac{L^B}{n} & \gamma^B + d \leq p \leq \tilde{\theta}^A \\ 1 & p \geq \tilde{\theta}^A \end{cases}$$

and

$$F_B(p) = \begin{cases} 0 & p \leq \gamma^B \\ 1 - \frac{(L^A+n)(\gamma^B+d)}{n(p+d)} + \frac{L^A}{n} & \gamma^B \leq p \leq \tilde{\theta}^B \\ 1 & p \geq \tilde{\theta}^B \end{cases}$$

F_A first order stochastically dominates F_B. See Figure 18.

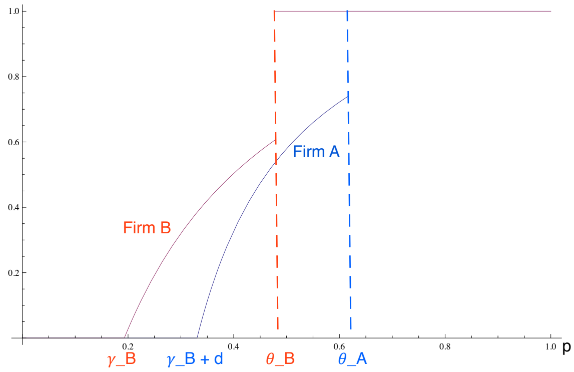


Figure 18: The distributions of firms' pricing strategies in the pricing subgame

Each firm has a mass point. Denote the probability at mass points by $m(A)$ and $m(B)$. For firm A, there is a mass point at $\tilde{\theta}^A$, and $m(A) = 1 - F_A(\tilde{\theta}^A) = \frac{L^B}{n}(\frac{1}{\tilde{\theta}^B} - 1) > 0$.[22] For firm B, the mass point is $\tilde{\theta}^B$, and $m(B) = 1 - F_B(\tilde{\theta}^B) = \frac{(L^A+n)(\gamma^B+d)}{n\tilde{\theta}^A} - \frac{L^A}{n}$. Recall that this equilibrium is under the case where $\gamma^B + d > \gamma^A$, so we have $m(B) > \frac{(L^A+n)\gamma^A}{n\tilde{\theta}^A} - \frac{L^A}{n} = \frac{L^A}{n\tilde{\theta}^A} - \frac{L^A}{n} > 0$.

Next I give an intuitive proof of the uniqueness of this mixed strategy equilibrium (F_A, F_B).

Suppose there exists another mixed strategy equilibrium (F'_A, F'_B), Firm B must get a higher expected profit than in the above equilibrium. This is because that Firm B does not accept any expected profit lower than L_2^B, which equals its secured profit. If Firm B's expected profit remains the same, the equilibrium (F'_A, F'_B) will be the same as the above one.

If Firm B gets a higher expected profit than L^B, the distribution F'_B must shift probabilities to higher prices than F_B. Then Firm A can undercut by shifting probabilities to prices just below Firm B's, and extract all the increased profits of Firm B. Therefore, Firm B cannot get any higher expected profit than L^B in the mixed strategy equilibrium, and (F_A, F_B) is the unique mixed strategy equilibrium. □

Proof of Proposition 2. (Condition of Uniqueness)

Proof. When both Firm A and Firm B satisfy PE, and Firm A has advantage in price competition against Firm B, the stage game is given in Table 2.

When $(L^A + n)(\gamma^B + d) - c \geq L^A$ is satisfied, advertising is a dominant strategy for Firm A. Firm B chooses not to advertise whenever Firm A advertises. Therefore, the unique equilibrium is that only Firm A advertises.

[22] According to the assumption of Beta-distributed beliefs, the mean $\tilde{\theta}^k = \frac{1+L^k}{2+T^k}$ is always strictly less than 1.

If $(L^A + n)(\gamma^B + d) - c < L^A$, both firms only advertise when the rival does not. Therefore, there are three equilibria, which are, only Firm A advertises, only Firm B advertises, and each firm randomizes advertising with a probability (λ^A, λ^B).

λ^A and λ^B are such that $[(L^A + n)(\gamma^B + d) - c]\lambda^B + [(L^A + n)\tilde{\theta}^A - c](1 - \lambda^B) = L^A$, and $(L^B - c)\lambda^A + [(L^B + n)\tilde{\theta}^B - c](1 - \lambda^A) = L^B$. □

Proof of Proposition 3: (Non-empty set of multiple equilibria)

Proof. Here I only prove $U - A \Rightarrow APC - A$, and it is similar for $U - B \Rightarrow APC - B$.

The condition $U - A$ is $(L^A + n)(\gamma^B + d) - c > L^A$.

$\Rightarrow (L^A + n)(\frac{L^B}{L^B+n} + \tilde{\theta}^A - \tilde{\theta}^B) > L^A$

$\Rightarrow \gamma^A - \tilde{\theta}^A < \gamma^B - \tilde{\theta}^B$, which is the condition $APC - A$

As long as $c > 0$, there always exist L^A and L^B such that $(L^A + n)(\gamma^B + d) > L^A > (L^A + n)(\gamma^B + d) - c$, which means that Firm A satisfies $APC - A$, i.e. $\gamma^A < \gamma^B + d$, but Uniqueness condition does not hold.

Therefore, the set $\{U - A\}$ is a strict subset of $\{APC - A\}$. □

Proof of Lemma 2. (A new firm always has advantage in price competition against an established firm.)

Proof. WLOG, here I prove for the case that Firm A is the established firm, or the incumbent. Consider the pricing subgame after both firms advertise.

Firm B has advantage in price competition if and only if $\gamma^B + d < \gamma^A$.

We have $\gamma^B = 0, d = \frac{1+L^A}{2+T} - \frac{1}{2}$, and $\gamma^A = \frac{L^A}{L^A+n}$.

Compare $LHS = \frac{1+L^A}{2+T} - \frac{1}{2}$ and $RHS = \frac{L^A}{L^A+n}$. Note that $T \geq n$.

At end points, $L^A = 0$ and $L^A = T$, we have $LHS < RHS$.

Both LHS and RHS are monotonically increasing: $\frac{\partial LHS}{\partial L^A} > 0, \frac{\partial RHS}{\partial L^A} > 0$.

Therefore, for all values of L^A, we have $LHS < RHS$. □

Table 7: Summary of Notations

Notation	Definition
θ^k	The product quality of firm k, defined as the *ex ante* probability that a random consumer would be satisfied with product k; $k = A$ or B
$\tilde{\theta}^k$	The belief of firm k's product quality
M^k	The advertising decision of firm k, a binary choice, takes value 1 or 0; $k = A$ or B
p^k	The price of firm k, $p^k \in \mathbb{R}_+$
n	The number of new consumers that can be informed by advertisement(s), $n \in \mathbb{N}$
c	The (fixed) cost of advertising, $c \in \mathbb{R}_+$
T^k	Total number of previous buyers of firm k's product
L^k	The loyal consumer base of firm k, made up by previous satisfied customers.
d	The difference between two firms' market beliefs: $d = \tilde{\theta}^A - \tilde{\theta}^B$
γ^k	Firm k's reservation price in competition, Firm k will not charge a price lower than this level in a price competition.
$\mathcal{R}_k$	Firm k's benefit of advertising
$\mathcal{R}_k^{CL}$	Firm k's benefit of advertising when its capacity limit is binding
$\bar{n}^k$	Firm k's capacity limit: $L^k + n^k + s^k = \bar{n}^k$